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THE TOTAL SOLUTE-WATER CORRELATION FUNCTION FOR LENNARD–JONES PARTICLES

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Dedicated to the memory of Professor Vladimir Šips

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A series of molecular dynamic simulations have been performed in order to evaluate the Lennard–Jones particle-water correlation function for solutes ranging in size from 0.3 to 1.2 nm. The results of these simulations have provided evidence for distinct orientational ordering of water molecules with respect to the solute. Furthermore, this orientational ordering is determined to be longer in range than the well-known radial ordering. Although the distance from the solute at which the correlation with water can be detected is independent of the size of the solute, the amplitude of the correlation function is sensitive to the size in the first hydration shell.

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1. Introduction

In the pioneering work in 1977, Pratt and Chandler [1] successfully evaluated several aspects of the hydrophobic hydration of small solutes in water. The only input specific to aqueous structure was the experimental angle-averaged water-water correlation function as measured by Narten and Levy [2].

More recently, Hummer and coworkers [3] extended and simplified the theory of hydrophobic hydration, based on angle-averaged correlation functions, in their "information theory" model for small solutes. Subsequently, Lum, Chandler and Weeks [4] proposed a unified theory of hydrophobicity for a whole range of solute sizes, using density fields with no explicit angular dependence.

It may appear surprising that these models can enjoy such a degree of success given their neglect of the specific tetrahedral coordination that is responsible for so many properties of fluid water. For example, in the studies of pure water, Errington and Debenedetti [5] demonstrated the coupling of the translational order parameter (the average distribution of the distance between neighbors) with the orientational order parameter (the extent to which neighboring molecules are at specific orientation with respect to each other), as function of density. In addition, neutron and X-ray diffraction experiments [6] on pure water indicate that orientational correlation decays more rapidly than positional correlation. Furthermore, the conventional wisdom regarding the hydrophobic effect is that hydration of small hydrophobic solutes depends on the formation of transient clathrate cages. More generally, it is believed that hydrophobic effect is governed by the response of the hydrogen bond network [7] to the presence of an inaccessible volume. The way in which the network will adjust to preserve the maximum number of bonds, depends greatly on the geometrical proprieties of the solute. Such a response requires pairs of water molecules not only to be optimally separated, but also to have a specific relative orientation.

The main aim of this study is to learn more about the properties of the angular response of aqueous solvent to the presence of a hydrophobic solute. This is possible through the analysis of total solute-solvent correlation functions, calculated here for the first time. The techniques employed allow the determination of main characteristics of the orientational correlation of water molecules around a hydrophobic particle as the size of the solute grows from microscopic toward mesoscopic scales.

2. Simulations

Using the GROMACS software package [8], molecular dynamics (MD) simulations were performed with a single Lennard–Jones (LJ) particle solvated in SPC/E [9] water using the SHAKE algorithm to constrain bond lengths and bond angles. A set of LJ particles was introduced into the force field with the parameter ϵ equal to 1 kJ/mol, and with σ in the range from 0.36 nm to 1.2 nm (σ = 0.36, 0.46, 0.57, 0.7, 0.8, 0.9, 1.0, 1.1 and 1.2 nm). The reaction field has been chosen for the treatment of long-range electrostatic effects. It has been shown previously [11] that the combination of SPC/E model with the reaction field (with a dielectric constant $\epsilon_{\mathrm{rf}} = 78.5$, long cutoffs and large boxes) reproduces the static properties of water most accurately for nonpolarizible systems. Specifically, this model offers the best reproduction of experimental data for the reorientational correlation time.

When the reaction field is used, the cutoff of the Coulumb potential is required to be longer than, or equal to the cutoff of the van der Waals potential. Since the interaction of the LJ particle with water is that of most interest, the cutoff of the van der Waals (r_{vdW}) and the Coulumb (r_{Coul}) potentials are defined in the following way

$$r_{\mathrm{cut}} = r_{\mathrm{vdW}} = r_{\mathrm{Coul}} = \frac{\sigma_{\mathrm{LJ}}}{2} + 3 \cdot \sigma_{\mathrm{H_2O}}. \tag{1}$$

Here $\sigma_{\mathrm{H_2O}} = 3.1$ nm. This implies that interaction of the LJ particle with its first three hydration shells is calculated directly.

