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FRESNEL AND FRAUNHOFER HOLOGRAMS GENERATED BY COMPUTER USING CIRCULAR BINARY MASK TECHNIQUES

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Instead of the rectangular binary masks introduced by A. W. Lohmann, D. P. Paris and B. R. Brown to construct holograms by a computer, the holograms may be computed using circular binary masks. The new holograms are equivalent to ordinary holograms only with regard to the image reconstruction. In this work, circular binary mask techniques have been established theoretically and verified experimentally.

1. Introduction

An ordinary hologram is produced by recording the interference of an object wave and a reference wave.

Lohmann and collaborators [1-3] showed how a computer-guided plotter can draw holograms using rectangular binary masks. This way for making holograms is sometimes advantageous because the object may be defined mathematically.

The holograms generated by a computer consist of many apertures on an opaque background where the binary character facilitates their production. Each aperture is positioned at a distance from a sampling point according to the phase of the computed wavefront. The height of the aperture is proportional to the module of the amplitude. Other rectangular aperture formats have also been used [1].

The non-rectangular apertures are also possible. A new variable-height circle is convenient for the mechanical plotter to draw, and we used it most often.

This work explains the priciple of circular binary Fraunhofer holograms generated by a computer using a deductive approach.

The selection of a proper format of this hologram is described, followed by the diffraction theory with and without approximations, then a comment on the computational and plotting producers is given and, finally, some experimental results are described.

2. Description of circular binary holograms

To produce a complex amplitude distribution, $O'(x',y')$, in the image plane of an object, $O(x,y)$, a properly designed binary hologram, $H(u,v)$, is illuminated with a tilted plane wave:

$$E(x_0,u) = \mathrm{e}^{2\pi \mathrm{i} x_0 u}, \tag{1}$$

where u and v are the reduced coordinates:

$$u = \frac{x_H}{\lambda f}, \qquad v = \frac{y_H}{\lambda f}, \tag{2}$$

and x_H and y_H are the actual coordinates in the hologram plane (see Fig. 1a).

The lens L_1, placed between the source plane and the hologram plane, produces the tilted plane wave (Eq. (1)).

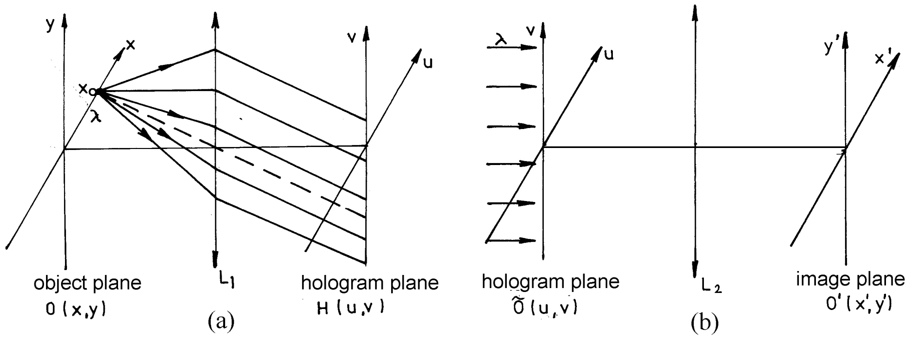


Fig. 1. Construction of an image from a binary hologram, illuminated by a point source.

A second lens, L_2, produces a two-dimensional Fourier transform $O'(x',y')$ of the complex amplitude:

$$O'(x',y') = \int\int \widetilde{O}(u,v)\mathrm{e}^{-2\pi \mathrm{i}(xu'+yv')}\mathrm{d}u\mathrm{d}v, \tag{3}$$

where the complex amplitude, $\widetilde{O}$, is produced by illuminating the binary hologram, $H(u,v)$, with the untilted plane wave, (see Fig. 1b).

