ISSN 1330–0008
CODEN FIZAE4

COUPLING OF MICROPULSATION WAVE MODES BY THE CORIOLIS FORCE AND CENTRIFUGAL FORCE IN THE EARTH'S EQUATORIAL PLANE

S. S. DE, S. K. DUBEY and M. DE

Centre of Advanced Study in Radio Physics and Electronics, University of Calcutta, Calcutta 700 009, INDIA

Received 5 March 2001; revised manuscript received 10 May 2001
Accepted 5 September 2001 Online 25 May 2002

Coupling of micropulsation wave modes by the Coriolis force and centrifugal force in the Earth's equatorial plane has been considered. The analytical form of electric field is derived for a particular coupled wave mode. To study the variation of electric field with geocentric distance, numerical analyses have been made which are presented graphically.

PACS numbers: 94.30.Ms, 94.20.Vv UDC 537.87

Keywords: Earth's equatorial plane, ionospheric disturbances, Coriolis force and centrifugal force, micropulsations, magnetic pulsations

1. *Introduction*

A remarkable variety of magnetic disturbances are observed as quasi-sinusoidal at the Earth's surface and on board of satellites by sensitive magnetometers. These are geomagnetic micropulsations, various types of which appear in a wide range from audio to very low frequency. Some of them are highly damped.

Solar streamers, geomagnetic micropulsations and ionospheric shock waves are generally discussed on the basis of the theory of outer atmospheric oscillations. As the frequency range corresponding to geomagnetic micropulsation is far below the gyrofrequency of positive ions, the magnetohydrodynamic waves are obtained in the ionised gas.

In connection with the studies of long-period micropulsations, dimensionless resonant frequencies of hydromagnetic modes have been calculated for a simple model plasmasphere including a lower ionosphere [1,2]. It has been concluded that the definite form of coupling between adjacent magnetic shells in the Alfvén mode depends on the ratio of the Hall conductivity to the Pedersen conductivity, known as Hall coupling. It is established that magnetospheric resonance and hydromagnetic (HM) waves are a cause of certain types of geomagnetic micropulsations. The damping of magnetospheric motions and HM wave propagations in the ionosphere are very important in the study of ionospheric roles during both quiet and disturbed periods. There are various damping mechanisms which may occur for geomagnetic pulsations. Hughes and Southwood [3] showed that such mechanism could be the coupling between adjacent flux tubes in the magnetosphere due to a compression of the magnetic field lines or due to mode coupling via the Hall conductance in the ionosphere. There could also be energy loss from the hydromagnetic wave modes to the plasma populations in the magnetosphere.

The investigation of magnetic pulsation is used as the indirect diagnostics of magnetospheric and solar wind conditions. Development of such an ability leads to a better physical understanding of the processes involved in the solar-wind–magnetosphere interaction. However, magnetic pulsations are attenuated and modified as they propagate through the ionosphere.

It is known that certain geomagnetic micropulsations could be hydromagnetic resonances [4,5]. Near the Earth, in the equatorial plane, Coriolis force and centrifugal force have important effects in the process of coupling of micropulsation wave modes.

Micropulsation activity obtained in the equatorial plane of the Earth is very much dependent on the time in the solar cycle and seasonal effect. These are also controlled by overall level of magnetic activity and controlled by particles and electromagnetic radiations emitted by the Sun. Micropulsation modes are known to be related to geomagnetic storm activity and have been connected with storm sudden commencement (SSC), storm sudden impulses (SSI) and low-frequency oscillations (Bays).

Various results of micropulsation observations in different period ranges have been summarised through statistically analysed data of different latitudes. The amplitude and periods of micropulsation activities from magnetograms are analysed, power spectra of these occurrences have been computed by means of rubidium-vapour magnetometer measuring the total field [6]. Statistically stable average spectra are observed from data collected from magnetograms and also through different geostationary satellites.

