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COHERENTLY PUMPED MICROMASER AS A MASER WITHOUT INVERSION

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In this paper, we use microscopic maser theory developed by Filipovicz, Javanaien and Meystre [Phys. Rev. A 34, 3077 (1986)] to investigate the properties of the coherently pumped two-level micromasers. We find that under right conditions, the initial atomic coherence locks the field phase and unlike ordinary lasers and masers there is no longer threshold. Consequently, the population inversion is not necessary.

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1. *Introduction*

Since the work of Jaynes and Cummings [1], the single-atom-cavity quantum electrodynamics has been studied extensively, as this simple model is able to exhibit clearly the discrete nature of light and quantum-mechanical aspects of its interaction with atoms. One of the marvelous experimental and theoretical systems that fulfills the idealized conditions of the Jaynes-Cummings model is the one-atom maser or micromaser [2-15]. An ordinary micromaser consists of a high-Q microwave cavity and a stream of excited Rydberg atoms which drive the field inside the cavity. The atomic beam is sufficiently sparse so that no more than one atom is in the cavity at any time. Because of their large dipole moment, Rydberg atoms couple strongly to the cavity mode. The single-atom maser oscillation has been demonstrated experimentaly [3,4].

The photon statistics of a micromaser shows interesting quantum effects. Examples include sub-Poissonian photon statistics [2,3,5] to the extreme case of a number state [9], quadrature noise reduction (squeezed state) [2,10–12] and trapping states [13]. The possibility of generating a coherent state of the micromaser [8] with injected atoms in a coherent superposition of the upper and lower states (atomic coherence), and the possibility of obtaining pure states of the field, the so-called tangent and cotangent states [14] (if the initial conditions are properly chosen), have also been predicted.

One of the key concepts in lasers and masers is population inversion. It is generally the case that a laser/maser requires population inversion in order to overcome the absorption from the lower level, since the gain is proportional to the population difference between the upper and lower levels of the lasing/masing transition. However, recent advances in the field of quantum optics and laser theory have shown that some laser systems may operate in the absence of population inversion [16-21]. The essential point for this possibility is to modify the emissive and absorptive profiles, with the help of a quantum-interference effect. In these laser systems, if atomic coherence is achieved, a kind of quantum interference is produced in lower levels, which leads to an absorption cancellation. A small population in the excited state can thus lead to net gain. Furthermore, micromaser, unlike conventional lasers and maser, works without population inversion. Masiak et al. [22] have studied an ordinary micromaser in which excited two-level atoms are injected into the micromaser cavity. By using an approximate description of the Fokker-Planck equation, as well as numerical calculations, they have shown that ordinary micromaser is a maser working without mean population inversion (defined as time average inversion over the interaction time). They believe that negative value of the mean inversion is a consequence of a destructive interference between propability amplitudes of photon emission for different Fock-states of the field in the cavity. Furthermore, they have shown that this effect does not depend on pumping statistics and of the kind of atomic transition (single or two photon).

The purpose of our paper is to study a micromaser with injected atomic coherence, i.e., a micromaser for which, unlike ordinary micromaser, active atoms are prepared initially in a coherent superposition of the upper and lower levels of the masing transition. Our approach is based on the microscopic maser theory developed by Filipovicz, Javanainen and Meystre [2]. We examine the steady-state behaviour of the micromaser field and by using a simple method, we derive an explicit expression describing steady-state field by which it is also possible to get some information about the field phase. We show that the maser action is possible even without population inversion. In fact, the initial atomic coherence leads to micromaser phase locking and furthermore provides a driving force, so that there is no threshold in the coherently pumped micromaser.

2. *Basic equations*

We consider a beam of monoenergetic two-level atoms injected at a rate r into a microwave cavity in which the atoms interact with the cavity mode for a finite time τ. It is assumed that $1/r$ follows a Poisson process (Poissonian pumping). The micromaser is usually operated in the regime in which there is at most only one atom in the cavity at any time and also in which the cavity damping time (C^{-1}) is much longer than the atom – field interaction time τ; that is the relaxation of the resonator field mode can be ignored while an atom is inside the cavity. Let t_N and $t_N+\tau$ be the entering and leaving times of $N^{\rm th}$ atom. The interaction of the cavity mode with the injected atom is described by the Hamiltonian

$$\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{A-F} \,. \tag{1}$$

Here, $\hat{H}_A$ and $\hat{H}_F$ describe the free atom and free field, respectively, and $\hat{H}_{A-F}$ describes the atom – field interaction in the dipole and rotating-wave approximations:

