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MODULATED ION-ACOUSTIC WAVE NEAR CRITICALNESS – A NEW NONLINEAR SCHRÖDINGER EQUATION IN A COLLISIONAL PLASMA

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Modulation of ion-acoustic waves in a slowly responding collisional plasma is analysed for some critical values of plasma parameters, angle of propagation, collisional frequency, ion-temperature and percentage of fast particles in the plasma. Electrons, which form the background, are assumed to be nonthermal. It is observed that at these critical values, the nonlinear ion-acoustic wave is described by the new form of nonlinear Schrödinger (NLS) equation containing higher-order and derivative-type nonlinearity. The condition for modulational stability is derived for some critical values of the plasma parameters, and lastly the solitary solution of the NLS equation is obtained and is found to vary somewhat for different critical values.

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1. Introduction

The study of modulational stability of ion-acoustic waves in a dispersive and weakly nonlinear plasma has been a topic of important research over the last few decades (e.g., see Refs. [1] and [2]). By incorporating the effects of harmonic generation and ponderomotive nonlinearities, many authors have derived the nonlinear Schrödinger (NLS) equation which governs the dynamics of nonlinear ion-acoustic wave packet [3,4]. Nonlinear parallel modulation of ion-acoustic waves in an unmagnetized plasma due to nonlinear interaction with slow-response quasi-static plasma was studied by Sukla [5]. The magnetized case was analysed by Bharathram and Sukla [6], Bharathram [7], and Alam and Roy Chowdhury [8]. The case of oblique

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modulation was studied by Chhabra [9]. Almost three decades ago, Hasegawa [10] pointed out that the ion-acoustic wave at a frequency smaller than the ion gyro frequency ($\Omega_\mathrm{i} = eB_0/(m_\mathrm{i}c)$) can decay due to the coupling with other tranverse effects. It was observed that the stability also depends on the angle of propagation. In this context, it can be mentioned that Vladimirov and Yu [11,12] studied the effect of collision on the evolution of ion-acoustic wave and Langmuir excitations in a different way. In our case we have introduced the collisions in a simple fashion.

In this paper, we study the effect of higher-order nonlinearity on the propagation of ion-acoustic wave in a slowly responding quasi-static plasma in the presence of nonthermal electrons and collisions in a constant magnetic field. The electrons are assumed to be nonthermal to take into account the effect of fast particles in the plasma [13,8]. The necessity for such considerations comes from Freja, Viking and other satellite observations [14,15] which were launched for measurements of various plasma parameters in space. These experiments were performed through the collaboration of NASA and ESA. We further assume the propagation to be at an angle to the magnetic field. The whole system is seen to behave in a very different way near a critical situation specified by special values of the angle of propagation (θ), the collision frequency (ν), the ion-temperature (T_i) and the percentage of fast particles β. The nonlinear equation so derived is a new kind of NLS equation involving higher nolinear term $\mid \psi \mid^2 \psi$ and also nonlinearities containing derivative of ψ. We then analyse the modulational stability by taking recourse to linearization and also obtain the exact form of the solitary wave. However, one should keep in mind the difference between our derivation of the new NLS equation and of the usual modified KdV case. Here our critical parameter values are fixed from completely different considerations. On the other hand, from the analysis of the linear dispersion relation, we have seen the growth and decay of the wave which explicitly shows the dependence of the cut-off on the values of β, the percentage of fast particles inside the plasma.

2. *Formulation*

We assume that the plasma consists of positive ions and electrons and that a hydrodynamic description is possible. The electrons form the background and are assumed to be nonthermal. Such a distribution of electrons was used by Mamun and Cairns [13] and by Alam and Roy Chowdhury [8] to take into account the effect of fast particles in the plasma. We also assume that there is a constant magnetic field $\vec{B}_0$ oriented along the z-axis. The equations governing the plasma can be written as

$$\frac{\partial n_\mathrm{i}}{\partial t} + \nabla \cdot (n_\mathrm{i} \vec{v}_\mathrm{i}) = 0\,, \tag{1}$$

$$\frac{\partial \vec{v}_\mathrm{i}}{\partial t} + (\vec{v}_\mathrm{i} \cdot \nabla)\vec{v}_\mathrm{i} = -\nabla\phi + \frac{\Omega_\mathrm{i}}{\omega_\mathrm{pi}}(\vec{v}_\mathrm{i} \times \vec{z}) - \frac{5}{2} T\,\nabla(n^{2/3}) + \frac{\nu}{\omega_\mathrm{pi}}\vec{v}_\mathrm{i}\,, \tag{2}$$

$$\nabla^2\phi = n_\mathrm{e} - n_\mathrm{i}\,, \tag{3}$$

$$n_\mathrm{e} = (1 - \beta\phi + \beta\phi^2)\exp(\phi)\,, \tag{4}$$

where Ω_i and ω_pi are, respectively, the ion-gyro and ion-plasma frequency, $T = T_\mathrm{i}/T_\mathrm{eff}$, T_eff is the constant temperature of free electrons and ν the collision frequency. n_i and n_e denote the ion and electron density, respectively, and $\vec{v}_\mathrm{i}$ the corresponding velocity of ions. Furthermore, the ions are considered hot and the electron distribution is assumed to be a nonthermal one, as used previously in works of Mamun and Cairns [13] and by Alam and Roy Chowdhury [8]. All physical quantities appearing in the above equations have been normalized by the following quantities:

