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# STRUCTURE OF RADIAL NULL GEODESICS IN MONOPOLE-CHARGED VAIDYA SPACETIME

S. S. ZADE[a,1], K. D. PATIL[b] and P. N. MULKALWAR[c]

[a]*Department of Mathematics, J. B. College of Science, Wardha (M. S.), India*
*E-mail address: sanjivzade@yahoo.com*

[b]*Department of Mathematics, B. D. College of Engineering, Sewagram, Wardha (M. S.), India* *E-mail address: kishordpatil@yahoo.com*

[c]*Department of Computer Science, Amolakchand Mahavidyalaya, Yavatmal (M. S.), India*

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The monopole-charged Vaidya model containing a naked singularity is analyzed for outgoing radial null geodesics. Using the Clarke and Krolak criteria, the curvature strength of naked singularity is examined to show that this is a strong curvature singularity, providing a counter example to cosmic censorship hypothesis (CCH). The graphs of the apparent horizons for different values of parameters have been drawn. An interesting feature which emerges is that, the monopole component pushes the apparent horizon towards the radial axis and thereby increases the radius of the apparent horizon in Vaidya collapse.

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## 1. Introduction

Naked singularities are one of the fascinating objects in classical general relativity. It is believed that they are formed by the gravitational collapse of massive star or by density fluctuations in the very early universe. It has been shown in Ref. [1] that under physically reasonable conditions, the gravitational collapse of a massive star must form singularities. But the important question is whether these singularities can be observed? In this respect, in 1969, R. Penrose made a celebrated proposal known as cosmic censorship hypothesis (CCH) which states that

[1]Corresponding author

under physically reasonable initial data, the gravitational collapse of the spacetime cannot yield a naked singularity. That is, if singularity forms, it must be covered by an event horizon of the gravity [2]. Although there is a long history of research on CCH, we are far away from its proof. On the contrary many counter examples to this hypothesis have been found. Various models studied in this respect include radiation [3 – 7], dust [8 – 12], perfect fluid [13, 14], strange quark matter [15, 16] etc.

One of the most important examples having naked singularities is the Vaidya solution representing an imploding (exploding) null dust fluid with spherical symmetry [17]. Papapetrou [18] first showed that this solution admits naked singularities and thus provides one of the earlier counter examples to CCH. Since then, this solution is being used to discuss the gravitational collapse of the null dust [19 – 23].

A. Wang introduced a more general family of Vaidya spacetimes which covers monopole solution, de Sitter and anti-de Sitter solutions and charged-Vaidya solutions as special cases [24]. In Ref. [25], gravitational collapse of monopole-Vaidya and charged-Vaidya has been discussed independently and it has been shown that the central singularities arising in these spacetimes are naked and strong curvature type. Since the energy-momentum tensor is linear in terms of the mass functions, a linear superposition of particular solutions is also a solution of the Einstein field equations [24]. Hence it would be interesting to investigate the nature of singularities arising in such a composite solution. Hence in the present work, we shall investigate the possibility of cosmic censorship violation in monopole-charged Vaidya spacetime. We also discuss the apparent horizon formation in this generalized Vaidya spacetime.

The paper is organized as follows. In Sec. 2, we give a brief review of the generalized Vaidya spacetimes. The nature of the singularities arising in monopole-charged Vaidya spacetimes is discussed in Sec. 3. In Sec. 4, we discuss the apparent horizons formed in the generalized Vaidya spacetimes. The final Sec. 5 summerises the implications and conclusions.

## 2. *Generalized Vaidya spacetimes*

The metric of spherically-symmetric generalized Vaidya spacetime is given by [24]

$$ds^2 = -\left[1 - \frac{2m(v,r)}{r}\right] dv^2 + 2dvdr + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \tag{1}$$

where $v$ is the advanced Eddington time coordinate ; $r$ is the radial coordinate which decreases towards the future along a ray $v=$ constant, with $0 < r < \infty$; $m(v,r)$ is the mass function, and represents the gravitational mass inside the sphere of radius $r$. $d\theta^2 + \sin^2\theta \, d\phi^2$ is a metric on unit 2-sphere.