$$\begin{aligned} \hat{H}_A &= \hbar\omega_0 \hat{\sigma}_N^z \,, \\ \hat{H}_F &= \hbar\omega \hat{a}^\dagger \hat{a} \,, \\ \hat{H}_{A-F} &= \hbar g (\hat{\sigma}_N^+ \hat{a} + \hat{\sigma}_N^- \hat{a}^\dagger) \,, \end{aligned} \tag{2}$$

where $\hat{a}(\hat{a}^\dagger)$ is the annihilation (creation) operator for the radiation field mode with frequency ω, obeying the commutation relation $[\hat{a},\hat{a}^\dagger]=1, \hat{\sigma}_N^z$ and $\hat{\sigma}_N^\pm$ are the usual Pauli spin operators for the two-level atom, g is the coupling constant and $\hbar\omega_0$ is the energy difference between the two atomic levels. For the sake of simplicity, in this paper we assume that the frequency of the cavity mode coincides with the atomic transition frequency. According to the microscopic maser theory [2], the time evolution of the density matrix $\hat{\rho}_F$ of the cavity mode, between two successive atomic injection, is governed by the map

$$\hat{\rho}_F(t_{N+1}) = \exp(\hat{L} t_p) \hat{T}(\tau) \hat{\rho}_F(t_N) \,, \tag{3}$$

where $t_p = t_{N+1} - t_N = 1/r$ is the mean interval between two consecutive arrival times of atoms in the cavity, and $\hat{L}$ is the Liouvillian operator which describes the coupling of the cavity mode to a thermal bath (cavity loss) and has the form [23]

$$\begin{aligned} \hat{L}\hat{\rho}_F &= -\frac{C}{2}(n_b+1)(\hat{a}^\dagger\hat{a}\hat{\rho}_F + \hat{\rho}_F\hat{a}^\dagger\hat{a} - 2\hat{a}\hat{\rho}_F\hat{a}^\dagger) \\ &\quad -\frac{C}{2}n_b(\hat{a}\hat{a}^\dagger\hat{\rho}_F + \hat{\rho}_F\hat{a}\hat{a}^\dagger - 2\hat{a}^\dagger\hat{\rho}_F\hat{a}) \,. \end{aligned} \tag{4}$$

Here, n_b is the average number of thermal photons in the cavity and C is the cavity decay rate. In Eq. (3), the gain operator $\hat{T}(\tau)$ is given by

$$\hat{T}(\tau)\hat{\rho}_F = \mathrm{Tr}_A[\hat{U}(\tau)\hat{\rho}_A(0) \otimes \hat{\rho}_F(0)\hat{U}^+(\tau)] \,, \tag{5}$$

where $\hat{\rho}_F(0)$ and $\hat{\rho}_A(0)$ describe the initial density operator of the field and the atomic system, respectively, Tr_A indicates partial trace over the Hilbert space of the two-level atom and $\hat{U}(\tau) \equiv \exp(-\mathrm{i}\hat{H}_{A-F}\tau/\hbar)$ is the time evolution operator in the interaction picture.

Now, we consider the following limits

$$Ct_p \ll 1, \qquad g\tau \ll 1 . \tag{6}$$

The first condition in (6) means that the decrease in photon number in a time between two successive atomic injection is very small. The second condition says that atom-field interaction is weak. Let us now rewrite Eq. (3) as

$$\hat{\rho}_F(t_{N+1}) = \{1 + [\exp(\hat{L}t_p) - 1]\}\{1 + [\hat{T}(\tau) - 1]\}\hat{\rho}_F(t_N) . \tag{7}$$

In the limit (6), we can write

$$[\exp(\hat{L}t_p) - 1] \approx Ct_p, \qquad [\hat{T}(\tau) - 1] \approx (g\tau)^2 .$$

Hence, by neglecting the product $[\exp(\hat{L}t_p) - 1][\hat{T}(\tau) - 1]$, we obtain

$$\hat{\rho}_F(t_{N+1}) = \{1 + [\exp(\hat{L}t_p) - 1]\} + [\hat{T}(\tau) - 1]\hat{\rho}_F(t_N) . \tag{8}$$

Next, we take into account the definition of the generator of the operator $\exp(\hat{L}t_p)$

$$\hat{L} = \lim_{t_p \to 0} \frac{\exp[(\hat{L}t_p) - 1]}{t_p} , \tag{9}$$

which for $Ct_p \ll 1$ allows us to reformulate Eq. (8) as

$$\hat{\rho}_F(t_{N+1}) = \hat{\rho}_F(t_N) + \hat{L}t_p\hat{\rho}_F(t_N) + [\hat{T}(\tau) - 1]\hat{\rho}_F(t_N) . \tag{10}$$

Under steady-state conditions, i.e. when $\hat{\rho}_F(t_N) = \hat{\rho}_F(t_{N+1}) \equiv \hat{\rho}_F^{ss}$, Eq. (10) takes the following form

$$(1 - \hat{L}t_p)\rho_F^{(ss)} = \hat{T}(\tau)\rho_F^{(ss)} . \tag{11}$$

This equation, which describes the behaviour of the micromaser field in the steady state, is the basis of our considerations.