The densities n_i and n_e by the unperturbed electron plasma density n_0, the velocity v_i by $c_s = \sqrt{T_\mathrm{eff}/m_\mathrm{i}}$, the potential ϕ by T_eff/e and length and time, respectively, by the effective Debye length $\lambda_\mathrm{D\ eff} = \sqrt{T_\mathrm{eff}/(4\pi n_0 e^2)}$ and the ion-plasma time period $\omega_\mathrm{pi}^{-1} = \sqrt{m_\mathrm{i}/(4\pi n_0 e^2)}$. Normalised field variables $n_\mathrm{j}, v_\mathrm{j}, \phi$ are all separated into their low- and high-frequency parts

$$n_\mathrm{j} = 1 + n_\mathrm{j}^\mathrm{l} + n_\mathrm{j}^\mathrm{h}\ \ ,\vec{v}_\mathrm{j} = \vec{v}_\mathrm{j}^{\,\mathrm{l}} + \vec{v}_\mathrm{j}^{\,\mathrm{h}}\ \ ,\phi = \phi^\mathrm{l} + \phi^\mathrm{h}\,,\quad \mathrm{j}\ =\ \mathrm{e},\ \mathrm{i}\,, \tag{5}$$

where the superscripts l and h stand, respectively, for low- and high-frequency components. We next assume that the higher-order derivatives of the low-frequency terms are negligible along with the fact that the average of high-frequency terms contributes only to the low-frequency terms. This type of approximation was used by Thornhill and Ter Haar [16] and others previously. Substituting (5) into Eqs. (1) to (4) and separating high- and low-frequency parts (keeping terms up to the 4^th order), we get

$$n_\mathrm{e}^\mathrm{h} = (1-\beta)\phi^\mathrm{h} + \phi^\mathrm{l}\phi^\mathrm{h} + \left(\frac{1+3\beta}{2}\right)(\phi^\mathrm{l})^2\phi^\mathrm{h} + \left(\frac{1+8\beta}{2}\right)(\phi^\mathrm{l})^3\phi^\mathrm{h}\,, \tag{6a}$$

$$n_\mathrm{e}^\mathrm{l} = (1-\beta)\phi^\mathrm{l} + \frac{1}{2}(\phi^\mathrm{l})^2 + \left(\frac{1+3\beta}{6}\right)(\phi^\mathrm{l})^3 + \left(\frac{1+8\beta}{24}\right)(\phi^\mathrm{l})^4\,, \tag{6b}$$

$$n_\mathrm{i}^\mathrm{h} = n_\mathrm{e}^\mathrm{h} - \nabla^2\phi^\mathrm{h}\,. \tag{7}$$

At this stage, it should be noted that we will be frequently using the condition of quasi-neutrality and quasi-static nature of plasma expressed by

$$n_\mathrm{i}^\mathrm{l} \approx n_\mathrm{e}^\mathrm{l}\ ,\quad v_\mathrm{i}^\mathrm{l} \approx v_\mathrm{e}^\mathrm{l} \approx 0\,. \tag{8}$$

These assumptions, along with (6), when used in the momentum equation, yield in the linear approximation

$$\frac{\partial v_\mathrm{ix}^\mathrm{h}}{\partial t} = \gamma\frac{\partial\phi^\mathrm{h}}{\partial x} + \omega_\mathrm{e} v_{\mathrm{i}y}^\mathrm{h} + \mu v_\mathrm{ix}^\mathrm{h}\,, \tag{9}$$

$$\frac{\partial v_{\mathrm{i}y}^{\mathrm{h}}}{\partial t} = \gamma \frac{\partial \phi^{\mathrm{h}}}{\partial y} - \omega_{\mathrm{e}} v_{\mathrm{i}x}^{\mathrm{h}} + \mu v_{\mathrm{i}y}^{\mathrm{h}} \,, \tag{10}$$

where $\gamma = -1 + \frac{2}{3}T(1-\beta)$, $\omega_{\mathrm{e}} = \Omega_{\mathrm{i}}/\omega_{\mathrm{pi}}$ and $\mu = \nu/\omega_{\mathrm{pi}}$. The equation of continuity then yields

$$\frac{\partial}{\partial t}[(1-\beta)\phi^{\mathrm{h}} - \nabla^2 \phi^{\mathrm{h}}] + \nabla \cdot \vec{v}_{\mathrm{i}}^{\,\mathrm{h}} = 0 \,. \tag{11}$$