Non-vanishing components of Einstein tensor are

$$G_0^0 = G_1^1 = -\frac{2m'}{r^2}, \qquad G_0^1 = \frac{2\dot{m}}{r^2}, \qquad G_2^2 = G_3^3 = -\frac{m''}{r}, \tag{2}$$

where

$$\{x^\mu\} = \{v, r, \theta, \phi\}\,,\ (\mu = 0,\,1,\,2,\,3),$$

and

$$\dot{m}(v,r) = \frac{\partial m}{\partial v}, \quad m' = \frac{\partial m}{\partial r}.$$

Einstein field equations are

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor, $\kappa$ is the gravitational constant and $T_{\mu\nu}$ is the energy momentum tensor (EMT) given by [24, 26, 27]

$$T_{\mu\nu} = T_{\mu\nu}^{(n)} + T_{\mu\nu}^{(m)}, \tag{4}$$

where

$$T_{\mu\nu}^{(n)} = \sigma l_\mu l_\nu, \tag{5}$$

$$T_{\mu\nu}^{(m)} = (\rho + P)\,(l_\mu n_\nu + l_\nu n_\mu) + P g_{\mu\nu}. \tag{6}$$

The part of the EMT $T_{\mu\nu}^{(n)}$ can be considered as the component of the matter field that moves along the null hypersurface $v$=constant and corresponds to the EMT of Vaidya null fluid. $T_{\mu\nu}^{(m)}$ is the EMT for a perfect fluid. $\sigma$ is the energy density of the Vaidya null radiation, $\rho$ and $P$ are respectively the energy density and pressure of a perfect fluid. $l_\mu$ and $n_\mu$ are null vectors given by

$$l_\mu = \delta_\mu^0, \quad n_\mu = \frac{1}{2}\left[1 - \frac{2m(v,r)}{r}\right]\delta_\mu^0 - \delta_\mu^1, \quad l_\lambda l^\lambda = n_\lambda n^\lambda = 0, \quad l_\lambda n^\lambda = -1. \tag{7}$$

Combining Eqs. (2) – (7), we obtain

$$\sigma = \frac{2\dot{m}}{\kappa r^2}, \qquad \rho = \frac{2m'}{\kappa r^2}, \qquad P = -\frac{m''}{\kappa r}. \tag{8}$$

In particular, when $\sigma = P = 0$, the solution reduces to the pure Vaidya solution with $m = m(v)$. Therefore, for the general case, we consider the EMT of Eq. (4) as a generalization of Vaidya solution.

The energy conditions for such fluids are given by [1, 24, 27]:

(i) The weak and strong energy conditions:

$$\sigma > 0, \quad \rho \geq 0, \quad P \geq 0, \quad (\sigma \neq 0). \tag{9}$$

(ii) The dominant energy conditions:

$$\sigma > 0, \quad \rho \geq P \geq 0, \quad (\sigma \neq 0). \tag{10}$$

The nature of the singularity that may form in the gravitational collapse can be determined by examining the behavior of the radial null geodesics defined by $ds^2 = 0,\ d\theta = d\phi = 0$.

To investigate the nature of the singularity, we follow the method described in Ref. [6]. Roughly speaking, naked singularities are the singularities that may be seen by the physically allowed observer. The central shell-focusing singularity (i.e. the one occurring at $r = 0$) is naked if the radial null geodesic equation admits one or more positive real root [28]. Using the null condition $K^a K_a = 0$, we obtain the equation of radial null geodesic for the metric (1) as

$$\frac{dK^v}{dk} + \left(\frac{m}{r^2} - \frac{m'}{r}\right)(K^v)^2 = 0, \tag{11}$$

$$\frac{dK^r}{dk} + \left(\frac{\dot{m}}{r} - \frac{m'}{r} + \frac{m}{r^2} + \frac{2mm'}{r^2} - \frac{2m^2}{r^3}\right)(K^v)^2 + 2\left(\frac{m'}{r} - \frac{m}{r^2}\right)K^v K^r = 0. \tag{12}$$

We note that $K^v$ and $K^r$ are functions of $v$ and $r$. We define the function $R(v,r)$ by

$$K^v = \frac{dv}{dk} = \frac{R(v,r)}{r}. \tag{13}$$

Then from the null conditions we obtain

$$K^r = \frac{R}{2r}\left[1 - \frac{2m(v,r)}{r}\right], \tag{14}$$

where $R$ satisfies the differential equation

$$\frac{dR}{dk} - \frac{R^2}{2r^2}\left(1 - \frac{4m}{r} + 2m'\right) = 0. \tag{15}$$

It is quite difficult to find an analytic solution of the above geodesic equation. To simplify the task, we need to choose the mass function $m(v,r)$ such that the equation becomes homogeneous, and can be solved in terms of elementary functions [29].