In mathematical terms, the binary function H satisfies the following equation:

$$\widetilde{O}(u,v) = const\ H(u,v) \cdot E(x_0,v). \tag{4}$$

The sampling theorem tells us that the size of the hologram is limited to:

$$|u| \leq \frac{\nu_{max}}{2} \qquad \text{and} \qquad |v| \leq \frac{\nu_{max}}{2}, \tag{5}$$

where ν_{max} represents the inverse of the resolution, $\Delta x = \Delta y$, in the object plane:

$$\nu_{max} = \frac{1}{\Delta x} = \frac{1}{\Delta y}. \tag{6}$$

The same resolution is performed in the image plane:

$$\Delta x = \Delta x' \qquad \text{and} \qquad \Delta y = \Delta y'. \tag{7}$$

Hence:

$$O'(x',y') = 0 \quad \text{outside of} \quad |x'| \leq \frac{\Delta x'}{2} \quad \text{and} \quad |y'| \leq \frac{\Delta y'}{2}, \tag{8}$$

and

$$\widetilde{O}(u,v) = 0 \quad \text{outside of} \quad |u| \leq \frac{\nu_{max}}{2}, \quad \text{and} \quad |v| \leq \frac{\nu_{max}}{2}. \tag{9}$$

Another function, $h(x',y')$, represents the diffracted amplitude from the hologram, $H(u,v)$, in the image plane where the image, $O'(x',y')$, is obtained.

In terms of binary holograms, $h(x',y')$ is proportional to $O'(x',y')$ [1-4]:

$$h(x',y') = \mathrm{rect}\left(\frac{x'}{\Delta x'}\right) \mathrm{rect}\left(\frac{y'}{\Delta y'}\right) \int\int H(u,v) \mathrm{e}^{2\pi \mathrm{i}[(x'+x_0)u+y'v]} \mathrm{d}u\mathrm{d}v, \tag{10}$$

where

$$\mathrm{rect}(z) = \begin{cases} 1 & \text{for} \quad |z| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

and

$$h(x',y') = const\ O'(x',y'). \tag{12}$$

For the circular binary hologram, that is introduced in this work, the function rect(z) is replaced by the function:

$$\mathrm{circle}(x',y') = \begin{cases} & \text{for} \quad x'^2 + y'^2 \leq r^2 \\ 0 & \text{otherwise} \end{cases} . \tag{13}$$

Hence:

$$h(x',y') = \mathrm{circle}(x',y') \int\int H(u,v) \mathrm{e}^{2\pi \mathrm{i}[(x'+x_0)u+y'v]} \mathrm{d}u \mathrm{d}v. \tag{14}$$

In other words, the rectangular aperture is replaced by a circular aperture of the known radius r.

This consideration is in concordance with the Huygens-Fresnel principle, stating that each point that is in touch with a wave front becomes a new radiating punctual source.

Usually, a binary hologram has the cell size of about 4 mm. To become a true hologram, it is necessary to reduce the binary hologram photographically. In this situation, at a physical point, each circular aperture is reduced to a physical point, while this is not the case for the rectangular aperture.

Since $O'(x',y')$ is different from zero only for the square of the size $\nu_{max} \times \nu_{max}$, its Fourier transform can be written as:

$$\widetilde{O}(u,v) = \sum_m \sum_n \widetilde{O}(\frac{m}{\Delta x}, \frac{n}{\Delta y}) \sin c(u \cdot \Delta x - m) \cdot \sin c(v \cdot \Delta y - n). \tag{15}$$

In other words, $\widetilde{O}(u,v)$ is completely specified by a set of complex parameters, $\widetilde{O}(m/\Delta x, n/\Delta y)$, which represent $\widetilde{O}$ at a mesh of points:

$$u = \frac{m}{\Delta x}, \qquad v = \frac{n}{\Delta y}. \tag{16}$$

A circular area of radius

$$r = \frac{1}{\Delta x} = \frac{1}{\Delta \nu}, \tag{17}$$

that surounds each of these sampling points, will be called a circular cell, or simply a cell. Here $\Delta\nu$ represents the size of the resolution element of the hologram that is acquired with the number of resolvable points in the image:

$$N^2 = (\Delta x \cdot \Delta \nu)^2. \tag{18}$$

In the present experiments, a circular cell with a variable radius and variable position was chosen for each point of the hologram (see Fig. 2).