From Eqs. (9) to (11), one obtains simply the dispersion relation

$$(1-\beta+k^2)\omega^3 - 2\mathrm{i}\mu(1-\beta+k^2)\omega^2 + [k^2\gamma - (\omega_{\mathrm{e}}^2+\mu^2)(1-\beta+k^2)]\omega - \mathrm{i}k^2\mu\gamma = 0 \,, \tag{12}$$

which indicates that the solution for either k or ω is complex, implying growth or decay of waves, depending on the sign of the imaginary part. Solving for k, we get

$$k^2 = \frac{\omega(1-\beta)(-\Omega^2 + \mathrm{i}2\mu\beta)}{\omega(\Omega^2+\gamma) - \mathrm{i}\mu(2\omega^2+\gamma)} \,.$$

For a wave with $\omega \gg \omega_{\mathrm{e}}$, the dispersion relation can be simplified,

$$\Re(k^2) = k_1^2 - k_2^2 = \frac{\omega^2(\beta-1)[(\omega^2+\mu^2)^2 + \mu^2\gamma]}{\omega^2(\omega^2-\mu^2+\gamma)^2 + \mu^2(2\omega^2+\gamma)^2} \,, \tag{13}$$

$$\Im(k^2) = 2k_1k_2 = \frac{\omega(1-\beta)(\omega^2+\mu^2)\mu\gamma}{\omega^2(\omega^2-\mu^2+\gamma)^2 + \mu^2(2\omega^2+\gamma)^2} \,, \tag{14}$$

where $k = k_1 + \mathrm{i}k_2$. Note that when ν, T_{i} and β are assumed equal to zero, we obtain the analytical expressions for k_1 and k_2 of Alam and Roy Chowdhury [8].

From Eqs. (13) and (14), one can analytically solve for k_1 and k_2. Fr example, we have

$$k_2^2 = \frac{\omega(1-\beta)}{X}[Y + \sqrt{Y^2+Z^2}] \,, \tag{15}$$

where

$$X = \omega^2(\omega^2-\mu^2+\gamma)^2 + \mu^2(2\omega^2+\gamma)^2 \,,$$

$$Y = \omega[(\omega^2+\mu^2)^2 + \mu^2\gamma], \qquad Z^2 = \mu^2\gamma^2(\omega^2+\mu^2)^2 \,. \tag{16}$$

The group velocities in the x and y directions can be calculated from Refs. [9] and [17] as

$$v_{\mathrm{g}x} = \frac{\partial}{\partial k_x}(\Re(\omega)), \qquad v_{\mathrm{g}y} = \frac{\partial}{\partial k_y}(\Re(\omega)) \,.$$

Explicit expressions for $v_{\mathrm{g}x}$ and $v_{\mathrm{g}y}$ are complicated and we do not reproduce them here. Now, to ascertain the growth and decay of the wave, we have plotted

k_2^2 versus ω in Figs. 1 and 2 for different values of β and μ and other plasma parameters. It is interesting to observe from Fig. 1 that the value of k_2 attains a maximum at a particular frequency ω, and this maximum value increases in the

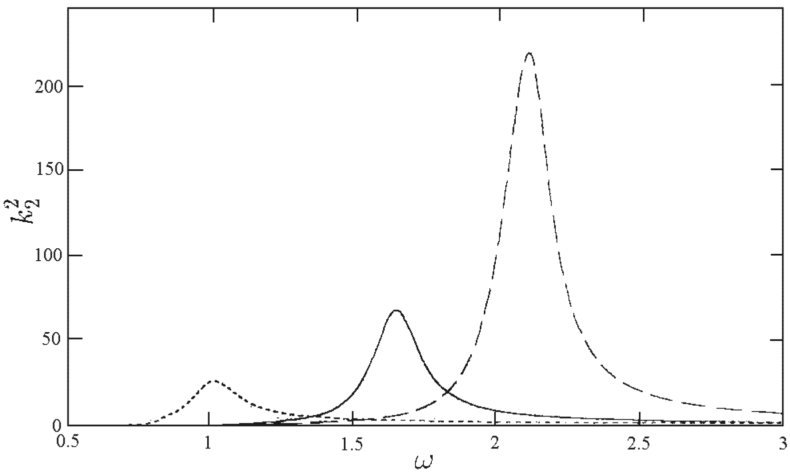

Fig. 1. Square of the imaginary part of k versus ω, showing the maximum growth and decay of the wave for the following values of the parameters: $\beta = 0.5$, $\mu = 0.2$, $T = 2.0$ (.......), $\beta = 0.5$, $\mu = 0.2$, $T = 0.0$ (——), $\beta = 0.0$, $\mu = 0.2$, $T = 2.0$ (– – –).