## 3. *Nature of singularities in monopole-charged Vaidya spacetime*

The mass function for the monopole solution is given by [24]

$$m_0(v,r) = \frac{ar}{2}, \tag{16}$$

where $a$ is an arbitrary positive constant.

For this solution, the physical quantities in Eq. (8) become

$$\rho = \frac{a}{\kappa r^2}, \quad \sigma = P = 0. \tag{17}$$

Monopoles are formed due to gauge-symmetry breaking and have many properties of elementary particles. Most of their energy is concentrated in a small region near the core [30]. Monopole has a mass that grows linearly with the distance from its core. It has been shown in Ref. [30] that when gravity is taken into account, the mass of monopole has an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin. It has been shown in Ref. [31] that this small gravitational potential is actually repulsive. It is interesting to note that, though the static solutions of regular global monopoles are always repulsive, solutions with an event horizon exist [32, 33].

The mass function for the charged Vaidya solution is given by [24]

$$m_1(v,r) = f(v) - \frac{q^2(v)}{2r}, \tag{18}$$

where $f(v)$ is the Vaidya linear mass and $q^2(v)$ is the electric charge at the advanced time $v$.

For the above mass function, the quantities in Eq. (8) become

$$\sigma = \frac{2}{\kappa r^3}\left[r\dot{f}(v) - q(v)\dot{q}(v)\right], \qquad \rho = P = \frac{q^2(v)}{\kappa r^4}. \tag{19}$$

In this case, $T^{(m)}_{\mu\nu}$ corresponds to the EMT of the electromagnetic field, $F_{\mu\nu}$, given by

$$F_{\mu\nu} = \frac{q(v)}{r^2}\left(\delta^0_\mu \delta^1_\nu - \delta^1_\mu \delta^0_\nu\right). \tag{20}$$

To get the analytic solution, we choose

$$f(v) = \frac{\lambda v}{2} \quad \text{and} \quad q^2(v) = \mu v^2. \tag{21}$$

As the EMT is linear in terms of the mass functions, a linear superposition of particular solutions is also a solution of the Einstein field equations [24]. In particular, the combination

$$m(v,r) = \frac{1}{2}\left(ar + \lambda v - \frac{\mu v^2}{r}\right), \tag{22}$$

where $\lambda$ and $\mu$ are positive constants, would represent the monopole-charged Vaidya solution.

With this mass function, the spacetime (1) becomes

$$ds^2 = -\left(1 - a - \frac{\lambda v}{r} + \frac{\mu v^2}{r^2}\right) dv^2 + 2dvdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{23}$$

Substituting the mass function (22) into Eq. (8), we obtain

$$\sigma = \frac{1}{\kappa r^2}\left(\lambda - \frac{2\mu v}{r}\right), \qquad \rho = \frac{1}{\kappa r^2}\left(a + \frac{\mu v^2}{r^2}\right), \qquad P = \frac{1}{\kappa r^4}\left(\mu v^2\right). \tag{24}$$

(i) For $\mu = 0$, the spacetime (23) reduces to monopole-Vaidya,
(ii) for $a = 0$, it reduces to charged-Vaidya and
(iii) for $\mu = a = 0$, it reduces to Vaidya null radiation.

The metric (23) is self-similar[2], admitting a homothetic killing vector $\xi^a$ given by

$$\xi^a = v\frac{\partial}{\partial v} + r\frac{\partial}{\partial r},$$

which satisfies

$$L_\xi\, g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab}\,, \tag{25}$$

where $L$ denotes the Lie derivative.

It can be seen that $\xi^a K_a$ is constant along radial null geodesics, i.e.

$$\xi^a K_a = vK_v + rK_r = S, \tag{26}$$

where $S$ is a constant.

Inserting the mass function (22) into Eq. (15), we obtain

$$\frac{dR}{dk} - \frac{R^2}{2r^2}\left(1 - a - \frac{2\lambda v}{r} + \frac{3\mu v^2}{r^2}\right) = 0. \tag{27}$$

Using Eqs. (13), (14) and (26), we obtain the following solution to the differential equation (27)

$$R = \frac{2S}{2 + (a-1)X + \lambda X^2 - \mu X^3}, \tag{28}$$

[2] A spherically symmetric spacetime is self-similar if $g_{tt}(ct, cr) = g_{tt}(t, r)$ and $g_{rr}(ct, cr) = g_{rr}(t, r)$ for every $c > 0$.

where we have defined $X = v/r$, and is known as a self-similarity variable.