All simulations were performed in the same manner: After the definition of cutoffs, the LJ molecule was centered in a cubic box of edge $d = 2.5 \cdot r_{\rm cut}$, assuring the presence of an adequate volume of bulk water. The number of water particles in the box was then determined, starting coordinates assigned randomly and periodic boundary conditions imposed. The number of water particles in the system was found to vary from 1309 (σ = 0.36 nm) to 2574 (σ = 1.2 nm). Subsequently, energy minimization (EM) was performed with the steepest descent algorithm with a tolerance of 0.1 kJ/mol. The resulting configuration was assigned initial velocities so as to reproduce the Maxwell distribution at 300 K. Standard conditions were maintained by the weak-coupling algorithms of Berendsen [10]. The temperature was scaled with a time constant of 100 fs. The pressure scaling was applied with a reference isotropic pressure of 1 bar, and a time constant of 1 ps. Neighbor lists were used, with searching inside $r_{\rm cut}$, and were updated every 10 steps. Coordinates and energies were saved every 50 steps. Each system was simulated for 1 ns in 200 000 steps, implying an integration step of 5 fs. To allow the full equilibration of the system, only the last 0.9 ns were used for the analysis.

3. Coordinate system

To uniquely describe the distance and the orientation of the water molecules with respect to the LJ particle, three coordinates are required (see Fig. 1). The first coordinate was chosen to be the radius vector connecting the center of the LJ particle and the center of the oxygen atom (r). An atractive manner for the description of the remaining two coordinates is to employ angles to specify the relative orientation of the LJ particle and a given water molecule. In the current work, the first angle (θ_1) is that defined by the center of the LJ particle, the center of the oxygen of the water molecule, and the mid-point of the line connecting the two hydrogen atoms (of the same water molecule). This coordinate will be referred to as the dipole angle. The second angle (θ_2) is chosen to be the angle that a normal to the plane of the water molecule is closing with the radius vector $\boldsymbol{r}$, in the center of the oxygen atom. This angle will be referred to as the angle of the normal.

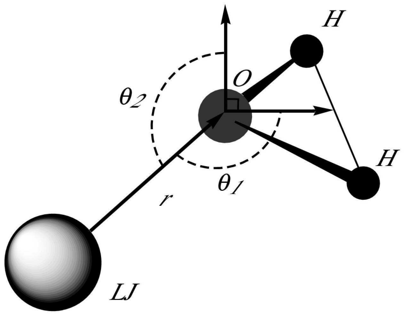

Fig. 1. The internal coordinate system of the LJ particle (shaded) and a water molecule (consisting of an oxygen (gray) and two hydrogens (black)). The distance between the LJ particle and the oxygen atom (denoted with O*) is presented with* r*. The dipole angle* θ_1 *and the angle of the normal* θ_2 *are also shown.*

The advantage of the above definition is that it respects the internal symmetry of the water molecule, as well as the symmetry of the system as a whole. Therefore, the dipole angle ranges from 0° to 180°, while the angle of the normal is in range from 0° to 90° which is sufficient to describe all possible orientations. It is useful to notice that $\cos\theta_1$ and $\cos^2\theta_2$ have the same periodicity as their respective defining angles. This property allows the spherical function of the first order for the dipole angle ($P_1(\theta_1) = \cos\theta_1$) and of the second order ($P_2(\theta_2) = \cos^2\theta_2 - 1$) for the angle of the normal, to be used as a basis for the total correlation function.

4. *Correlation functions*

Using the coordinates defined in the previous section, the total LJ particle-water correlation function ($g_{\text{LJ-w}}$) can be written as

$$\begin{aligned} g_{\text{LJ-w}}(r,\theta_1,\theta_2) &= g_{\text{LJ-w}}(r,\langle\cos\theta_1\rangle,\langle\cos^2\theta_2\rangle) \\ &= g_{\text{LJ-w}}(r,\langle P_1(\theta_1)\rangle,\langle P_2(\theta_2)\rangle). \end{aligned} \tag{2}$$

The $g_{\text{LJ-w}}$ is finally a function of a distance from the LJ particle and the angular parts equal the mean values of the belonging spherical function at a particular distance.