Hydromagnetic waves in the period range 150 to 600 s (PC5) may be produced by the interaction of the solar wind with the magnetospheric volume. Particularly, near the dawn and dusk meridians, the turbulence-modified solar wind at the magnetopause produces tangential stress which can give rise to waves generated by the Kelvin-Helmholtz instability. An increase in the solar wind pressure causes compression of the magnetosphere, observed as a sudden commencement (SC) on a magnetogram, and as a result, certain parts of the magnetosphere oscillate due to the impulse.

Some workers [7–10] have noted that amplitudes of waves in the period range 40 to 150 s (Pi2) are maximal in the auroral zone, although some of the later data indicate a sub-auroral maximum amplitude [11]. Apart from times of very low magnetic activity ($K_p = 0$), there are some records which show that the waveforms and power spectra of Pi2, observed in the auroral zone, are different from those observed simultaneously at mid and low latitudes [12]. Supporting evidence for this latitude dependence is known [11]. Usually a predominant period can be found in the wave-trains observed at mid and low latitudes, while the waveforms of auroral zone Pi2 are irregular. It has been concluded that power spectra are of a random-noise type [12].

In the auroral regions, charged particles are accelerated to very high energies by electric fields parallel to the magnetic field. The effect of parallel electric field has a stabilising effect on the electrostatic ion cyclotron instability and reduces the energy of resonant particles [13].

It is found that the gradual growth of wave amplitude with time initiates a convective plasma instability within the magnetosphere rather than an externally applied impulse. This event was detected as an almost pure transverse wave at college, Alaska, which is quite close to the geomagnetic field line that links ATS 1 satellite to the Earth's surface [14].

Experimentally, many micropulsation studies have suggested that the magnetospheric system has certain resonances through the frequency of occurrence [15]. Under resonant condition, during wave-particle interaction, particles can lose energy to the wave so that it is amplified. Particles can also gain energy from the wave giving thereby Landau damping. Also amplification occurs within the plasmasphere [16]. The electric field plays an important role in the dynamics of the plasma in the plasmasphere as well as in the magnetosphere and in the solar wind.

In this paper, some theoretical investigations are carried out in the space near the Earth in the equatorial plane to determine the electric field and the influence of Coriolis force and centrifugal force in the process of coupling of micropulsation wave modes. The expression of electric field in the equatorial plane has been deduced for a specific wave mode, considering the influence of Coriolis force and centrifugal force. Numerical analyses are made for the coupled wave mode to study the variation of electric field with geocentric distance.

2. *Mathematical formulation*

The equations giving the dynamics are chosen as

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}\,, \tag{1}$$

$$\nabla \times \vec{H} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi e}{c}\,(n_i \vec{v}^{(1)} - n_i \vec{v}^{(2)})\,, \tag{2}$$

$$\nabla \cdot \vec{E} = 4\pi e\,(n_i - n_e)\,, \tag{3}$$

$$\nabla \cdot \vec{H} = 0\,. \tag{4}$$

The equations of motion can be written as

$$m^{(1)}\left[\frac{\partial \vec{v}^{(1)}}{\partial t}+2\vec{\omega}_0\times\vec{v}^{(1)}+\vec{\omega}_0\times(\vec{\omega}_0\times\vec{r})\right]=e[\vec{E}+\vec{v}^{(1)}\times\vec{B}_0]\,, \tag{5}$$

$$m^{(2)}\left[\frac{\partial \vec{v}^{(2)}}{\partial t}+2\vec{\omega}_0\times\vec{v}^{(2)}+\vec{\omega}_0\times(\vec{\omega}_0\times\vec{r})+(\vec{v}^{(2)}\cdot\nabla)\vec{v}^{(2)}\right]=-e[\epsilon\vec{E}+\vec{v}^{(2)}\times\vec{B}_0]\,, \tag{6}$$