3. *Coherently pumped micromaser*

So far our considerations were rather general. We now concentrate on the coherently pumped micromaser. It is assumed that all atoms are initially prepared in a same coherent superposition of the upper level $\mid a\rangle$ and the lower level $\mid b\rangle$. The wave function of the N^{th} atom has the form

$$\big|\psi_A^{(N)}\big\rangle = \alpha \exp(\mathrm{i}\varphi)\big|a_N\big\rangle + \beta\big|b_N\big\rangle , \tag{12}$$

where α, β and φ are real numbers and $\alpha^2 + \beta^2 = 1$. According to Eq. (5), after one atom has passed through the micromaser cavity, the gain term describing the atom-field coupling reads

$$(\hat{T}(\tau)\hat{\rho}_F)_{N=1} = \mathrm{Tr}_A\big[\hat{U}_{N=1}(\tau)\hat{\rho}_A^{(N=1)} \otimes \hat{\rho}_F^{(N=0)}\hat{U}^+_{N=1}(\tau)\big]\,. \tag{13}$$

For two atoms, we have

$$(\hat{T}(\tau)\hat{\rho}_F)_{N=2} = \mathrm{Tr}_A\big[\hat{U}_{N=2}(\tau)\hat{\rho}_A^{(N=2)} \otimes \hat{\rho}_F^{(N=1)}\hat{U}^+_{N=2}(\tau)\big] \tag{14}$$

and generally, after N atoms have passed, we have

$$(\hat{T}(\tau)\hat{\rho}_F)_{N} = \mathrm{Tr}_A\big[\hat{U}_{N}(\tau)\hat{\rho}_A^{(N)} \otimes \hat{\rho}_F^{(N-1)}\hat{U}^+_{N}(\tau)\big]\,. \tag{15}$$

Since $\hat{\rho}_A^{(N)} = \big|\psi_A^{(N)}\rangle\langle\psi_A^{(N)}\big|$ and $\hat{\rho}_F^{(N-1)} = \big|\psi_F^{(N-1)}\rangle\langle\psi_F^{(N-1)}\big|$, we can rewrite Eq. (15) in terms of the photon number state $\mid n\rangle$

$$(\hat{T}(\tau)\hat{\rho}_F)_{N} = \tag{16}$$
$$\mathrm{Tr}_A\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Big\{\hat{U}_N(\tau)\big|\psi_A^{(N)}\rangle \otimes \big|n\rangle\langle n\big|\psi_F^{(N-1)}\rangle\Big\}\Big\{\langle\psi_F^{(N-1)}\big|m\rangle\langle m\big| \otimes \langle\psi_A^{(N)}\big|\hat{U}_N^+(\tau)\Big\}\Big]\,.$$

Here, $\big|\psi_F^{(N-1)}\rangle$ is the wave function of the micromaser field after passing of $(N-1)$ atoms through the cavity. It is easy to show that the time evolution operator $\hat{U}_N(\tau)$ can be expressed in the form

$$\hat{U}_N(\tau) \equiv \exp\left(-\mathrm{i}\frac{\hat{H}_{A-F}\tau}{\hbar}\right) = \exp[-\mathrm{i}g\tau(\hat{\sigma}_N^+\hat{a} + \hat{\sigma}_N^-\hat{a}^\dagger)] \tag{17}$$
$$= \cos(g\tau\sqrt{1/2 + \hat{\sigma}_N^z + \hat{a}^\dagger\hat{a}}) - \mathrm{i}\frac{\sin(g\tau\sqrt{1/2 + \hat{\sigma}_N^z + \hat{a}^\dagger\hat{a}})}{\sqrt{1/2 + \hat{\sigma}_N^z + \hat{a}^\dagger\hat{a}}}(\hat{\sigma}_N^+\hat{a} + \hat{\sigma}_N^-\hat{a}^\dagger)\,.$$