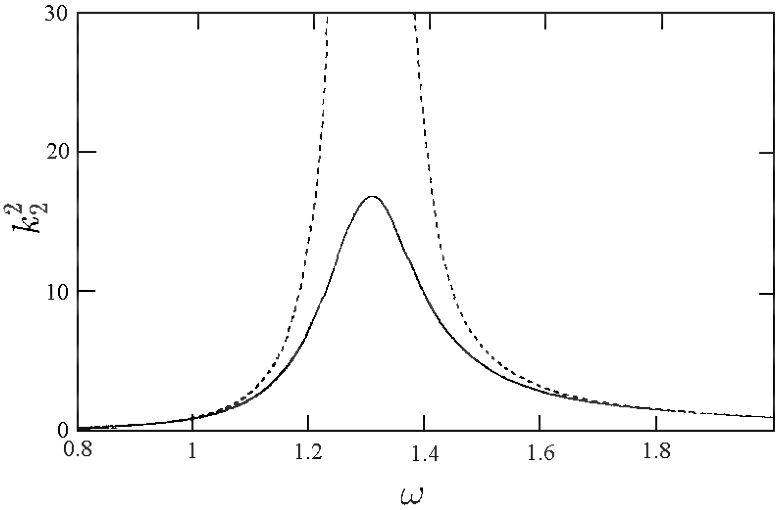

Fig. 2. The same as in Fig. 1 for ion-temperature T equal to zero: $\beta = 0.8$, $\mu = 0.0$, $T = 0.0$ (.......), $\beta = 0.8$, $\mu = 0.2$, $T = 0.0$ (——).

case when one of the above parameters is equal to zero. Ultimately, the value of k_2 gets saturated for large ω. Figure 2 shows that in the absence of collisions, the growth of the wave is fast and reaches the same maximum value in the range $1.2 < \omega < 1.5$ and for $\omega > 1.4$. The values of k_2 in both cases fall off and attain a saturation for $\omega > 1.6$. Due to this phenomenon, the waves get damped after travelling a finite distance inside the plasma and ultimately may be cut-off, i.e., the propagation stops. Figures 1 and 2 explicitly display the nature of decay for various parameter values. On the other hand, keeping the nonlinear terms, we get from the high-frequency component of the momentum equation as

$$\frac{\partial \vec{v}_{\rm i}^{\,\rm h}}{\partial t} = \gamma\nabla\phi^{\rm h} + \omega_{\rm e}(\vec{v}_{\rm i}^{\,\rm h}\times\vec{z}) + \gamma_1\nabla(\phi^{\rm l}\phi^{\rm h}) + \gamma_2\nabla[(\phi^{\rm l})^2\phi^{\rm h}] + \gamma_3\nabla[(\phi^{\rm l})^3\phi^{\rm h}] + \mu\vec{v}_{\rm i}^{\,\rm h}\,, \quad (17)$$

whereas the low-frequency part leads to

$$\phi^{\rm l} = \frac{1}{2\eta}\langle v_{{\rm i}x}^{\rm h}\rangle^2\,, \quad (18)$$

where

$$\gamma_1 = \frac{2}{9}\alpha(2\alpha + 2\beta - \beta^2)\,, \qquad \gamma_2 = \frac{1}{6}\alpha(1 + 7\beta)\,, \qquad \gamma_3 = \frac{1}{27}\alpha(1 - 20\beta + 6\beta^2)\,,$$

$$\eta = -1 + \frac{5T}{2}(5 - \beta - \beta^2)\,, \qquad \alpha = \frac{-5T}{2}\,.$$

We further assume that the modulational amplitudes $v_{{\rm i}x}^{\rm h}$ and $v_{{\rm i}y}^{\rm h}$ vary slowly with respect to y, so that from (9) and (10) we have

$$v_{{\rm i}x}^{\rm h} = -\gamma\delta_1\phi^{\rm h}\,, \qquad v_{{\rm i}y}^{\rm h} = -\gamma\delta_2\phi^{\rm h}\,,$$

with

$$\delta_1 = \frac{{\rm i}[(\mu + {\rm i}\omega)k_x - \omega_{\rm e}k_y]}{(\mu + {\rm i}\omega)^2 + \omega_{\rm e}^2}\,, \quad \delta_2 = \frac{{\rm i}[(\mu + {\rm i}\omega)k_y - \omega_{\rm e}k_x]}{(\mu + {\rm i}\omega)^2 + \omega_{\rm e}^2}\,, \quad \delta_3 = \frac{\gamma^2}{2\eta}\delta_1^2\,.$$