where the angular velocity of the Earth $\vec{\omega}_0$ has components $(0,0,\omega_0)$, $\vec{B}_0$ is the magnetic field with components $(0,0,B_0)$, $\vec{r}$ is the radius vector from the centre of the Earth, $\vec{E}$ the induced electric field, $\vec{H}$ the induced magnetizing field, e the electronic charge, c the velocity of light in free space, ϵ the electrical permittivity of the medium, $m^{(1)}$ the mass of the ion, $m^{(2)}$ the mass of the electron, n_i the ion number density, n_e the electron number density, $\vec{v}^{(1)}$ the velocity of the ion, and $\vec{v}^{(2)}$ is velocity of the electron. Here, superscript 1, and superscript 2 are for the ions and electrons, respectively. One can express

$$\vec{v}_{\perp}^{(s)}=\vec{v}_{x}^{(s)}+j\vec{v}_{y}^{(s)}\qquad \vec{v}_{\parallel}^{(s)}=\vec{v}_{z}^{(s)}\hat{e}_3,\quad (s=1,2)$$

where, $\hat{e}_3=\vec{B}_0/|\vec{B}_0|$. The suffixes $\perp$ and $\parallel$ are used to denote vectors perpendicular and parallel to $\vec{B}_0$, respectively. For any vector $\vec{\rho}$ which is perpendicular to $\vec{B}_0$,

$$\hat{e}_3\times\vec{\rho}=j\vec{\rho}\,. \tag{7}$$

The electric current can be written as

$$\vec{J}=e(n_i\vec{v}^{(1)}-n_i\vec{v}^{(2)})\,. \tag{8}$$

Assuming time dependence of the form $\mathrm{e}^{\mathrm{i}\omega t}$, one obtains

$$\nabla\times(\nabla\times\vec{E})=\frac{\omega^2}{c^2}\,\vec{E}-\frac{4\pi\mathrm{i}\omega}{c^2}\,\vec{J}\,, \tag{9}$$

$$\vec{v}_{\parallel}^{(1)}=-\frac{\mathrm{i}}{\omega_m^{(1)}}\,(e\vec{E}_{\parallel}+m^{(1)}\omega_0^2\vec{r}_{\parallel})\,, \tag{10}$$

$$\vec{v}_{\parallel}^{(2)}=\frac{\mathrm{i}}{\omega_m^{(2)}}\,(e\epsilon\vec{E}_{\parallel}-m^{(2)}\omega_0^2\vec{r}_{\parallel})\,, \tag{11}$$

$$\vec{v}_{\perp}^{(1)}=\frac{1}{m^{(1)}}\,\frac{e\vec{E}_{\perp}+m^{(1)}\omega_0^2\vec{r}_{\parallel}}{\mathrm{i}\omega+j(2\omega_0+\omega_1)}\,, \tag{12}$$

$$\vec{v}_{\perp}^{(2)}=\frac{1}{m^{(2)}}\,\frac{-e\epsilon\vec{E}_{\perp}+m^{(2)}\omega_0^2\vec{r}_{\parallel}}{\mathrm{i}\omega+j(2\omega_0+\omega_1)}\,, \tag{13}$$

$$\vec{J}_{\|} = -\frac{\mathrm{i}e}{\omega B_0}\,(n_i\omega_1 + \epsilon n_e\omega_2)\vec{E}_{\|} - \frac{\mathrm{i}e\omega_0^2}{\omega}(n_i - n_e)\vec{r}_{\|}\,, \tag{14}$$

$$\vec{J}_{\perp} = \left[\frac{\mathrm{i}\omega}{\omega_1} + j\left(\frac{2\omega_0}{\omega_1} - \frac{\omega^2}{\omega_1^2}\right)\right]\left[\frac{e}{B_0}\,(n_i + \epsilon n_e)\vec{E}_{\perp} + \frac{\omega_0^2}{B_0}\,(n_i m^{(1)} - n_e m^{(2)})\vec{r}_{\|}\right]\,, \tag{15}$$