The matrix elements of this operator are

$$\begin{aligned}
\langle a_N; n \mid \hat{U}_N(\tau) \mid a_N; m\rangle &= \cos(g\tau\sqrt{n+1})\delta_{n,m} \\
\langle b_N; n \mid \hat{U}_N(\tau) \mid b_N; m\rangle &= \cos(g\tau\sqrt{n})\delta_{n,m} \\
\langle a_N; n \mid \hat{U}_N(\tau) \mid b_N; m\rangle &= -\mathrm{i}\sin(g\tau\sqrt{n+1})\delta_{n,m-1} \\
\langle b_N; n \mid \hat{U}_N(\tau) \mid a_N; m\rangle &= -\mathrm{i}\sin(g\tau\sqrt{n})\delta_{n,m+1}\,.
\end{aligned} \tag{18}$$

By applying Eqs. (12) and (18) in Eq. (16), we can get the following relation for the matrix elements of the gain term

$$
\begin{aligned}
\langle n\big|(\hat{T}(\tau)\hat{\rho}_F)_N\big|m\rangle \;=\;& \Big[\alpha^2\cos(g\tau\sqrt{n+1})\cos(g\tau\sqrt{m+1}) \\
&+\;\beta^2\cos(g\tau\sqrt{n})\cos(g\tau\sqrt{m})\Big]\rho_F^{(N-1)}(n,m) \qquad (19)\\
&+\;\beta^2\sin(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\rho_{_F}^{(N-1)}(n+1,m+1)\\
&+\;\alpha^2\sin(g\tau\sqrt{n})\sin(g\tau\sqrt{m})\rho_{_F}^{(N-1)}(n-1,m-1)\\
&+\;\mathrm{i}\alpha\beta\Big[\exp(\mathrm{i}\varphi)\cos(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\rho_{_F}^{(N-1)}(n,m+1)\\
&+\;\exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{n})\sin(g\tau\sqrt{m})\rho_{_F}^{(N-1)}(n,m-1)\\
&-\;\exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\rho_{_F}^{(N-1)}(n+1,m)\\
&-\;\exp(\mathrm{i}\varphi)\sin(g\tau\sqrt{n})\cos(g\tau\sqrt{m})\rho_{_F}^{(N-1)}(n-1,m)\Big]\,.
\end{aligned}
$$

On the other hand, according to Eq. (4), the matrix elements of the loss operator $\hat{L}\hat{\rho}_{_F}$ are

$$
\begin{aligned}
\langle n\big|\hat{L}\hat{\rho}_{_F}\big|m\rangle \;=\;& C(n_b+1)[\sqrt{(n+1)(m+1)}\rho_{_F}(n+1,m+1)-\frac{1}{2}(n+m)\rho_{_F}(n,m)]\\
&+\;Cn_b[\sqrt{nm}\rho_{_F}(n-1,m-1)-\frac{1}{2}(n+m+2)\rho_{_F}(n,m)]\,. \qquad (20)
\end{aligned}
$$

By using Eqs. (11), (19) and (20), one finds

$$
\begin{aligned}
&\Big\{\alpha^2\big(1-\cos(g\tau\sqrt{n+1})\;\cos(g\tau\sqrt{m+1})\big)+\beta^2(1-\cos(g\tau\sqrt{n})\cos(g\tau\sqrt{m})\\
&\quad+\;\frac{Ct_p}{2}n_b(n+m+2)+\frac{Ct_p}{2}(n_b+1)(n+m)\Big\}\rho_{_F}^{(ss)}(n,m) \qquad (21)\\
&-\;\Big\{Ct_pn_b\sqrt{nm}+\alpha^2\sin(g\tau\sqrt{n})\sin(g\tau\sqrt{m})\Big\}\;\rho_{_F}^{(ss)}(n-1,m-1)\\
&-\;\Big\{\;Ct_p(n_b+1)\sqrt{(n+1)(m+1)}+\beta^2\sin(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\Big\}\\
&\qquad\rho_{_F}^{(ss)}(n+1,m+1)\\
&-\;\mathrm{i}\alpha\beta\Big\{\exp(\mathrm{i}\varphi)\cos(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\rho_{_F}^{(ss)}(n,m+1)\\
&\quad+\;\exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{n})\sin(g\tau\sqrt{m})\rho_{_F}^{(ss)}(n,m-1)\\
&\quad-\;\exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{m+1})\sin(g\tau\sqrt{n+1})\rho_{_F}^{(ss)}(n+1,m)\\
&\quad-\;\exp(\mathrm{i}\varphi)\cos(g\tau\sqrt{m})\sin(g\tau\sqrt{n})\rho_{_F}^{(ss)}(n-1,m)\Big\}=0\,.
\end{aligned}
$$