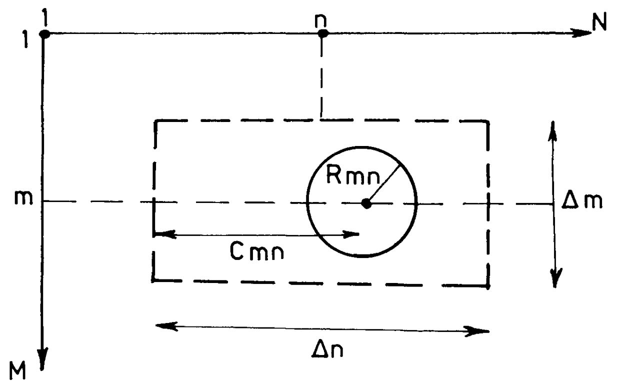


Fig. 2. Structure of a cell in the circular binary hologram. The circular opening has the radius R_{mn} and it is displaced from the center of the cell by the distance C_{mn}.

R_{mn} is the radius related to the value of the calculated amplitude, C_{mn} is its relative shift off to the phase and this circular cell is enclosed in a rectangular area of dimensions:

$$\frac{\Delta x_H}{\lambda f} \times \frac{\Delta y_H}{\lambda f}. \tag{19}$$

The amplitude and the phase characterize each point of the hologram $H(u,v)$:

$$H(u,v) = A(u,v)\mathrm{e}^{\mathrm{i}\phi(u,v)}, \tag{20}$$

which consists of $N \times M$ complex numbers; N and M represent the dimensions of the object matrix and of the hologram matrix.

3. Computational and plotting procedures

The Fraunhofer hologram was made using the Fourier transformation with the Cooley-Tukey algorithm [5]. This reduces the computer time by a factor of the order of $4\log_2(N)/N^2$, as compared to the more conventional Fourier transform programs. In this case, where the number of points $N \times M$ was mostly 64×64, this meant a reduction by factor of about 170.

From the amplitude spectrum, $A(u,v) = A_{mn}$, where $m = 1,2,\ldots,M$ and $n = 1,2,\ldots,N$, the maximum value, A_{max}, was determined, and with this maximum value, the radius of each cell was calculated:

$$R_{mn} = \frac{A_{mn}}{A_{max}}\Delta n_A, \tag{21}$$

with the spectrum phase, $\Phi(u,v) = \Phi_{mn}$. Since 2π is the maximum value of the phase, the removal of the cell was calculated using the relation:

$$C_{mn} = \frac{\Phi_{mn}}{2\pi}\Delta n_{\phi} \tag{22}$$

inside the area $\Delta m \times \Delta n$.

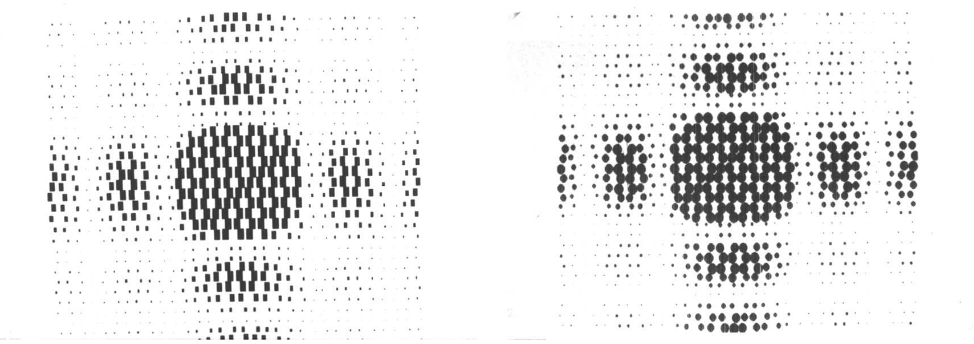
The value Δn_A corresponds to the maximum amplitude A_{mn} that represents the maximum number of points that will be plotted to draw a circular area, and Δn_{ϕ} represents the number of points which establish the removal of the cell corresponding to the hologram point with a maximum phase. Hence:

$$\Delta n = \Delta n_a + \Delta n_{\phi}. \tag{23}$$

Usually we used $\Delta n_A = 15$ points and $\Delta n_{\phi} = 10$ points.

4. Experimental results

When the drawing a hologram, the cell size was usually about 4 mm, either for the rectangular binary hologram (Fig. 3a), or for the circular binary hologram (Fig. 3b).



