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CORRELATION BETWEEN DIFFRACTION OF LIGHT BY CIRCULAR APERTURE AND CLOSE-RANGE INTERACTION ENERGY OF TWO CHARGED SPHERES

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Dedicated to Professor Kseno Ilakovac on the occasion of his 70th birthday

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It is shown that the coefficients A_k of angles θ_k, corresponding to the minima of light intensity in the diffraction by a circular aperture, can successfully be described by a function which appears in the electrostatic interaction energy between two uniform surface-charged spheres. The coefficient A_k defines the angle θ_k by $\sin\theta_k = A_k\lambda/d$, $k = 1, 2, 3, \ldots$, where λ is the wavelength of light and d is the diameter of the aperture. These coefficients may be correlated with a dimensionless function $f_{b/R}$, encountered in the relation of the interaction energy terms of two spheres of equal radii R, with uniformly-distributed surface charges q and q' and at a mutual distance b. The total interaction energy, i.e. the Coulomb potential energy, is $W = qq'/(4\pi\epsilon_0 b)$, where ϵ_0 is the permitivity of vacuum. This energy contains two terms, a positive one $W^{(+)} = W f_{b/R}$ and negative one $W^{(-)} = W(1 - f_{b/R})$. The correlation between A_k and the function $f_{b/R}$ is given by $A_k - k = u + v f_{b/R,k}$, where u and v are constants. Coefficients A_k obtained by this correlation agree with those defined by the diffraction method within an error of 10^{-3}% at $k = 1$, and gradually diminishing to 10^{-4}% at $k = 10$.

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1. Introduction

A correlation of the electrostatic interaction energy of various systems with the intrinsic quantized redshift of quasars was found recently [1]. The electrostatic systems studied were two uniformly charged spheres of equal radii R at a mutual distance b, a point charge and uniform infinite line charge, two parallel uniform infinite line charges and two parallel infinite uniformly charged planes. Similar exponential functions describe quite different phenomena, which, at the first sight, appear unrelated. The functions have the form $z_n = K_1 F^n - 1$ for redshift and $N_k = K_2 F^k$ for the characteristic coefficients of the interaction energy for the electrostatic systems considered. K_1 and K_2 are constants and n and k are integer numbers. Both functions have nearly the same value of factor $F \approx 1.23$. These results are an indication that these diverse systems are subjected to analogous physical laws.

It was also shown that some other systems may be characterized by the same factor F, which appears in the power law, as, for example, in gravitational systems $m_s = K_3 M^F$, where m_s is the mass of all satellites which orbit around the central mass M [2]. Similarly, the atomic $A - Z$ correlation was found to be of the form $A = \alpha Z^\beta$ [2–5], where A is the atomic weight, Z atomic number, and α is constant, while the parameter β is presumably $\sqrt{F}$ [2].

In this work we extend the investigation to the relation of the interaction energy between two charged spheres [1] with the diffraction of light by a circular aperture.

2. *Diffraction by a circular aperture*

The theory of diffraction of light by a circular aperture is well-known. Only a brief outline is given here. The intensity of diffracted light at an angle θ to the normal to the aperture, is given by [6],

$$I(\theta) \propto \pi^2 r^4 \left(\frac{J_1(2m)}{m} \right)^2 , \tag{1}$$

where $J_1(2m)$ is the Bessel function of the first kind of the order unity, r is the radius of the aperture and m is a parameter defined by

$$m = \frac{\pi r \sin\theta}{\lambda} , \tag{2}$$

where λ is the wavelength of light. The minimum of intensity at an angle θ is determined by $J_1(2m)/m = 0$, and the minima are given by the following values of m: 1.916, 3.508, 5.087, 6.662,... Consequently, the dark fringes, or dark circles, occur at the value of θ given by

$$\sin\theta = 1.220 \frac{\lambda}{d}, \quad 2.233 \frac{\lambda}{d}, \quad 3.238 \frac{\lambda}{d}, \quad 4.241 \frac{\lambda}{d}, \ldots \tag{3}$$

If k is defined as an integer number corresponding to a dark circle, then Eqs. (3) may be written in the form

$$\sin\theta_k = A_k \frac{\lambda}{d} \tag{4}$$

with $k = 1, 2, 3, ..., \infty$. Thus, the associated coefficients A_k are:

$$(1 + 0.220), \quad (2 + 0.233), \quad (3 + 0.238), \quad (4 + 0.241), ...$$

or generally

$$A_k = (k + C_k)\,, \tag{5}$$

where $C_k = (2m_k/\pi) - k$. The subscript k is added to the value of m.