where $\omega_1 = eB_0/m^{(1)}$ and $\omega_2 = eB_0/m^{(2)}$. Writing

$$(\vec{E})_{\perp} = E_x + jE_y \qquad \text{and} \qquad \vec{E}_{\|} = E_z\hat{e}\,, \tag{16}$$

one obtains

$$[\nabla\times(\nabla\times\vec{E})]_{\perp} = [\alpha E_x + \mathrm{i}\beta E_y + \gamma z] + j[\alpha E_y - \mathrm{i}\beta E_x + \mathrm{i}z\delta]\,, \tag{17}$$

where

$$[\nabla\times(\nabla\times\vec{E})]_{\|} = \left[\frac{\epsilon\omega^2}{c^2} - \frac{4\pi e}{c^2 B_0}(n_i\omega_1 + \epsilon n_e\omega_2)\right]E_z - \frac{4\pi e\mathrm{i}\omega_0^2}{c^2}\,(n_i - n_e)z\,, \tag{18}$$

with

$$\alpha = \frac{\omega^2}{c^2}\left[1 + \frac{4\pi e}{c^2 B_0}\,(n_i + \epsilon n_e)\right]\,,$$

$$\beta = \frac{\omega^2}{c^2}\left[\frac{4\pi e}{B_0\omega\omega_1}\,\left(2\omega_0 - \frac{\omega^2}{\omega_1}\right)(n_i + \epsilon n_e)\right]\,,$$

$$\gamma = \frac{\omega^2}{c^2}\left[\frac{4\pi\omega_0^2}{B_0\omega_1}\,(n_i m^{(1)} - n_e m^{(2)})\right]\,,$$

$$\delta = \frac{\omega^2}{c^2}\left[\frac{4\pi\mathrm{i}\omega_0^2}{B_0\omega_1}\,\left(2\omega_0 - \frac{\omega^2}{\omega_1^2}\right)(n_i m^{(1)} - n_e m^{(2)})\right]\,.$$

For a plane wave, field components depend only on the coordinate z. Hence from (17), one can derive the following two equations

$$\frac{\mathrm{d}^2 E_x}{\mathrm{d}z^2} + \alpha E_x + \mathrm{i}\beta E_y = \gamma z\,, \tag{19}$$

$$\frac{\mathrm{d}^2 E_y}{\mathrm{d}z^2} + \alpha E_y - \mathrm{i}\beta E_x = \mathrm{i}z\delta\,. \tag{20}$$

From (18), the solution for the z-component of the electric field can be deduced as

$$E_z = \frac{4\pi e\omega_0^2(n_i - n_e)B_0 z}{\epsilon\omega^2 B_0 - 4\pi e(n_i\omega_1 + \epsilon n_e\omega_2)}\,. \tag{21}$$

Assuming further

$$F_+ = E_x + \mathrm{i}E_y, \qquad F_- = E_x - \mathrm{i}E_y \,, \tag{22}$$

one can get

$$\frac{\mathrm{d}F_+}{\mathrm{d}z^2} + (\alpha+\beta)F_+ = -(\gamma-\delta)z \,, \tag{23}$$

$$\frac{\mathrm{d}F_-}{\mathrm{d}z^2} + (\alpha-\beta)F_+ = -(\gamma+\delta)z \,. \tag{24}$$

The dispersion relation has been derived as

$$K^2 = \frac{\omega^2}{c^2} + \frac{4\pi e\omega^2}{c^2 B_0\omega_1}(n_i + \epsilon n_e) \pm \frac{4\pi e\omega}{c^2 B_0 \omega_1}\left(2\omega_0 - \frac{\omega^2}{\omega_1}\right)(n_i+\epsilon n_e) = \alpha \pm \beta \,. \tag{25}$$

The nature of the dispersion can be known from the roots of (25). The possible combinations of K would be (K_1, K_2), (K_1, K_4), (K_2, K_3) and (K_3, K_4), where $K_1 = +\sqrt{\alpha+\beta}$, $K_2 = +\sqrt{\alpha-\beta}$, $K_3 = -\sqrt{\alpha+\beta}$ and $K_4 = -\sqrt{\alpha-\beta}$.