The radial part (Fig. 2), usually called the radial distribution function (rdf), is the probability of finding a molecule of water at a distance r from the center of the LJ particle. The present results are in excellent agreement with previous work [12] for the range of sizes in which they overlap (up to $\sigma = 0.75$ nm). In addition, the information shown in Fig. 2 extends the previously characterized rdfs to larger sizes of LJ particle. After the second shallow minimum all rdfs approach their asymptotic value very rapidly, reflecting the insensitivity of water, in this limit, to the presence of LJ particle. The height of the first maximum is found to scale with the volume of the LJ particle. This scaling can be seen from the fit (dotted line) shown in Fig. 2.

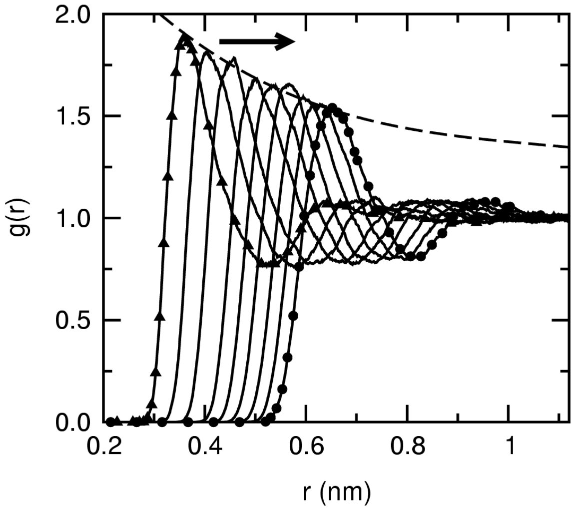

Fig. 2. The radial distribution function for all simulated systems. The symbols represents the result of simulations and the line is averaging over successive sets of three data points. The triangles and the circles indicate results of the simulation for the LJ particle with $\sigma = 0.36$ nm *and* $\sigma = 1.2$ nm, *respectively. This presentation will be maintained throughout the manuscript.*

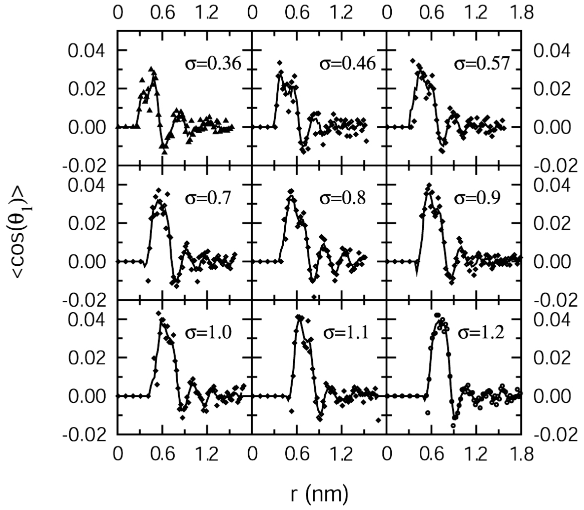

Fig. 3. The correlation functions for the dipole angle coordinate. The size of the LJ particle is indicated in the upper right corner of every graph. The symbols represents the result of simulations and the line is averaging over successive sets of three data points.

Figure 3 shows the results for the correlation of the dipole angle (with symbols), separately for each size of LJ particle. The data presented are clearly associated with a relatively large amount of noise. The noise in the first maximum is due to the low level of sampling in the vicinity of the LJ particle. As the number of molecules in the first hydration shell increases almost linearly with increasing size of the LJ particle (data not presented), the averages over time are worst at small distances and for small LJ particles. The noise appearing at large distances is only apparent for this coordinate as the absolute amplitude of the correlation of the dipole angle is itself small. However, even with the presence of strong noise, two maximuma and two minimuma can be clearly discerned for the dipole angle correlation function.