The limiting value $C_\infty = 0.25$ is obtained for $k = \infty$. Thus, according to Eqs. (3–5), it follows that $C_k + 1 = 1.220;\ 1.233;\ 1.238;\ 1.241; \ldots\ 1.25$.

A new dimensionless quantity may be introduced

$$\Delta C_k = C_k - C_1 = \frac{2(m_k - m_1)}{\pi} - (k - 1)\,. \tag{6}$$

The integer numbers k and numerical values of m_k, A_k, C_k and ΔC_k are given in Table 1. The ΔC_k dependence on k is also shown in Fig. 1. The required data for m (now denoted by m_k) are taken from Ref. [7].

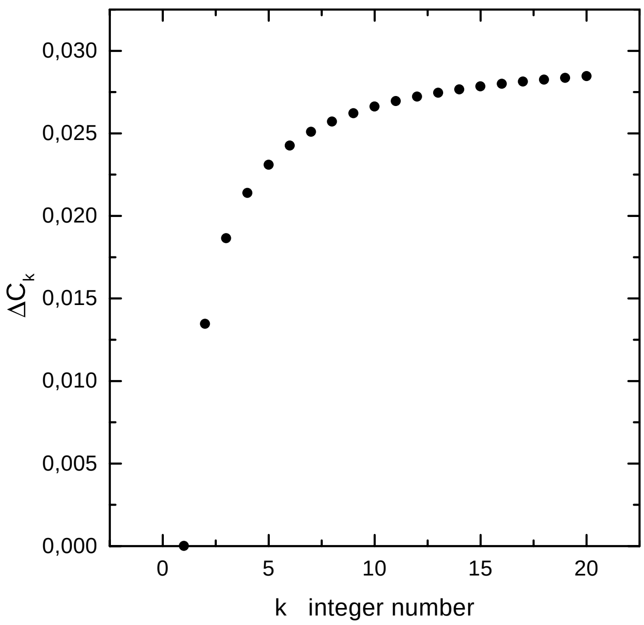

Fig. 1. Dimensionless quantity $\Delta C_k = C_k - C_1$, defined by Eq. (6), as a function of integer number k (the order of dark circle).

TABLE 1. Data for minima of diffraction by a circular aperture: k - order of dark circle, $m = m_k$ - parameter defined by Eq. (2), A_k - coefficient in Eq. (4) for the diffraction angle θ_k, C_k - quantity defined by Eq. (5), ΔC_k - quantity defined by Eq. (6).

k	m_k	$A_k = 2m_k/\pi$	$C_k = 2m_k/\pi - k$	$\Delta C_k = C_k - C_1$
1	1.91586	1.21967	0.21967	0.00000
2	3.50779	2.23313	0.23313	0.01346
3	5.08674	3.23832	0.23832	0.01865
4	6.66185	4.24106	0.24106	0.02139
5	8.23532	5.24276	0.24276	0.02309
6	9.80793	6.24392	0.24392	0.02425
7	11.38004	7.24476	0.24476	0.02509
8	12.95184	8.24539	0.24539	0.02572
9	14.52342	9.24589	0.24589	0.02622
10	16.09484	10.24629	0.24629	0.02662
11	17.66616	11.24662	0.24662	0.02695
12	19.23739	12.24690	0.24690	0.02723
13	20.80855	13.24713	0.24713	0.02746
14	22.37966	14.24733	0.24733	0.02766
15	23.95073	15.24751	0.24751	0.02784
16	25.52177	16.24766	0.24766	0.02799
17	27.09278	17.24780	0.24780	0.02813
18	28.66377	18.24792	0.24792	0.02825
19	30.23473	19.24803	0.24803	0.02836
20	31.80568	20.24812	0.24812	0.02845
∞	∞	∞	0.25000	0.03033