When $\beta = 0$, one obtains $K_1 = K_2$, $K_3 = K_4$ and $\omega^2 = \omega_0\omega_1$, and there will be no coupling of different micropulsation wave modes.

For the (K_1, K_2) mode of the dispersion relation, the field components are obtained as

$$\begin{aligned} E_x = \; & \frac{z}{2\beta}\Big[\big\{(\cos K_1 z + \cos K_2 z)(\gamma\cos K_1 z + \delta\cos K_2 z) \\ & -(\sin K_1 z - \sin K_2 z)(\gamma \sin K_1 z - \delta \sin K_2 z) - p\beta\big\}^2 \\ & +\big\{(\cos K_1 z - \cos K_2 z)(\gamma\cos K_1 z - \delta\cos K_2 z) \\ & +(\sin K_1 z + \sin K_2 z)(\gamma\sin K_1 z + \delta\sin K_2 z)\big\}\Big]^{1/2} , \end{aligned} \tag{26}$$

$$\begin{aligned} E_y = \; & \frac{z}{2\beta}\Big[\big\{(\cos K_1 z + \cos K_2 z)(-\gamma\sin K_1 z + \delta\sin K_2 z) \\ & +(\sin K_1 z - \sin K_2 z)(\gamma\cos K_1 z + \delta\cos K_2 z)\big\} \\ & +\big\{(\cos K_1 z - \cos K_2 z)(\gamma\sin K_1 z + \delta\sin K_2 z) \\ & -(\sin K_1 z + \sin K_2 z)(\gamma\cos K_1 z - \delta\cos K_2 z) - q\beta\big\}^2\Big]^{1/2} , \end{aligned} \tag{27}$$

$$E_z = \frac{z}{2\beta}\,\frac{8\pi e\beta\omega_0^2(n_i - n_e)B_0}{\epsilon\omega^2 B_0 - 4\pi e(n_i\omega_1 - \epsilon n_e\omega_2)} \,, \tag{28}$$

where

$$p = \frac{2(\alpha\gamma+\beta\delta)}{\alpha^2-\beta^2} \,, \qquad q = \frac{2(\beta\gamma+\alpha\delta)}{\alpha^2-\beta^2} \,.$$

The resultant field can been derived from the above field components as

$$|E| = [E_x^2 + E_y^2 + E_z^2]^{1/2} \,. \tag{29}$$

Similarly, for (K_1, K_4), (K_2, K_3) and (K_3, K_4), the resultant fields can be calculated.

3. Results and discussion

Power spectra derived from the micropulsation activity are complex. They reveal certain peaks which are localised in space. This suggests that the energy has propagated in a mode which is guided. Other spectral peaks have a maximum amplitude at a particular latitude which can be observed over a wide range of latitude and longitude, which suggests that the propagation has been in a mode which is more nearly isotropic.