The situation being considered here is that for $v < 0$, the spacetime is the monopole solution with $f(v) = 0,\ q(v) = 0$. The radiation is focused into a central singularity at $r = 0,\ v = 0$ of growing mass $f(v)$ and $q(v)$. At $v = T$, say, the radiation is turned off. For $v > t$, the exterior spacetime settles to the Schwarzschild field embedded in a monopole field. Thus for $v = 0$ to $v = T$, the metric is monopole-charged-Vaidya, whereas for $v > T$ it is a monopole-Reissner-Nordström solution.

To analyze the nature of the singularity, we consider the radial null geodesics for the metric (23) defined by $ds^2 = 0$, taking into account the condition $d\theta = d\phi = 0$.

Radial null geodesics for the metric (23) are given by

$$\frac{dv}{dr} = \frac{2}{\left(1 - a - \dfrac{\lambda v}{r} + \dfrac{\mu v^2}{r^2}\right)}. \tag{29}$$

It can be observed that the above differential equation has a singularity at $r = 0,\ v = 0$.

Let $X_0$ denote the limit of the function $X$ as one approaches the singularity along the radial null geodesic. Then $X_0$ represents the tangent to the outgoing geodesics [28], i.e.

$$X_0 = \lim_{\substack{v \to 0 \\ r \to 0}} X = \lim_{\substack{v \to 0 \\ r \to 0}} \frac{v}{r} = \lim_{\substack{v \to 0 \\ r \to 0}} \frac{dv}{dr} \tag{30}$$

Using Eq. (29), the above equation reduces to the algebraic equation

$$\mu X_0^3 - \lambda X_0^2 + (1 - a) X_0 - 2 = 0. \tag{31}$$

The above equation decides the nature of the singularity. If the equation has a real and positive root, then there exist future directed radial null geodesics originating from $r = 0,\quad v = 0$. In this case the singularity will be naked. If the equation has no real and positive root, the singularity will be covered and the collapse ends into a black hole.

The nature of the root of this equation can be determined from the following rule in the *Theory of equations: Every equation of odd degree has at least one real root whose sign is opposite to that of its last term, the coefficient of the first term being positive.*

Using the above rule one can easily check that Eq. (31) has at least one real and positive root. Thus the central singularity arising in monopole-charged Vaidya collapse is naked.

## *Strength of the singularity*

The main importance of determining the strength of the singularity is due to the fact that the CCH does not need to rule out the possibility of the occurrence of the weak naked singularity [34].

A singularity is said to be strong if the collapsing objects do get crushed to a zero volume at the singularity, and a weak one if they do not. If the singularity is not strong, then it may not be considered as a physically realistic singularity.

Following Clarke and Krolak [35], a sufficient condition for a singularity to be strong, in the sense of Tipler [36], is that at least along one radial null geodesic (with affine parameter $k$) we must have

$$\lim_{k\to 0} k^2\Psi = \lim_{k\to 0} k^2 R_{ab}K^aK^b > 0, \tag{32}$$

where $K^a$ is the tangent to the null geodesics and $R_{ab}$ is the Ricci tensor.

Using Eqs. (13) and (14), we obtain

$$k^2 R_{ab}K^aK^b = k^2\left[\frac{2\dot{m}}{r^2}\,(K^v)^2\right] \tag{33}$$

$$= (\lambda - 2\mu X)\left(\frac{kR}{r^2}\right)^2. \tag{34}$$

As the singularity is approached, $k \to 0$, $r \to 0$ and $X \to X_0$. Hence, using the L'Hospital's rule, we find that

$$\lim_{k\to 0}\left(\frac{kR}{r^2}\right) = \frac{1}{1 - a - \lambda X_0 + \mu X_0^2}. \tag{35}$$

Then Eq. (34) yields

$$\lim_{k\to 0} k^2 R_{ab}K^aK^b = \frac{\lambda - 2\mu X_0}{\left(1 - a - \lambda X_0 + \mu X_0^2\right)^2}. \tag{36}$$

Hence the singularity will be a strong if

$$\lambda - 2\mu X_0 > 0. \tag{37}$$

Using numerical method, we find that for $\lambda = 0.1,\ \mu = 0.001$ and $a = 0.5$, one of the roots of Eq. (31) is $X_0 = 94.9562$. For this set of values

$$\lambda - 2\mu X_0 > 0.$$

Thus the naked singularity arising in this case is a strong curvature one.