3. Interaction energy between two uniformly charged spheres

Consider now uniform charge distributions in vacuum, on surfaces of two spheres of equal radii R, with charges q and q' and a centre-to-centre distance b. The charges are fixed in order to avoid polarization effects. It is well-known that the energy W_V of the electric field E in a volume V is given by [8]

$$W_V = \frac{\epsilon_0}{2} \int E^2 \mathrm{d}V \,, \tag{7}$$

where ϵ_0 is the permitivity of vacuum. The resultant electric field E_r of the two charged spheres, at a given point of space, is given by

$$E_r^2 = E^2 + E'^2 + 2EE' \cos\theta_0 \,. \tag{8}$$

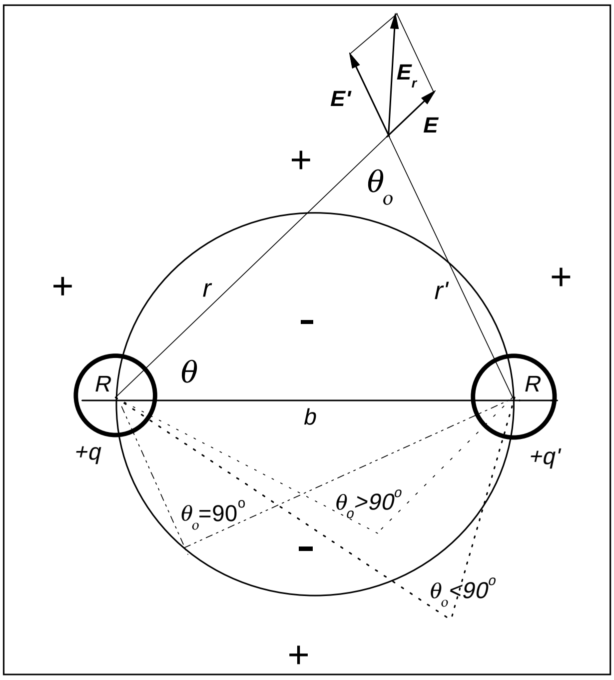

Fig. 2. Regions of positive (+) and negative (–) interaction energy between two uniform spherical charge distributions of radii R and centre-to-centre distance b. Positive charges q and q' produce electric fields $\boldsymbol{E}$ and $\boldsymbol{E}'$ at distances r and r', respectively. The negative energy is confined within the sphere of radius $b/2$, with the central point midway between the centres of the spheres.

The symbols are indicated in Fig. 2. Energy of the electric field, from Eqs.(7) and (8), is now

$$W_V = \frac{\epsilon_0}{2} \int E^2 \mathrm{d}V + \frac{\epsilon_0}{2} \int E'^2 \mathrm{d}V + \epsilon_0 \int E E' \cos\theta_0 \mathrm{d}V \,. \tag{9}$$

The first two terms are the self-energies of charges q and q' and the third term is the interaction energy which may be written as

$$W = \frac{1}{2} \frac{qq'}{4\pi\epsilon_0} \int \sin\theta \, \mathrm{d}\theta \int \frac{(r - b\cos\theta)\mathrm{d}r}{(b^2 - 2rb\cos\theta + r^2)^{3/2}} \,. \tag{10}$$

Integration of Eq. (10) over all space gives the well-known Coulomb potential energy $W = qq'/(4\pi\epsilon_0 b)$. However, Eq. (10) contains two terms: a positive one, $W^{(+)}$ and a negative one, $W^{(-)}$. It is due to the factor $\cos\theta_0$ which is negative within the sphere of radius $b/2$, with charges q and q' located at the poles of that sphere, while it is positive outside that sphere. Details of the calculation may be found in Ref.

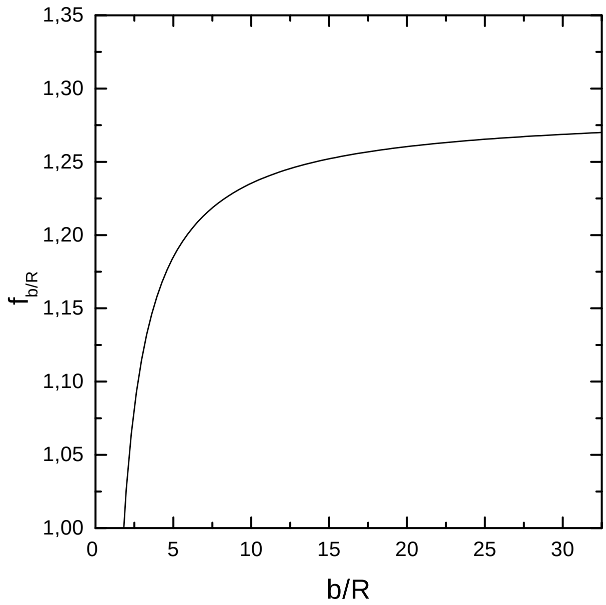

Fig. 3. Dimensionless function $f_{b/R} = W^{(+)}/W$, defined by Eq. (11), as a function of b/R, where b is the distance between two charged spheres, both of radius R (see Fig. 2).