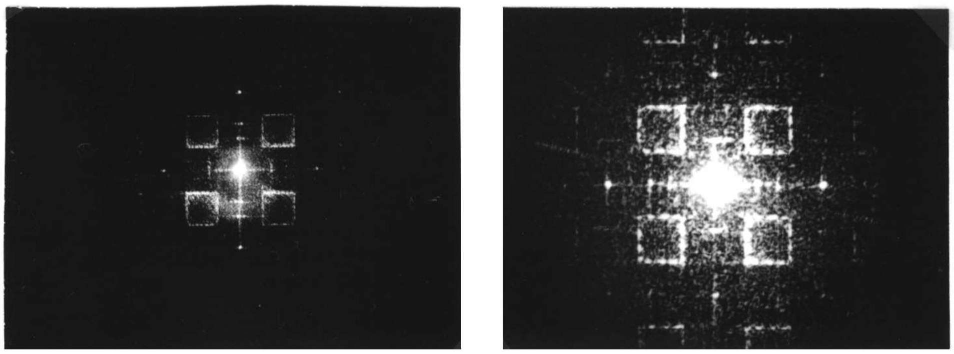
(a) (b)

Fig. 3. Plotting of a) a rectangular binary hologram and b) a circular binary hologram.

Both drawings were reduced photographically by a factor of 100 using an objective lens with $f = 50$ mm. The achieved holograms were recorded on photo-material LP 2 ORWO plates, appearing fairly sharp under the microscope.

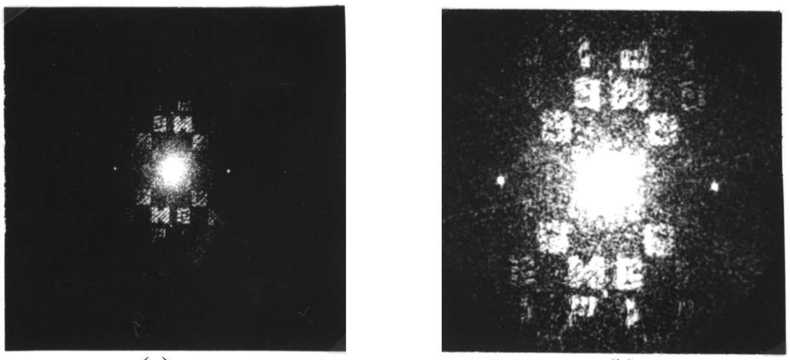
In the reconstruction of the images, the light of a He-Ne laser was used. Both holograms reconstructed the same image, a square (Figs. 4a,b). The image produced by the circular binary hologram (Fig. 4a) is clearer than the image produced by the rectangular binary hologram (Fig. 4b).

Another example is the hologram of some letters; again, the image produced by the circular binary hologram (Fig. 5a) is clearer than the image produced by the rectangular binary hologram (Fig. 5b).



(a) (b)

Fig. 4. Image (a square) produced by a) the circular binary hologram, b) the rectangular binary hologram.



(a) (b)

Fig. 5. Another image (some letters) produced by a) the circular binary hologram, b) the rectangular binary hologram.

Both pairs of holograms were calculated using a matrix of the dimensions 128×128.

5. Conclusions

One can reconstruct images from circular binary Fraunhofer holograms. This type of hologram contains circular cells in place of rectangular cells introduced by A. W. Lohmann, B. R. Brown and D. P. Paris.

Each circular cell has an area proportional to the amplitude and it is removed by a distance proportional to the phase.

The images from the two types of holograms are not identical; the images produced by the circular binary hologram are clearer than the images produced by the rectangular binary hologram.

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FRESNELOVI I FRAUNHOFEROVI HOLOGRAMI NAČINJENI RAČUNALOM PRIMJENOM TEHNIKE BINARNIH MASKI

Umjesto pravokutnih binarnih maski koje su uveli A. W. Lohmann, D.P. Paris i B. R. Brown radi računalne konstrukcije holograma, hologrami se mogu izračunati primjenom kružnih maski. Novi su hologrami ekvivalentni običnim hologramima samo u pogledu rekonstrukcije slika. U ovom se radu daje teorijska osnova i eksperimentalna potvrda tehnike kružnih binarnih maski.