Finally, elimination of all variables in favour of $\phi^{\rm h}$ leads to

$$\left[\frac{\partial^2}{\partial t^2}(1-\beta-\nabla^2)+\gamma\nabla^2\right]\phi^{\rm h} + \left[\delta_3\frac{\partial^2}{\partial t^2}+a_1\nabla^2\right] |\,\phi^{\rm h}\,|^2\,\phi^{\rm h} + \left[a_2\frac{\partial^2}{\partial t^2}+a_3\nabla^2\right] |\,\phi^{\rm h}\,|^4\,\phi^{\rm h}$$
$$+\left[b_2\frac{\partial^2}{\partial t^2}+b_3\nabla^2\right] |\,\phi^{\rm h}\,|^6\,\phi^{\rm h} - \frac{\partial}{\partial x}\left[c_1\phi^{\rm h}+c_2\,|\,\phi^{\rm h}\,|^2\,\phi^{\rm h}+c_3\,|\,\phi^{\rm h}\,|^4\,\phi^{\rm h}+c_4\,|\,\phi^{\rm h}\,|^6\,\phi^{\rm h}\right]$$
$$+\frac{\partial}{\partial y}\left[d_1\phi^{\rm h} + d_2\,|\,\phi^{\rm h}\,|^2\,\phi^{\rm h} + d_3\,|\,\phi^{\rm h}\,|^4\,\phi^{\rm h} + d_4\,|\,\phi^{\rm h}\,|^6\,\phi^{\rm h}\right] = 0\,. \quad (19)$$

This is the required nonlinear Schrödinger equation of $\phi^{\rm h}$. The notations used in Eq. (19) are given in the Appendix A.

We now assume that the nonlinear interaction of the ion-acoustic wave with the slow response of plasma gives rise to an envelope of waves whose amplitude varies with time and space scales much more slowly than those of ion-acoustic oscillations. So we set

$$\phi^{\rm h} = \epsilon^{\frac{1}{2}}\phi^{\rm h}(\xi,\tau)\exp({\rm i}k_x x + {\rm i}k_y y - {\rm i}\omega t) + c.c.\ , \quad \xi = \epsilon(x - v_g t)\ , \quad \tau = \epsilon^2 t\,.$$

From the coefficient of $\epsilon^{\frac{3}{2}}$, we get

$$A\frac{\partial \phi^{\rm h}}{\partial \xi} + B \mid \phi^{\rm h} \mid^2 \phi^{\rm h} = 0\,, \tag{20}$$

where A and B are given in the Appendix A.

We now consider the case for some values of the plasma parameters β, μ, θ and T for which the coefficients of Eq. (20), the steady-state equation, vanish. The values thus calculated from $A = 0$ and $B = 0$ are said to be critical values of the plasma parameters on the assumption that $\delta_3 \neq 0$, because otherwise $A = 0$ gives $1 - \beta + k^2 = 0$, and again we do not have a steady-state equation.

For waves with $\omega \gg \omega_{\rm e}$, the critical values can be calculated from the following equations for a particular frequency or wave number

$$[k_1(\mu^2 - 2\omega^2) + k_2\omega\mu](\gamma\cos\theta) + \omega\omega_{\rm e}k_2(\gamma\sin\theta) = 2(\omega^2 - \mu^2)\omega v_g(1 - \beta + k_1^2 - k_2^2) + 8\mu\omega^2 v_g k_1 k_2, \tag{21}$$

$$3\mu\omega k_1 + k_2(\mu^2 - \omega_e^2)](\gamma\cos\theta) - k_1\omega\omega_{\rm e}(\gamma\sin\theta) = -4\mu\omega^2 v_g(1 - \beta + k_1^2 - k_2^2) - 4\omega v_g k_1 k_2(\mu^2 - \omega^2), \tag{22}$$

$$k_1 = \frac{\omega^2}{\omega_{\rm e}\sqrt{(1-\beta)\gamma'}}, \quad k_2 = \frac{-\omega\mu}{\omega_{\rm e}\sqrt{(1-\beta)\gamma'}}, \quad \gamma' = -\gamma\,. \tag{23}$$

Equations (21) to (23), together with the dispersion equations (13) and (14), actually define the region of the critical values for a particular frequency or wave number in the plasma. In particular, for a frequency near μ, the critical values can be calculated from the following equations

$$T = \frac{4}{5(1-\beta)}\left(\frac{16}{5}\omega_e^2(1-\beta)^2 - 1\,,\right) \tag{24}$$

$$\cos\theta = \frac{7}{32\omega_{\rm e}(1-\beta)^{3/2}}\left[\frac{16\omega_e^2(1-\beta)^2 - 5}{64\omega_e^2(1-\beta)^2 - 5}\right] \quad (25)\,,$$

$$\mu = \frac{1}{2}\left[1 + \frac{4}{3}\left(\frac{16}{5}\omega_e^2(1-\beta)^2 - 1\right)\right]^{1/2} \quad (26)\,,$$