[1]. Here we give the final expressions

$$W^{(+)} = \frac{qq'}{4\pi\epsilon_0 b}\left[\frac{1}{2} + \frac{b}{R} - \frac{\pi}{4} + \arcsin\sqrt{1 - \left(\frac{R}{b}\right)^2} - \sqrt{\left(\frac{b}{R}\right)^2 - 1}\right] = W f_{b/R}\,, \tag{11}$$

$$W^{(-)} = W(1 - f_{b/R})\,. \tag{12}$$

The dependence of the function $f_{b/R}$ on b/R is shown in Fig. 3. For $b/R = 2$, the spheres are in contact. For $b/R \to \infty$, the charged spheres may be considered as point charges; then, Eqs.(11) and (12) reduce to [1]

$$W^{(+)}_{b/R,\infty} = \left(\frac{1}{2} + \frac{\pi}{4}\right) W = (1.28539816\ldots)\, W\,, \tag{13}$$

$$W^{(-)}_{b/R,\infty} = \left(\frac{1}{2} - \frac{\pi}{4}\right) W = (-0.28539816\ldots)\, W\,. \tag{14}$$

It is shown in Ref. [1] that the coefficient 0.28539816... may be expressed as $F(F - 1)$, resulting explicitly in the factor $F = \frac{1}{2}(1 + \sqrt{\pi - 1}) = 1.2317... \approx 1.23$.

4. *Normalization procedure*

Although the properties of diffraction of light by an aperture, defined, e.g., by Eq. (6), and those of the interaction energy between two charged spheres, given by Eq. (11), have not evidently a formal mathematical analogy, an inspection of graphs in Fig. 1 and Fig. 3, where b/R is analogous to k, and $f_{b/R}$ to ΔC_k, respectively, indicate a probable similarity. It suggests that discrete points in Fig. 1, properly normalized, could be fitted to the curve in Fig. 3.

For the comparison, one has to determine the step $\Delta(b/R)$ which will be compatible with the step $\Delta k = 1$. This means that for the equidistant distribution of points along the k-axis in Fig. 1, one should find also equidistant values of the analogous quantity b/R in Fig. 3, within the interval where analogous functions exist. It is evident that the boundary values are:

$$\Delta C_1 = 0 \quad \text{at } k = 1 \quad \text{and} \quad \Delta C_\infty = 0.03033 \quad \text{at } k = \infty$$
$$f_{b/R,2} = 1.02975 \quad \text{at } b/R = 2 \quad \text{and} \quad f_{b/R,\infty} = 1.28540 \quad \text{at } b/R = \infty\,.$$

Thus, one should obtain the normalized quantity $\Delta C'_k = f_{b/R,k}$, including the boundary values, i.e.

$$\Delta C'_\infty = f_{b/R,\infty} = f_{b/R,2} + p\Delta C_\infty\,, \tag{15}$$

or

$$1.28540 = 1.02975 + p\ 0.03033\,.$$

It determines the value of the normalization factor $p = 8.42888$, resulting in

$$\Delta C'_k = 1.02975 + 8.42888\Delta C_k\,. \tag{16}$$

The values of k and $\Delta C'_k$ are listed in Table 2, columns 1 and 2.

Now, it is possible to find relationship between the quantities k and b/R. For $k = 1$, an equivalent boundary value is $(b/R)_1 = 2$. For $k > 1$, it is expected that

$$(b/R)_k = (b/R)_1 + (k-1)\Delta(b/R)\,. \tag{17}$$

For example, at $k = 20$, using Eqs. (16) and (11),

$$\Delta C'_{20} = 1.26955 = \left[\frac{1}{2} + \frac{b}{R} - \frac{\pi}{4} + \arcsin\sqrt{1 - \left(\frac{R}{b}\right)^2} - \sqrt{\left(\frac{b}{R}\right)^2 - 1}\right] = f_{b/R,20}.$$