The correlation functions belonging to the angle of the normal are presented in Fig. 4. All curves exhibit a very fast growth before reaching a sharp first maxi-

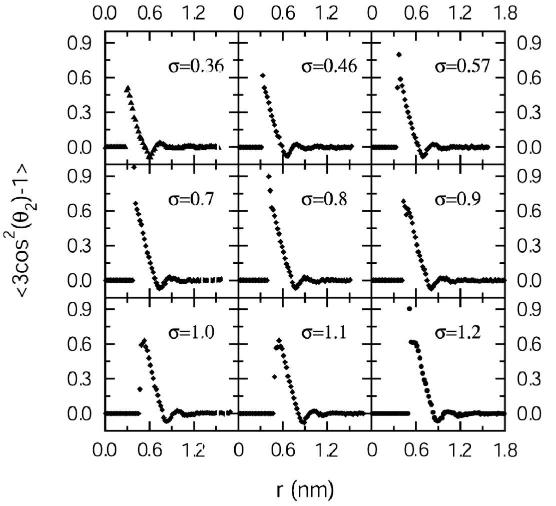

Fig. 4. Correlation functions of the angle of the normal coordinate for all simulated sizes. The size of the LJ particle increases from the graph in the upper left towards the bottom right corner, as indicated.

mum. Another feature common to all sizes of the LJ particle is the presence of a much weaker, but still well defined, second maximum. After a very shallow second minimum, all correlation vanishes. Once again, there is some noise in the first maximum, particularly in close proximity to the hard core of the LJ particle. On the other hand, as was the case for the rdfs, the noise at larger distances is negligible since the amplitude of structuring is at least an order of magnitude larger than for the dipole angle.

A very informative way to compare and contrast the correlation functions shown in Figs. 2−4 is to shift the origin to the surface of each LJ particle and overlay the various curves. Such an analysis is presented in Fig. 5. In this context, the surface (or the limit of the soft core) of each LJ particle is taken to be at the first maximum of the corresponding rdf.

This realignment of the functions clarifies many important features of the correlation functions. The first and most striking point is that the discernable structure for the angular correlation functions extends much deeper into bulk water than it does for the radial distribution functions. This result indicates a greater degree of ordering of water around a hydrophobic particle than has previously been thought.

In the standard definition, the first minimum of the rdf defines the limit of the first hydration shell. For all LJ particles investigated, this limit occurs at 0.15 nm from the surface of the particle. The correlation functions for both angular coordinates exhibit their first

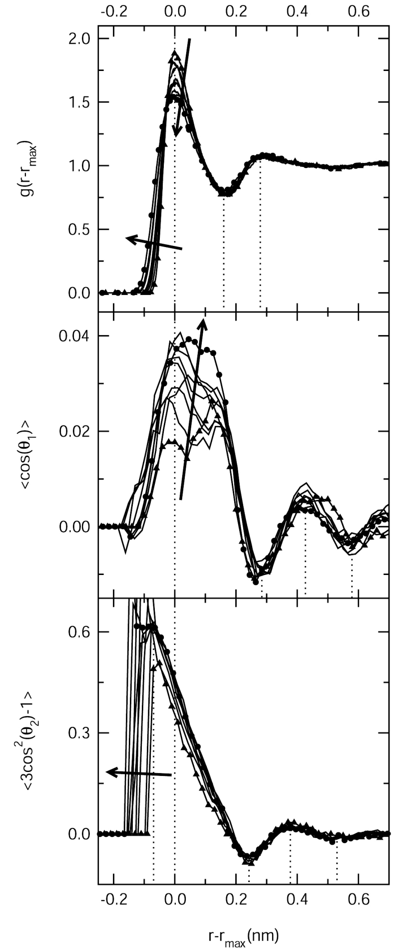

Fig. 5. The LJ particle-water correlation as a function of the distance from the surface of the particle for the radial (up), dipole angle (middle) and the normal angle (down) coordinate. The range and the overall structure is independent on the size of the LJ particle. The amplitude of the rdf decreases with increasing the size of LJ particle, whereas the dipole angle correlation function shows opposite behaviour.