This equation describes the steady-state behaviour of a coherently pumped micromaser that operates on two level-atoms. It is clear that the coupling between the diagonal matrix elements $\rho_{_F}^{(ss)}(n,n)$ and the off-diagonal elements $\rho_{_F}^{(ss)}(n,n\pm 1) = \rho_{_F}^{(ss)*}(n\pm 1,n)$ occurs only when the atomic coherence $\alpha\beta$ is present. If the micromaser is pumped by unpolarized atoms, ($i.e., \alpha\beta = 0$) then the off-diagonal elements don't occur, and consequently the field phase is always random. However, atoms prepared in a coherent superpostion of their states before entering the micromaser cavity create nonvanishing off-diagonal elements, that is they create a preferred field phase. Now, by setting $n = m = n_0$ in Eq.(21), we obtain

$$
\begin{aligned}
& \Big\{\alpha^2 \sin^2(g\tau\sqrt{n_0+1}) + \beta^2 \sin^2(g\tau\sqrt{n_0}) + Ct_p n_b(n_0+1) + Ct_p(n_b+1)n_0\Big\} \\
& P_{_F}^{(ss)}(n_0) \\
- \quad & \Big\{Ct_p n_b n_0 + \alpha^2 \sin^2(g\tau\sqrt{n_0})\Big\} P_{_F}^{(ss)}(n_0-1) \qquad (22) \\
- \quad & \Big\{Ct_p(n_b+1)(n_0+1) + \beta^2 \sin^2(g\tau\sqrt{n_0+1})\Big\} P_{_F}^{(ss)}(n_0+1) \\
- \quad & \mathrm{i}\alpha\beta\Big\{\exp(\mathrm{i}\varphi)\cos(g\tau\sqrt{n_0+1})\sin(g\tau(\sqrt{n_0+1})\rho_{_F}^{(ss)}(n_0,n_0+1) \\
+ \quad & \exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{n_0})\sin(g\tau\sqrt{n_0})\rho_{_F}^{(ss)}(n_0,n_0-1) \\
- \quad & \exp(-\mathrm{i}\varphi)\cos(g\tau\sqrt{n_0+1})\sin(g\tau\sqrt{n_0+1})\rho_{_F}^{(ss)}(n_0+1,n_0) \\
- \quad & \exp(\mathrm{i}\varphi)\cos(g\tau\sqrt{n_0})\sin(g\tau\sqrt{n_0})\rho_{_F}^{(ss)}(n_0-1,n_0)\Big\} = 0\,.
\end{aligned}
$$

Here n_0 and $P_F^{(ss)}(n_0) = \rho_{_F}^{(ss)}(n_0,n_0)$ are the steady-state mean photon number, and the normalized photon number and normalized photon number distribution, respectively. It is easy to see from Eq. (22) that this is satisfied when

$$
\begin{aligned}
& \frac{1}{2}\Big\{\sin^2(g\tau\sqrt{n_0})\Big[\alpha^2 P_{_F}^{(ss)}(n_0-1) - \beta^2 P_{_F}^{(ss)}(n_0)\Big] - Ct_p(n_b+1)n_0 P_{_F}^{(ss)}(n_0) \\
+ \quad & Ct_p n_b n_0 P_{_F}^{(ss)}(n_0-1)\Big\} \\
- \quad & \frac{1}{2}\mathrm{i}\alpha\beta\exp[\mathrm{i}(\varphi-\theta_0)]\sin(2g\tau\sqrt{n_0})\ \left|\rho_{_F}^{(ss)}(n_0-1,n_0)\right| = 0\,, \qquad (23)
\end{aligned}
$$

where θ_0 is the steady-state micromaser field phase and $\rho_{_F}^{(ss)}(n_0-1,n_0)$ is defined by

$$
\rho_{_F}^{(ss)}(n_0-1,n_0) = \left|\rho_{_F}^{(ss)}(n_0-1,n_0)\right|\ \exp(-\mathrm{i}\theta_0)\,. \qquad (24)
$$

Equation (23) is just detailed balancing of the photon number flux (Fig. 1). Now for the purpose of study the steady-state behaviour of the micromaser field, we treat independently the field and atomic pumping of the field. In terms of atomic and field operators, this means that we uncorrelate the field and atomic operators, i.e., we make the approximation

$$
\langle\hat{\sigma}_z\hat{a}\rangle \approx \langle\hat{\sigma}_z\rangle\langle\hat{a}\rangle_{ss} = (\alpha^2-\beta^2)\sum_n \sqrt{n}\rho_F^{(ss)}(n,n-1). \qquad (25)
$$