Collecting the terms occurring as coefficients of $\epsilon^{5/2}$, we get

$$\mathrm{i}\frac{\partial \phi^{\mathrm{h}}}{\partial \tau} + P\frac{\partial^2 \phi^{\mathrm{h}}}{\partial \xi^2} + Q \mid \phi^{\mathrm{h}} \mid^2 \frac{\partial \phi^{\mathrm{h}}}{\partial \xi} + R(\phi^{\mathrm{h}})^2 \frac{\partial \phi^{h*}}{\partial \xi} + S \mid \phi^{\mathrm{h}} \mid^4 \phi^{\mathrm{h}} = 0\,. \tag{27}$$

The expressions for P, Q, R and S are given in the Appendix B.

Equation (27) is the new NLS equation with a higher-order nonlinearity. This new type of NLS equation is a combination of derivative NLS and the ordinary NLS equation with a higher-order nonlinearity.

3. *Modulational stability*

To proceed with the analysis of modulational instability, we set

$$\phi^{\mathrm{h}} = \rho^{1/2} \exp\left(\mathrm{i}\int^{\xi} \frac{\sigma}{2P}\mathrm{d}\xi\right), \tag{28}$$

where ρ is the amplitude and the $\sigma/(2P)$ phase .

Substituting (28) into (27), we get

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial}{\partial \xi}(\rho\sigma) + \frac{Q-R}{P}(\rho^2\sigma) = 0\,, \tag{29}$$

$$\frac{\partial \sigma}{\partial \tau} + \sigma\frac{\partial \sigma}{\partial \xi} = 4PS\frac{\partial \rho}{\partial \xi} + P(Q+R)\frac{\partial^2 \rho}{\partial \xi^2} + \rho^2\frac{\partial}{\partial \xi}\left(\rho^{-1/2}\frac{\partial}{\partial \xi}(\rho^{-1/2}\frac{\partial}{\partial \xi})\right). \tag{30}$$

For the analysis of stability, we linearise the above equations by setting

$$\rho = \rho_0 + \delta\rho\exp(\mathrm{i}\overline{k}\xi - \mathrm{i}\overline{\omega}\tau) = \rho_0 + \delta\overline{\rho}\,, \quad \sigma = \sigma_0 + \delta\sigma\exp(\mathrm{i}\overline{k}\xi - \mathrm{i}\overline{\omega}\tau) = \sigma_0 + \delta\overline{\sigma}$$

Whence from the condition for the existence of non-zero values of $\delta\rho$ and $\delta\sigma$, we get

$$\begin{vmatrix} \mathrm{i}(\overline{k}\sigma_0 - \overline{\omega}) + \frac{2(Q-R)}{P}\rho_0\sigma_0 & \mathrm{i}\overline{k}\rho_0 + \frac{Q-R}{P}\rho_0^2 \\ P\overline{k}(4\mathrm{i}S - (Q+R)\overline{k} - \mathrm{i}\overline{k}^2 P\rho_0^{-1}) & -\mathrm{i}(\sigma_0\overline{k} - \overline{\omega}) \end{vmatrix} = 0\,. \tag{31}$$

As the value of β increases from 0 to 1, we find at the critical values given by Eqs. (24) to (26) that $|P| \gg |Q|$, $|R|$ and $|P| > |S|$. In that case, Eq. (31) reduces to

$$\overline{\omega} \approx \sigma_0\overline{k} - \mathrm{i}\overline{k}\sqrt{P(P\overline{k}^2 - 4\rho_0 S)}\,, \tag{32}$$

from which we have ($\overline{\omega} = \overline{\omega}_1 + \mathrm{i}\,\overline{\omega}_2$)

$$\overline{\omega}_2 \approx \pm \frac{\overline{k}}{\sqrt{2}} \left[N_1 + \sqrt{N_1^2 + N_2^2} \right] . \tag{33}$$

Here

$$N_1 = (P_1^2 - P_2^2)\overline{k}^2 - 4\rho_0(P_1 S_1 - P_2 S_2) \,,$$

$$N_2 = 2\left[P_1 P_2 \overline{k}^2 - 2\rho_0(P_1 S_1 + P_2 S_2) \right] \,,$$

with $P = P_1 + \mathrm{i}P_2$, $S = S_1 + \mathrm{i}S_2$, etc.

From Eq. (33), we find that the modulational wave is always stable. From Fig. 3, it is clear that as β increases, the stability of the wave increases.