minimum deeper into the bulk water than this value. For the dipole angle, this minimum is calculated to occur at 0.28 nm, while the corresponding value for the normal angle is 0.24 nm. Such distance is more in accordance with the position of the center of the second hydration shell as defined by rdfs. Interestingly, all extrema of the dipole angle correlation function are shifted toward the bulk by approximately 0.04 nm with respect to those associated with the normal angle.

Surprisingly, the correlation functions for all three coordinates exhibit a behaviour that is qualitatively independent of the size of the LJ particle. This effect is well known for the rdfs, but it is interesting that it persists for the angular coordinates of the total correlation function. If all three coordinates are taken together, provided one takes the surface as the origin, it seems that the range of the potential of solute-solvent interaction is virtually independent of the size of the LJ particle. Some quantitative size dependencies do, however, arise and warrant some brief discussion.

As mentioned previously, the height of the first maximum in the rdfs decreases with increasing particle size. This is a reflection of the known weakening of the solute-solvent interaction and results in a reduced density of particles in the first hydration shell as the hydrophobic particle becomes larger. At the same time, as the volume of the LJ particle grows, it becomes more accessible to water molecules. This is reflected in the more prominent tail that penetrates further into the soft core of the growing particle.

Although somewhat obscured by noise, the height of the first maximum of the dipole angle correlation function shows a distinct enhancement as the LJ particle becomes larger, indicating an interplay between the translational degrees of freedom and the dipole angle degree of freedom. In more densely packed shells, a typical water molecule has, on the average, more neighbors with which to form hydrogen bonds. This implies a more extensive orientational space within which a water molecule can find convenient orientations. As the number of neighboring water molecules becomes smaller, the corresponding orientational space is reduced and, in order to preserve the network, the water molecules correlate more strongly with respect to their orientation.

The part of the correlation function associated with the angle of the normal appears to be mostly independent of the size of the LJ particle. This indicates that the presence of a LJ particle immediately restricts the orientation of water molecules with respect to the normal coordinate. Furthermore, these results suggest that this degree of freedom can be decoupled from the remaining two. The resultant correlation function is therefore is a product of two parts,

$$g_{\text{LJ-w}}(r,\theta_1,\theta_2) = g(r,\theta_1)g(\theta_2). \tag{3}$$

5. *Concluding remarks*

The calculation of the total solute-solvent correlation function allows a quantitative understanding of the solvent orientational degrees of freedom. This is particularly true in the sense that the calculated correlations can be associated with an angle-dependent effective potential between the solute and the solvent. A natural extension of the calculations presented herein would be the elucidation of the influence of the angle-dependent correlation function on the free energy of hydration. It would also be of a great interest to analyze in details the hydrogen-bond network in the first hydration shell. Such an analysis would elucidate proprieties of the transient clathrate cage that is assumed to form around small hydrophobic molecules.

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Potpuna korelacijska fukcija za meudjelovanje vode s otopljenim nositeljem Lennard-Jones potencijala

Da bi se izvrijednila potpuna korelacijska funkcija izmeu vode i otopljene estice koja meudjeluje s okolinom preko Lennard Jonesovog potencijala (LJ estica), izveden je niz molekularno-dinamikih simulacija. Promjer LJ estica je izabran u rasponu od 0.3 do 1.2 nm. Rezultati simulacija pokazuju jasno orijentacijsko ureenje molekula vode u odnosu na LJ esticu. Nadalje, utvreno je da to orijentacijsko ureenje puno duljeg dosega od dosada dobro poznatog radijalnog ureenja. Naeno je da su udaljenosti izmeu LJ estice i molekula vode na kojoj postoje korelacije neovisne o veliini LJ estice. Amplituda korelacijskih funkcije je naprotiv osjetljiva na veliinu LJ estice, posebice u prvoj hodratacijskoj eliji.