## 4. *Discussion on apparent horizons*

When a large amount of mass is contained in a in a small region of a spacetime, a trapped surface forms around it. Therefore, as the matter collapses under the influence of a gravitational force, there is a possibility of the formation of a trapped surface as the collapse proceeds. If this happens, then on a sufficiently late-time spatial surface, there will be a boundary that separates the trapped region from the normal region. This boundary is known as the apparent horizon. For the spacetime (23) the apparent horizon is given by

$$(1-a)r^2 - \lambda v r + \mu v^2 = 0 \tag{38}$$

i.e.

$$r_{\pm} = \frac{\lambda v \pm \sqrt{\lambda^2 v^2 - 4(1-a)\mu v^2}}{2(1-a)} \tag{39}$$

$$= \frac{v\left(\lambda \pm \sqrt{\lambda^2 - 4(1-a)\mu}\right)}{2(1-a)}, \tag{40}$$

where $r_+$ and $r_-$ denote, respectively, the *outer apparent horizon* and the *inner Cauchy horizon*.

We will assume that $\lambda^2 > 4(1-a)\mu$ as an initial condition. At the equality, these two horizons coincide and for $\lambda^2 < 4(1-a)\mu$ they are absent, and the singularity is visible to an external observer. In other words, if one chooses the monopole component $a$ and electric charge parameter $\mu$ in such a way that the inequality

$$\lambda^2 < 4(1-a)\mu$$

holds, then the discriminant in Eq. (40) becomes negative and no horizons form, hence the CCH would be violated. Thus the final outcome of the collapse, a naked singularity or a black hole, depends sensitively on the monopole component and the electric charge parameter.

For the particular case $\lambda = 0.1$, $\mu = 0.001$, $a = 0.25$, the equations for the outer apparent horizon and inner Cauchy horizon for the monopole-charged Vaidya solution are $r_+ = (1/8.1670)v$ and $r_- = (1/91.8330)v$, respectively. If we remove the monopole and the charge fields (i.e. $a = \mu = 0$ ), then the solution reduces to the Vaidya solution, and the equation for the apparent horizon is $r = (1/10)v$.

To see the effect of the effect of monopole field on the gravitational collapse of Vaidya and charged Vaidya spacetime we give two different sets of graphs in Fig. 1 and 2.

From the equations of apparent horizons for various values of the parameters $\lambda, \mu$ and $a$, it is observed that the introduction of the monopole field increases the radius of the trapped region, i.e. increases the size of the black hole in Vaidya collapse. It has also been observed that, with the increase in the magnitude of the monopole field, the radius of the trapped region increases.