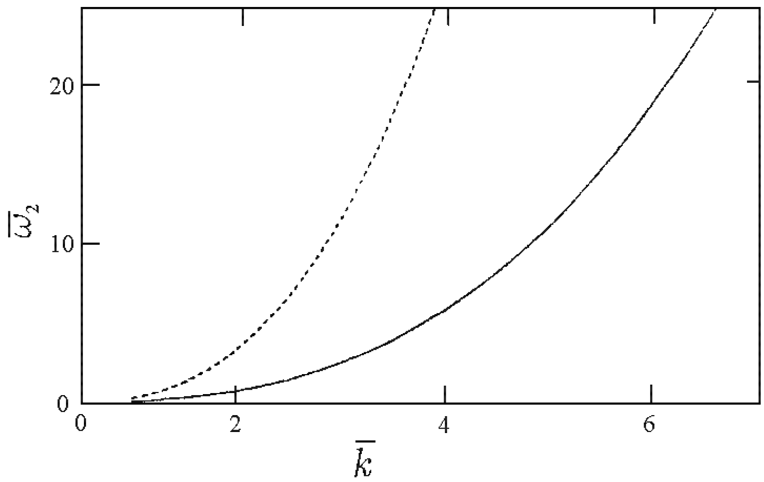

Fig. 3. The imaginary part of $\overline{\omega}$ versus $\overline{k}$, showing the stability of the modulational wave at the critical values: $\beta = 0.75$, $\mu = 0.5 \times 10^3$, $T = 0.0$, $\theta = \pi/2$ (.......), $\beta = 0.5$, $\mu = 0.5 \times 10^3$, $T = 0.0$, $\theta = \pi/2$ (——) .

4. Solitary wave solution

Solitary wave solutions of Eq. (27) can be obtained by substituting

$$\phi^{\mathrm{h}}(\xi, \tau) = \psi(\xi, \tau) \exp(\mathrm{i}K\xi - \mathrm{i}\Omega\tau)$$

in Eq. (27) and then equating the real and imaginary parts. We get

$$\psi_\tau + P_2\psi_{\xi\xi} + 2P_1 K\psi_\xi + (Q_2 + R_2)\psi_\xi\psi^2 + (Q_1 - R_1)K\psi^3 + S_2\psi^5 - P_2 K^2\psi = 0 \,, \tag{34}$$

$$P_1\psi_{\xi\xi}-2P_2K\psi_\xi+(\Omega-P_1K^2)\psi+(Q_1+R_1)\psi_\xi\psi^2+(R_2-Q_2)K\psi^3+S_1\psi^5=0\,. \quad (35)$$

Combining Eqs. (34) and (35), we obtain

$$\psi_\tau+\Theta_1\psi_\xi+\Theta_2\psi_{\xi\xi}-\Theta_3\psi-\Theta_4\psi^3-\Theta_5\psi^5=0\,, \quad (36)$$

where where expressions for $\Theta_1,...,\Theta_5$ are given in Appendix B.

We now set $x=\xi-\Theta_1\tau$ so that we have from (36)

$$\Theta_2\psi_{xx}-\Theta_1\psi-\Theta_4\psi^3-\Theta_5\psi^5=0\,, \quad (37)$$

which upon integration leads to

$$\psi=\pm\left(\frac{2a}{\sqrt{(b^2-4ac)}\cosh(2\sqrt{a}(\xi-\xi_0-2PK\tau))-b}\right)^{1/2}. \quad (38)$$

Finally, we get

$$\phi^{\mathrm{h}}(\xi,\tau)=\psi(\xi,\tau)\,\exp(\mathrm{i}K\xi-\mathrm{i}\Omega\tau)\,. \quad (39)$$

The expressions for a, b and c are

$$a=\frac{P_2K^2(Q_1+R_1)+(\Omega-P_1K^2)(Q_2+R_2)}{P_1(Q_2+R_2)-P_2(Q_1+R_1)}\,,\quad b=\frac{(R_1^2+R_2^2)-(Q_1^2+Q_2^2)}{2[P_1(Q_2+R_2)-P_2(Q_1+R_1)]}\,,$$

$$c=\frac{S_1(Q_2+R_2)-S_2(Q_1+R_1)}{3[P_1(Q_2+R_2)-P_2(Q_1+R_1)]}\,. \quad (40)$$

Incidentally, it may be mentioned that nowadays it is possible to obtain N-soliton solutions of such a NLS equation by the Hirota's method. One substitutes $\phi=G/F$, where G is a complex function and F is real, and they are given by

$$G=\rho_0\exp(\mathrm{i}kx-\mathrm{i}\omega t)(1+\epsilon g_1+\epsilon^2 g_2+\dots\,,\qquad F=1+\epsilon f_1+\epsilon^2 f_2+\dots\,.$$

Usually a truncation is possible and one gets a specified number of solutions depending on the number of terms retained. We do not go into further details here, because such type of equations have already been studied from the N-soliton point of view by Lakshmanan et al. [18] and Radhakrishnan et al. [19]. Also, it is a fact that the N-soliton structures are not of interest in plasma physics due to the difficulties in their experimental observations.