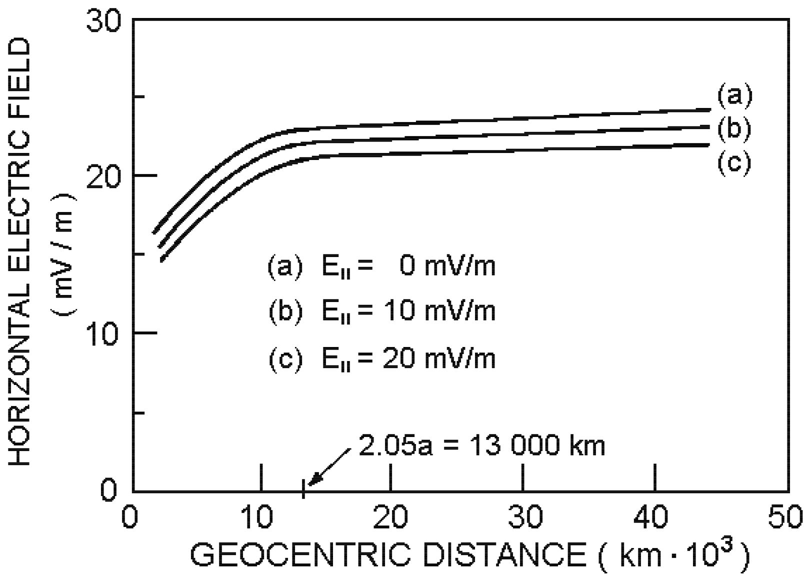

Fig. 1. Variation of the horizontal electric field with geocentric distance for different values of $E_\parallel$.

In the equatorial plane, the influence of the Coriolis force and centrifugal force in the coupling of micropulsation wave modes can be obtained from the study of the dispersion relation (25). The electric field for the (K_1, K_2) mode at different geocentric distances has been evaluated from (29). The results are shown in Fig. 1. Numerical analyses reveal that $E_\parallel$ is very small in comparison to $E_\perp$. It is also found that for micropulsations in the range of the period 21 to 42 seconds, these forces are important within a distance $2.05\,a$ (approximately) from the Earth's centre. Here a is the radius of the Earth. The results of analysis reveal that for certain periods and magnetic field strengths, coupling of different micropulsation wave modes will be absent.

References

[1] W. Allan and F. B. Knox, Planet. Space Sci. **27** (1979) 79.

[2] A. H. Craven and J. A. Lawrie, Planet. Space Sci. **27** (1979) 211.

[3] W. J. Hughes and D. J. Southwood, J. Geophys. Res. **81** (1976) 3234.

[4] J. W. Dungey, Ionos. Res. Lab. Pennsylvania State Univ. Sci. Rep. No. 69 (1954).

[5] T. Tamao, Rep. Ionos. Space Res. Japan **18** (1964) 16.

[6] T. J. Herron, J. Geophys. Res. **72** (1967) 759.

[7] J. A. Jacobs and K. Sinno, J. Geophys. Res. **65** (1960a) 107.

[8] J. A. Jacobs and K. Sinno, Geophys. J. **3** (1960b) 333.

[9] O. M. Raspopov, Geomag. and Aeron. **8** (1968) 257.

[10] L. T. Afanasyeva, O. M. Raspopov, R. V. Shchepetnov, V. K. Koshelevskiy and M. D. Nazarov, Geomag. and Aeron. **10** (1970) 600.

[11] J. C. Gupta and R. J. Stenning, Can. J. Phys. **49** (1971) 2338.

[12] H. Fukunishi and T. Hirasawa, Rep. Ionos. Space Res. Japan (1970) 24.

[13] V. K. Bajaj and M. S. Tiwari, Indian J. of Radio and Space Phys. **21** (1992) 53.

[14] L. J. Lanzerotti and N. A. Tartaglia, J. Geophys. Res. **77** (1972) 1934.

[15] T. Saito, Sci. Rep. Tohoku Univ., fifth series, Geophysics. **14** (1962) 81.

[16] R. Gendrin, Space Sci. Rev. **11** (1970) 54.

VEZANJE MIKROPULSNIH VALNIH MODOVA CORIOLISOVOM I CENTRIFUGALNOM SILOM U EKVATORIJALNOJ RAVNINI ZEMLJE

Razmatramo vezanje mikropulsnih valnih modova Coriolisovom i centrifugalnom silom u ekvatorijalnoj ravnini Zemlje. Izveli smo analitički izraz za električno polje posebnih valnih modova. Za proučavanje promjena električnog polja s geocentričnom udaljenošću, načinili smo numeričke račune i ishode prikazali grafički.