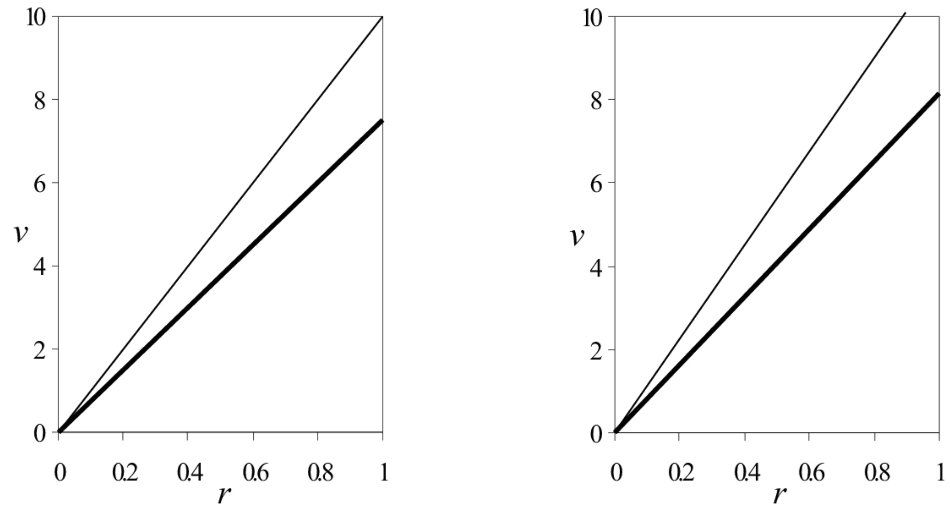


*Fig. 1. (Left) Apparent horizons for Vaidya solution for $v = 10r$ (thin line) and monopole Vaidya solution for $v = 7.5r$ (thick line). (Right) Outer apparent horizons for charged Vaidya solution for $v = 11.2702r$ (thin line) and monopole-charged Vaidya solution for $v = 8.1670r$ (thick line). In all four cases, $\lambda = 0.1$, $\mu = 0.001$ and $a = 0.25$ was assumed.*

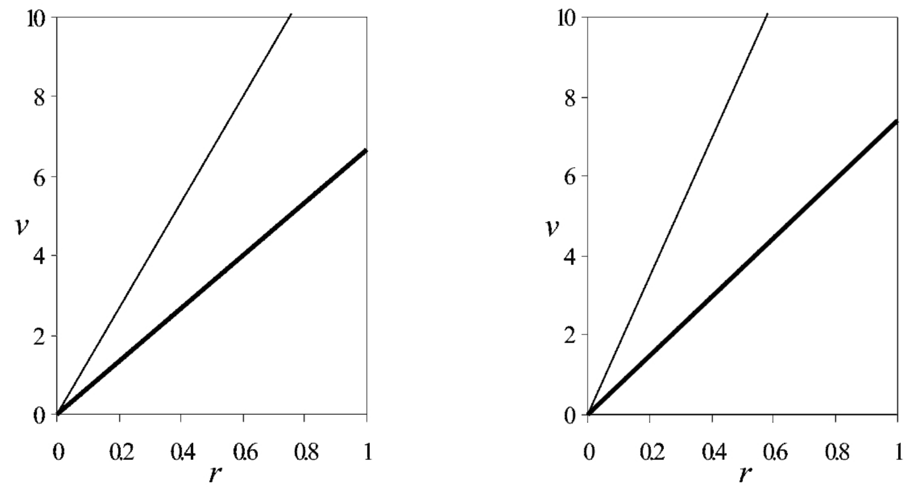


*Fig. 2. (Left) Apparent horizons for Vaidya solution for $v = 13.3333r$ (thin line) and monopole Vaidya solution for $v = 6.6667r$ (thick line). (Right) Outer apparent horizons for charged Vaidya solution for $v = 11.2702r$ (thin line) and monopole-charged Vaidya solution for $v = 8.1670r$ (thick line). In all four cases, $\lambda = 0.075$, $\mu = 0.001$ and $a = 0.5$ was assumed.*

## 5. Conclusion

One of the most important issues concerning the nature of spacetime singularities is that of cosmic censorship hypothesis. A rigorous formulation and proof for CCH is not available, hence, examples showing the occurrence of naked singularities remain important to arrive at provable formulation for the hypothesis. We have investigated the possibility of cosmic censorship violation in a composite solution – the monopole-charged-Vaidya solution. It is found that the central singularities arising in monopole-charged Vaidya spacetimes are naked, but nakedness of such singularities sensitively depends upon the monopole component ($a$) and electric charge parameter ($\mu$).

Using the Clarke and Krolak criteria [35], we have analyzed the strength of singularity and have shown that the naked singularities found in this composite solution is gravitationally strong. Thus the gravitational collapse of monopole-charged Vaidya spacetimes contradicts the CCH.

We have analyzed the trapped surfaces formation in the generalized Vaidya spacetime. The presence of monopole component can, in principle, change the boundary of the trapped region in Vaidya spacetimes. The introduction of monopole field increases the radius of the trapped region, thereby increases the size of the black hole in Vaidya collapse. It is also observed that, with increase in the magnitude of the monopole field, the radius of the trapped region increases. This might be the effect of repulsive force exerted by the monopoles against the gravity.

References

[1] S. W. Hawking and G. F. R. Ellis, *The large scale structure of spacetime*, Cambridge University Press, Cambridge (1973).

[2] R. Penrose, Riv. Nuovo Cimento **1** (1969) 252.

[3] W. A. Hiscok, L. G. Williams and D. M. Eardley, Phys. Rev. D **26** (1982) 751.