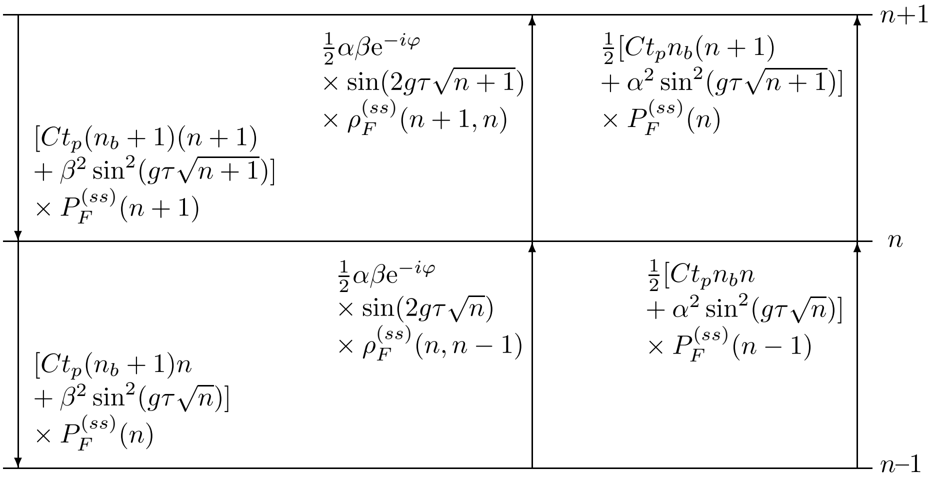


Fig. 1. Flow of probability for finding n_0 photons in a coherently pumped micromaser.

Physically, this corresponds to treating the field classically. When $n_0 \gg 1$, these are good approximations. In fact in this limit the dominant contribution to the expression (25) comes from

$$\begin{aligned}(\alpha^2-\beta^2)\sqrt{n_0}\rho_F^{(ss)}(n_0,n_0-1) &= (\alpha^2-\beta^2)\sqrt{n_0}e^{-\mathrm{i}\theta_0} \mid \rho_F^{(ss)}(n_0,n_0-1)\mid \\ &\approx (\alpha^2-\beta^2)e^{-\mathrm{i}\theta_0}\sqrt{n_0}P_F^{(ss)}(n_0)\,, \end{aligned} \tag{26}$$

where we have made use of

$$\mid \rho_F^{(ss)}(n_0,n_0-1)\mid \approx \mid \rho_F^{(ss)}(n_0-1,n_0)\mid \approx P_F^{(ss)}(n_0) \approx P_F^{(ss)}(n_0-1)\,. \tag{27}$$

(Note that the photon number distribution is normalized to unity). By using this approximation, Eq. (23) takes the following form

$$(\alpha^2-\beta^2)\sin^2(g\tau\sqrt{n_0})-\mathrm{i}\alpha\beta[\cos(\varphi-\theta_0)+\mathrm{i}\sin(\varphi-\theta_0)]\sin(2g\tau\sqrt{n_0})-\frac{C}{r}n_0 = 0\,. \tag{28}$$

This equation leads to two real equations in the steady-state

$$\alpha\beta\cos(\varphi-\theta_0)\sin(\sqrt{N_0}) = 0; \qquad \alpha\beta\neq 0 \tag{29a}$$

and

$$\frac{2}{\sqrt{N_0}}[(\alpha^2-\beta^2)\sin^2(\frac{\sqrt{N_0}}{2})+\alpha\beta\sin(\varphi-\theta_0)\sin(\sqrt{N_0})] = \frac{C}{A}\sqrt{N_0}\,, \tag{29b}$$

where $N_0 = 4g^2\tau^2 n_0$ and $A = 2rg^2\tau^2$ are the normalized mean photon number and linear gain coefficient, respectively. From Eq. (29a), we have

$$\theta_0 = \varphi \pm \frac{\pi}{2} \quad ; \quad \sin(\sqrt{N_0}) \neq 0 \,. \tag{30}$$

With the positive sign, the stability of micromaser field is not satisfied. This means that if we let $\theta_0 = \theta_0 + \delta\theta$, only the solution

$$\theta_0 = \varphi - \frac{\pi}{2} \tag{31}$$

will satisfy $\mathrm{d}(\delta\theta_0)/\mathrm{d}t < 0$ and is thus stable. This shows that, because of the presence of the initial atomic coherence, the micromaser phase is locked to a particular value and the maser becomes a phase sensitive device. It is improtant to note that the phase locking occurs only for those values of $\sqrt{N_0}$ such that $\sqrt{N_0} = 2g\tau\sqrt{n_0} \neq m\pi$ (m integer). In other words, if $\sqrt{N_0} = m\pi$, then the phases of the micromaser field diffuse freely, even if there exists initial atomic coherence ($\alpha\beta \neq 0$). By substituting Eq. (31) into Eq. (29b), we obtain

$$\frac{2}{\sqrt{N_0}}[(\alpha^2 - \beta^2)\sin^2(\frac{\sqrt{N_0}}{2}) + \alpha\beta\sin(\sqrt{N_0})] = \frac{C}{A}\sqrt{N_0} \,. \tag{32}$$