This equation has to be solved numerically for b/R. It turns out to be $(b/R)_{20} = 31.564$, and it follows that $(b/R)_{20} = (b/R)_1 + (20-1)\Delta(b/R) = 2 + 19\Delta(b/R) = 31.564$. This defines $\Delta(b/R) = 1.556$ as an approximate value, and

$$\left(\frac{b}{R}\right)_k = 2 + 1.556(k-1)\,. \tag{18}$$

TABLE 2. Data for the normalized quantity $\Delta C'_k$ and the associated value $f_{b/R,k}$: k - order of dark circle, $\Delta C'_k$ - dimensionless quantity defined by Eq. (16), $(b/R)_k$ defined by Eq. (18), $f_{b/R,k}$ - value defined by Eqs. (11) and (18), $f_{b/R,k}$ (last column) - value defined by Eq. (11) at $(b/R)_k = m_k$ (see Table 1).

k	$\Delta C'_k$	$(b/R)_k$	$f_{b/R,k}$	$f_{b/R,k}$ $((b/R)_k = m_k)$
1	1.02975	2.000	1.02975	1.01794
2	1.14320	3.556	1.14384	1.14187
3	1.18695	5.112	1.18727	1.18678
4	1.21004	6.668	1.21027	1.21020
5	1.22437	8.224	1.22452	1.22461
6	1.23415	9.780	1.23423	1.23437
7	1.24123	11.336	1.24126	1.24143
8	1.24654	12.892	1.24659	1.24677
9	1.25076	14.448	1.25078	1.25096
10	1.25413	16.004	1.25414	1.25432
11	1.25691	17.560	1.25692	1.25709
12	1.25927	19.116	1.25923	1.25940
13	1.26121	20.672	1.26120	1.26136
14	1.26289	22.228	1.26290	1.26305
15	1.26441	23.784	1.26437	1.26452
16	1.26567	25.340	1.26566	1.26580
17	1.26685	26.896	1.26681	1.26694
18	1.26787	28.452	1.26782	1.26795
19	1.26879	30.008	1.26874	1.26886
20	1.26955	31.564	1.26955	1.26967
∞	1.285398	∞	1.285398	1.285398

The values of $(b/R)_k$ and $f_{b/R,k}$ are given in Table 2, columns 3 and 4, respectively, together with $\Delta C'_k$, column 2, calculated from Eq.(16). The dependence of the function $f_{b/R}$ on b/R (solid curve) and of $\Delta C'_k$ on k, for $\Delta k = 1$ (open circles), is shown in Fig. 4. Standard deviation of the points from the curve is 0.00015.

The values of $(b/R)_k$ in Table 2, column 3, are very close to the values of m_k in Table 1, column 2. Therefore, one may use m_k as a very good approximation of $(b/R)_k$, and calculate the corresponding values of $f_{b/R,k}$ using Eq.(11). They are listed in Table 2, column 5. However, the standard deviation increases now to 0.0013, because the first point in the graph deviates significantly. Without that point the deviation is 0.00023.

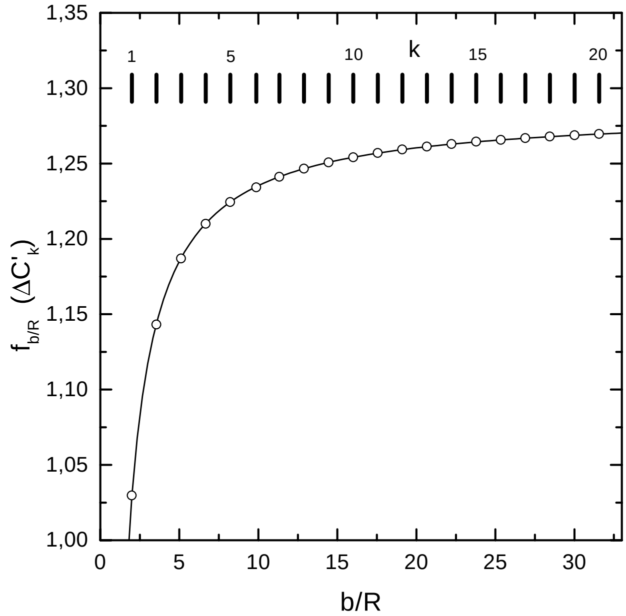

Fig. 4. Function $f_{b/R}$ versus b/R (solid curve, the same as in Fig. 3), with normalized $\Delta C'_k$ values (open circles) defined by Eq. (16). At the top of the diagram are indicated the values of $(b/R)_k$, corresponding to numbers k, according to Eq. (18).