[4] K. Rajagopal and K. Lake, Phys. Rev. D **35** (1987) 1531.

[5] J. P. S. Lemos, Phys. Rev. D **59** (1999) 044020.

[6] I. H. Dwivedi and P.S. Joshi, Class. Quantum Grav. **6** (1989) 1599.

[7] K. D. Patil and U. S. Thool, Int. J. Mod. Phys. D **15** (11) (2006) 1977.

[8] D. Christodoulou, Commun. Math. Phys. **93** (1984) 171.

[9] D. M. Eardley and L. Smarr, Phys. Rev. D **19** (1979) 2239.

[10] P. S. Joshi and I. H. Dwivedi, Phys. Rev. D **47** (1993) 5357.

[11] K. D. Patil, Phys. Rev. D **67** (2003) 024017.

[12] K. D. Patil, Int. J. Mod. Phys. D **15** (2) (2006) 251.

[13] T. Harada and H. Maeda, Phys. Rev. D **63** (2001) 0804022.

[14] H. Iguchi, T.Harada and F. C. Mena, Class. Quantum Grav. **22** (2005) 841.

[15] T. Harko and K. S. Cheng, Phys. Lett. A **266** (2000) 249.

[16] S. G. Ghosh and Naresh Dadhich, Gen. Relativ. Gravit. **35** (2003) 359.

[17] P. C. Vaidya, Proc. Indian Acad. Sci. A **33** (1951) 264.

[18] A. Papapetrou, in *A Random walk in Relativity and Gravitation*, eds. N. Dadhich, J. K. Rao, J. V. Naralikar and C. V. Vishveshwara Wiley, New York, (1985).

[19] K. S. Govindo and M. Govender, Phys. Rev. D **68** (2003) 024034.

[20] K. D. Patil and S. S. Zade, Int. J. Mod. Phys. D. **15** (9) (2006) 1359.

[21] T. Harko, Phys. Rev. D **68** (2003) 064005.

[22] Erik Schnetter and Badri Krishnan, Phys. Rev. D **73** (R) (2006) 021502.

[23] Ishai Ben-Dov, Phys. Rev. D **75** (2007) 064007; gr-qc / 0611057 (2006).

[24] Anzhong Wang and Yumei Wu, Gen. Relativ. Gravit. Vol. **31** (1) (1999) 107.

[25] K. D. Patil, R. V. Saraykar and S. H. Ghate, Pramana, J. Phys. **52** (1999) 553.

[26] J. F. Villas da Rocha, Int. J. Mod. Phys. D **11** (1) (2002) 113.

[27] V. Husain, Phys. Rev. D **53** (1996) R 1759.

[28] P. S. Joshi, *Global Aspects in Gravitation and Cosmology*, Oxford, Clarendon (1993).

[29] K. Lake and T. Zannias, Phys. Rev. D **43** (1991) 1798.

[30] M. Barriola and A. Vilenkin, Phys. Rev. Lett., **63**, 341 (1989)

[31] Diego Harari and Carlos Lousto, Phys. Rev. D **42** (1990) 2626.

[32] S. L. Liebling, Phys. Rev. D **60** (1999) 061502.

[33] U. Nukamendi and D. Sudarsky, Class. Quantum Grav. **17** (2000) 4051.

[34] B. C. Nolan, Phys. Rev. D **60** (1999) 024014.

[35] C. J. S. Clarke and A. Krolak, J. Geom. Phys. **2** (1986) 127.

[36] F. J. Tipler, Phys. Lett. A **64** (1977) 8.

## STRUKTURA RADIJALNIH NULTIH GEODETSKIH LINIJA U VAIDYA-OVOM PROSTORU-VREMENU S MONOPOLIMA

Analiziraju se izlazne radijalne geodetske linije za Vaidya-ov model s monopolima koji sadrži golu singularnost. Primjenom Clarke-Krolakovih kriterija ispituje se jakost zakrivljenosti gole singularnosti kako bi se utvrdila jaka singularnost, što daje protuprimjer hipotezi o kozmičkoj cenzuri. Prikazuju se dijagrami prividnih horizonata za niz vrijednosti parametara. Zanimljivost ishoda računa jest da monopolna komponenta potiskuje prividni horizont prema radijalnoj osi i tako povećava polumjer prividnog horizonta u Vaidya-ovom urušavanju.