This relation shows the role of the initial atomic coherence: $\alpha\beta$ acts as a "driving force" and, consequently, there is no longer a threshold in the coherently pumped micromaser. This leads to an interesting phenomenon: population inversion (i.e; $\alpha^2 > \beta^2$) is not necessary. To see the threshold behaviour of a coherently pumped micromaser more clearly, one can consider the following special case. If the initial atomic variables are

$$\beta^2 = 1 - \alpha^2 = C/A, \quad \alpha\beta = 0 \ \text{(incoherently pumped micromaser)}\,, \tag{33}$$

then according to (32), we have

$$\left(1 - 2\frac{C}{A}\right)\sin^2\left(\sqrt{N_0}/2\right) = (C/2A)N_0 \,. \tag{34}$$

This equation yields a positive nonzero solution for normalized mean photon number N_0 if $A > 2C$ (threshold condition). So according to (33), one sees that population inversion ($\beta^2 < 0.5$) is necessary. But in the case of coherently pumped micromaser ($\alpha\beta \neq 0$), Eq. (32) results in

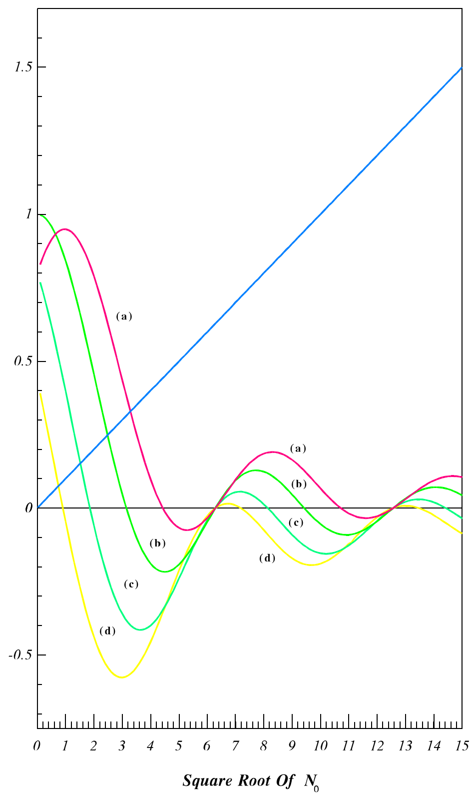
$$\left(1 - 2\frac{C}{A}\right)\sin^2(\sqrt{N_0}/2) + \sqrt{\frac{C}{A}(1 - \frac{C}{A})}\sin(\sqrt{N_0}) = (C/2A)N_0 \,, \tag{35}$$

and in order to obtain a positive nonzero solution for N_0 one should have

$$\left(2\frac{C}{A} - 1\right) < 2\sqrt{\frac{C}{A}(1 - \frac{C}{A})}\cot(\sqrt{N_0}/2) \,. \tag{36}$$

As can be seen, in this case the requirement of threshold condition is removed since even for $A < 2C$ the inequality (36) is satisfied. Furthermore, we have $\beta^2 > 0.5$. This means that in the presence of atomic coherence, masing without inversion is possible and there is no threshold .

In general, Eq.(32) can be solved graphically. Figure 2 plots its left-hand side (representing the effect of the gain medium) as a function of $\sqrt{N_0}$ for $\alpha^2 > \beta^2$, $\alpha^2 = \beta^2$ and $\alpha^2 < \beta^2$, and plots its right-hand side (representing cavity loss) as a function of $\sqrt{N_0}$ and for fixed value of C/A. N_0's are determined by the cross