5. Discussion and conclusion

We have shown that the coefficients A_k, defining the minima of intensity of diffracted light by a circular aperture, may be expressed, according to Eqs. (3) and (4), as in Eq. (5),

$$A_k = k + C_k, \qquad k = 1, 2, 3 \ldots, \infty \,.$$

The values of $C_k = 2m_k/\pi - k$ are limited to an interval (0.22, 0.25). The difference $\Delta C_k = C_k - C_1$ (Eq. (6)) depends on k (see Fig. 1) in a very similar way as the function $f_{b/R}$ on b/R, appearing in the interaction energy between two charged spheres (Eq. (11)) (see Figs. 2 and 3). ΔC_k has to be normalized to a new value $\Delta C'_k$, in order to achieve a numerical agreement with $f_{b/R}$. Thus, one should have

$$\Delta C'_k = f_{b/R,k} \,, \tag{19}$$

where $f_{b/R,k}$ is equal to $f_{b/R}$ at $(b/R)_k = 2 + (k-1)\Delta(b/R)$, according to Eq. (17). The above agreement is achieved by the following transformation

$$\Delta C'_k = f_{b/R,2} + (f_{b/R,\infty} - f_{b/R,2}) \frac{\Delta C_k}{\Delta C_\infty} \,, \tag{20}$$

using Eq. (15). The final relationship of A_k, obtained from Eqs. (5) and (20), is

$$A_k = k + C_k = k + C_1 + \frac{f_{b/R,k} - f_{b/R,2}}{f_{b/R,\infty} - f_{b/R,2}} \Delta C_\infty \,, \tag{21}$$

or

$$A_k - k = u + v f_{b/R,k} \,, \tag{22}$$

with

$$u = C_1 - \frac{f_{b/R,2}}{f_{b/R,\infty} - f_{b/R,2}} \Delta C_\infty \qquad \text{and} \qquad v = \frac{\Delta C_\infty}{f_{b/R,\infty} - f_{b/R,2}} \,.$$

Equation (22) in numerical form reads

$$A_k = k + 0.09750 + 0.11864 \, f_{b/R,k} \,. \tag{23}$$

The correlation of calculated values of A_k, using Eq. (23), with those A_k determined from the theory of diffraction is shown and listed in Fig. 5. The standard deviation is only 0.00002.

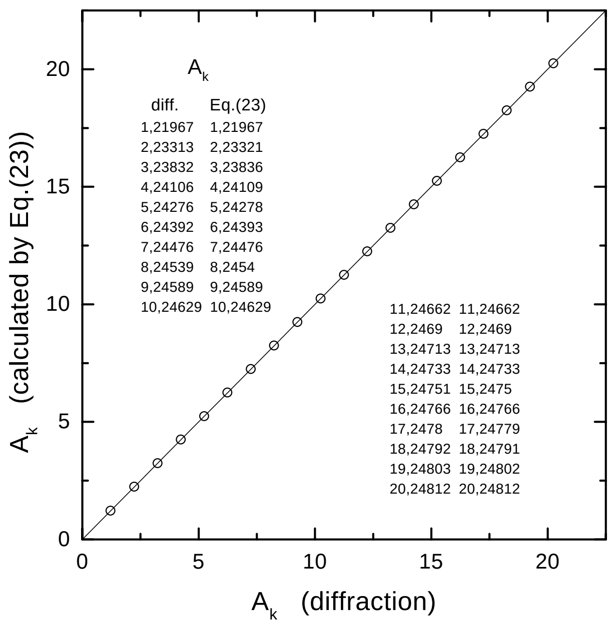

Fig. 5. Comparison of the values of coefficients A_k obtained by diffraction and those calculated by Eq. (23). The table of numerical data is also shown.

However, if one prefers to use Eq. (22) in order to find the best fit to diffraction data of the first 20 minima, then with $A_k - k = C_k$, the coefficients u and v may be determined regardless of the boundary values used above. Obviously, $f_{b/R,k}$ has to be calculated at those values of $(b/R)_k$ which will ensure the lowest standard deviation of coefficients u and v at the minimum of χ^2 value. It means that the optimum value of $\Delta(b/R)$ may be found by changing it around an approximate value 1.556, used in Eq. (18). The minimum standard deviation of A_k, calculated using Eq. (22), from the theoretical one, is obtained for $\Delta(b/R) = 1.5435$, as can be seen in Fig. 6. The new equation is

$$A_k = k + 0.097369 + 0.118762 f_{b/R,k}\,, \tag{24}$$

reproducing the diffraction coefficient A_k with standard deviation of only 7×10^{-6}.