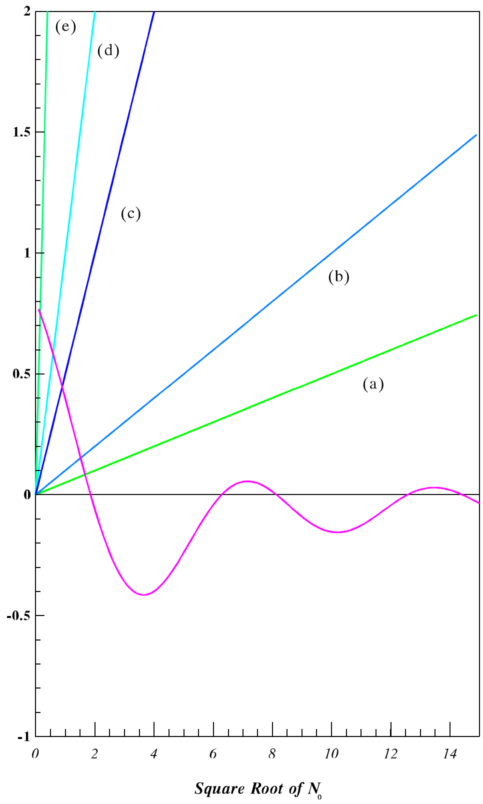


Fig. 2. The left- and right-hand sides of equation (32) as functions of $\sqrt{N_0}$, *for* $C/A = 0.1$ *and* (a) $\alpha^2 = 0.8$, (b) $\alpha^2 = 0.5$, (c) $\alpha^2 = 0.2$ *and* (d) $\alpha^2 = 0.05$.

Fig. 3. (right). The left- and right-hand sides of equation (32) as functions of $\sqrt{N_0}$, *for* $\alpha^2 = 0.2$ *and* (a) $C/A = 0.05$, (b) $C/A = 0.1$, (c) $C/A = 0.5$, (d) $C/A = 1$ *and* (e) $C/A = 5$.

point of a gain curve and the straight loss-line. As is seen, even without population inversion (i.e. with $\alpha^2 < \beta^2$), there exists a positive solution for $\sqrt{N_0}$. Figure 3 plots the left- and right-hand sides of Eq. (32) as a functions of $\sqrt{N_0}$, for fixed value of α^2, but for various values of C/A. This shows that there is no threshold for coherently pumped micromaser. Here we want to point out that at $\sqrt{N_0} = (2m+1)\pi$ (m integer), the second term in Eq. (32) becomes zero and this leads to recovery of the maser threshold and, consequently, the population inversion is still required. In other words, in the case of $\alpha^2 < \beta^2$, the gain curve(s) and loss straight line(s) cross each other at $\sqrt{N_0} \neq (2m+1)\pi$. This is shown in Figs. 2 and 3. Furthermore, at $\sqrt{N_0} = 2m\pi$ (trapping state) the gain is always equal to zero since atoms neither emit nor absorb photons (the intersection points of the gain curves with x-axis in Fig. 2), while the right-hand side of Eq. (32) is not equal to zero (provided that $C \neq 0$). Thus, we find that in the limit of $n_0 \gg 1$ (coherent field) the trapping states of the micromaser field don't occur at all. This result is in full agreement with the experimental result obtained by Weidinger et al. [24]. Of course, under the trapping condition and the condition of weak interaction, a coherent state with amplitude $(2\alpha/\beta)g\tau$ can be reached in the steady-state regime of a lossless micromaser [14].

Finally, it should be noted that in Ref. [22] the criterion for masing without inversion in an excited pumped micromaser is that the mean inversion as a function of interaction time becomes negative, while in our treatment, the relevant criterion is given on the basis of the existence of a positive solution for the steady-state mean photon number in the absence of inversion. Of course, these two criteria are conceptually similar. It is necessary to point out that, in agreement with Ref. [22], we believe that the occurrence of masing without inversion is a consequence of a kind of destructive interference. In our approach, this interference occurs between the atomic dipole and the cavity field yielding a null of the transition probability corresponding to photon absorption. This cancellation arises from the atomic coherence.

4. *Conclusion*

We have studied the main properties of the field generated in a coherently pumped micromaser cavity that operates on two-level atoms. The microscopic maser theory has been applied to find the explicit expression of the detailed-balance steady-state photon distribution. We have found that under certain conditions, the initial atomic coherence locks the field phase and unlike ordinary lasers and masers, there is no longer threshold and consequently, the population inversion is not necessary.

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KOHERENTNO PUMPAN MIKROMASER KAO MASER BEZ INVERZIJE

Primijenjujemo mikroskopsku teoriju masera koju su razvili Filipovicz, Javanaien i Meystre [Phys. Rev. A 34, 3077 (1986)] radi istraživanja svojstava koherentno pumpanih dvorazinskih mikromasera. Nalazimo da u povoljnim uvjetima, početna atomska koherencija veže fazu polja, te za razliku od običnih masera i lasera, nemamo praga i stoga inverzija zaposjednutosti nije potrebna.