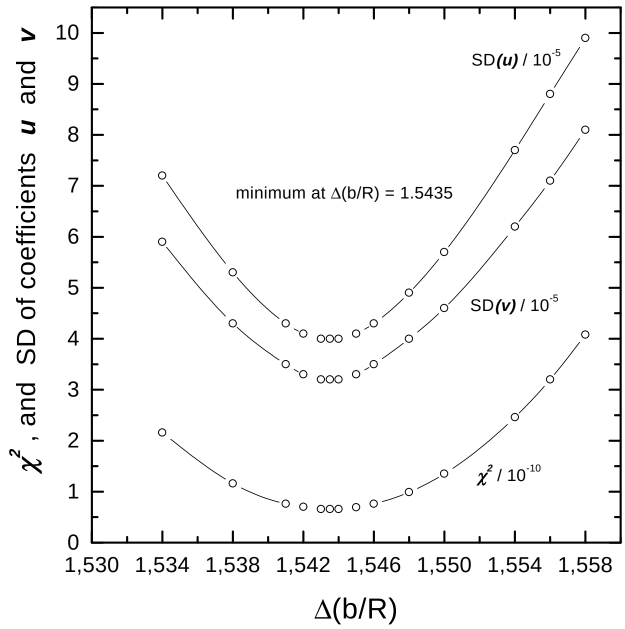

Fig. 6. Standard deviation (SD) of coefficients u and v in Eq. (22) and the χ^2 values, as the results of the best fit of A_k for particular values of $\Delta(b/R)$. The optimum value of $\Delta(b/R) = 1.5435$ is determined from the minima of the curves.

From Eq. (24), with $(A_k - k) = C_k$ (Eq. (5)), it follows that at $k = \infty$

$$f_{b/R,\infty} = 8.420202\, C_\infty - 0.819867\,.$$

Due to $C_\infty = 0.25$, it results in $f_{b/R,\infty} = 1.28540$, equal to the coefficient of the positive interaction energy term between two point charges defined by Eq. (13), as it must be according to performed normalization procedure. Thus, $1 + F(F-1) = 1.28540$, i.e., $F(F-1) = 0.28540$, leading again to $F = 1.2317 \approx 1.23$, supports the factor F also in the case of diffraction phenomena.

It is instructive to consider a modified system of two uniformly charged spherical distributions in which one of them is replaced by a point charge. The system is treated in Ref. [1]. Here, it is sufficient to give the final results. The positive interaction energy term is $W^{(+)} = (qq'/(4\pi\epsilon_0 b))f_{b/R}$, using the same symbols as before. However, $f_{b/R}$ has now slightly different form when compared to that in Eq. (11), i.e.,

$$f_{b/R} = \frac{1}{2}\left[\frac{1}{2} + \frac{b}{R} + \arcsin\sqrt{1-\left(\frac{R}{b}\right)^2} - \sqrt{\left(\frac{b}{R}\right)^2 - 1}\right]. \qquad (25)$$

Using again the boundary values:

$\Delta C_1 = 0$	at $k = 1$	and	$\Delta C_\infty = 0.03033$	at $k = \infty$
$f_{b/R,1} = 1$	at $b/R = 1$	and	$f_{b/R,\infty} = 1.28540$	at $b/R = \infty$

one obtains the normalization equation

$$\Delta C'_k = 1 + 9.409759\Delta C_k\,. \qquad (26)$$

By using Eq. (26) at $k = 20$, one obtains $\Delta C'_{20}$ which has to be equal to $f_{b/R,20}$ for $(b/R)_{20}$. Using Eq.(17), this value determines the step $\Delta(b/R)$ equal to 0.691.

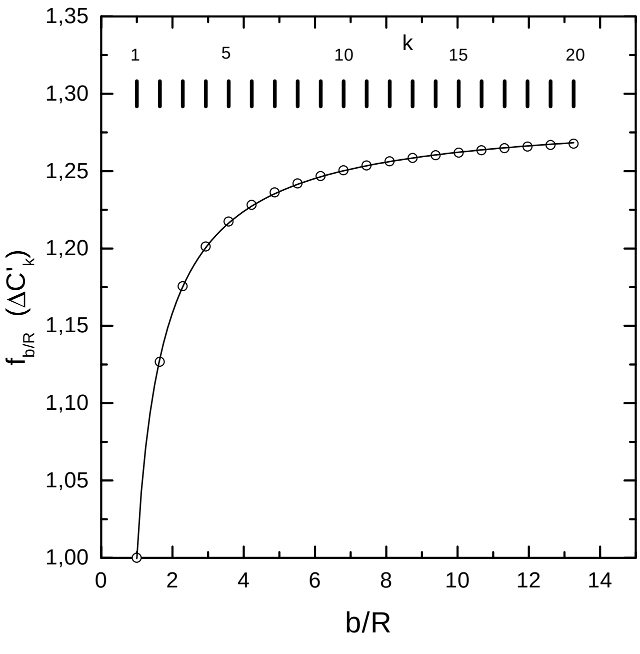

Fig. 7. The values $f_{b/R}$ versus b/R (solid curve) for the system of charged sphere and a point charge, with normalized values $\Delta C'_k$ (open circles). The optimum value $\Delta(b/R) = 0.645$, corresponding to $\Delta k = 1$, has been used.

However, if Eq. (22) is used together with $A_k - k = C_k$, and all values C_k are subjected to the fitting procedure, then the optimum value becomes $\Delta(b/R) = 0.6450$. Using this optimum value, an agreement of $\Delta C'_k$ with $f_{b/R,k}$ is determined at the same accuracy as that for the system of two uniformly charged spheres. The associated points and the curve are presented in Fig. 7.

For two charged spheres, the corresponding optimum value is found to be $\Delta(b/R) = 1.5435$. It is interesting to note that 1/1.5435=0.6479 is very close to 0.6450. Thus, the optimum values of $\Delta(b/R)$ are related by $\Delta(b/R)_{\text{sphere}-\text{sphere}} \approx 1/\Delta(b/R)_{\text{sphere}-\text{point}}$, for the electrostatic systems of two charged spheres and a charged sphere and a point charge. At present, there is no explanation for that correlation, and it may be just a coincidence. Moreover, if the values of $\Delta(b/R)$ are determined by the values associated to $k = 20$ for both systems, then 1.556 is not close to 1/0.691=1.447. However, from the physical point of view, the optimum values are more acceptable, because the intensity of bright circles of diffracted light falls off very quickly with increasing order of diffraction. The central bright spot contains 84% of the light flux incident on the aperture, the first bright circle only 1.74%, the second 0.41%, etc. In this way, 20 bright circles (or dark circles) are sufficient to be included in the first approximation for the calculations presented in this work. Of course, it should be pointed out that deeper physical reasons for a correlation of diffraction of light by a circular aperture and the interaction energy of electrostatic systems still remain to be revealed. Investigations in that direction are in course.

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POVEZANOST OGIBA SVJETLOSTI KRUŽNIM OTVOROM I ENERGIJE MEĐUDJELOVANJA DVIJU BLIZIH NABIJENIH KUGLI

Pokazuje se da se koeficijenti A_k kutova θ_k, koji odgovaraju minimumima jakosti svjetlosti u difrakciji na kružnom otvoru, mogu uspješno opisati funkcijom koja se javlja u elektrostatskoj energiji međudjelovanja između dviju jednoliko površinski nabijenih kugli. Koeficijent A_k određuje kut θ_k relacijom $\sin\theta_k = A_k\lambda/d$, $k = 1, 2, 3, \ldots$, gdje je λ valna duljina svjetlosti a d promjer otvora. Ti se koeficijenti mogu povezati s bezdimenzijskom funkcijom $f_{b/R}$, koja se javlja u izrazu za energiju međudjelovanja dviju kugli jednakih polumjera R, naboja q i q', a nalaze se na udaljenosti b. Ukupna energija međudjelovanja, tj. Coulombova potencijalna energija, iznosi $W = qq'/(4\pi\epsilon_0 b)$, gdje je ϵ_0 permitivnost vakuuma. Ta energija sadrži dva dijela, pozitivan $W^{(+)} = W f_{b/R}$ i negativan $W^{(-)} = W(1 - f_{b/R})$. Povezanost A_k i funkcije $f_{b/R}$ dana je relacijom $A_k - k = u + v f_{b/R,k}$, gdje su u i v konstante. Koeficijenti A_k dobiveni tom relacijom slažu se s onima određenim difrakcijom uz odstupanje od 10^{-3}% za $k = 1$ koje se postupno smanjuje na 10^{-4}% za $k = 10$.