5. *Conclusions*

In above presented analysis, we have found that in the presence of collisions, the wave number k turns out to be complex in the linear dispersion relation. The wave amplitude increases when any one of the parameters β, μ and T is neglected, and it gets saturated for large values of ω. Due to this phenomenon, the waves get damped after travelling a finite distance inside the plasma and ultimately may be cut-off, i.e., the propagation stops.

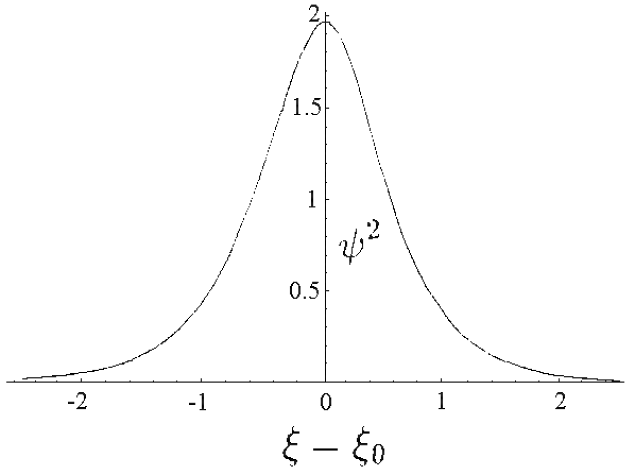

Fig. 4. Variation of the amplitude (ψ^2) of the solitary solution with space varialbe ($\xi - \xi_0$) at the critical values $\beta = 0.75$, $\mu = 0.5 \times 10^3$, $T = 0.0$, $\theta = \pi/2$.

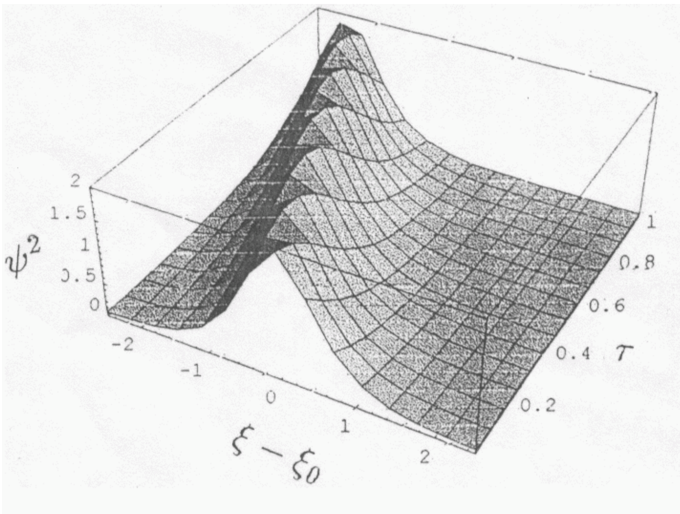

Fig. 5. The three-dimensional view of the amplitudes for the same critical values as in Fig. 4.

The nonlinear ion-acoustic wave is described by the new NLS equation with higher-order and derivative-type nonlinearities. It originates is due to the critical values of the plasma parameters β, θ, μ and T. It also exhibits a different form of solitons at different critical values – one of the usual type and other one the 'hole' type. The soliton forms are shown in Figs. 4 to 7. It is observed that the modulational wave is stable even at some critical values of the plasma parameters, and the stability increases as the percentage of fast particles increases.

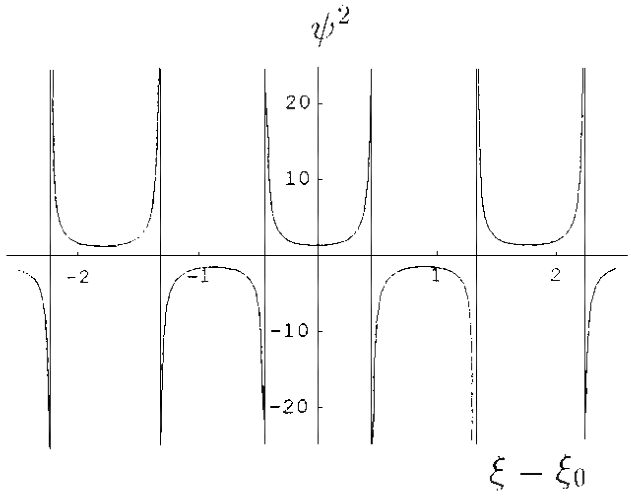

Fig. 6. The same variation as in Fig. 4 showing the different forms of soliton at different critical values $\beta = 0.5, \mu = 0.5 \times 10^3, T = 0.0, \theta = \pi/2$.

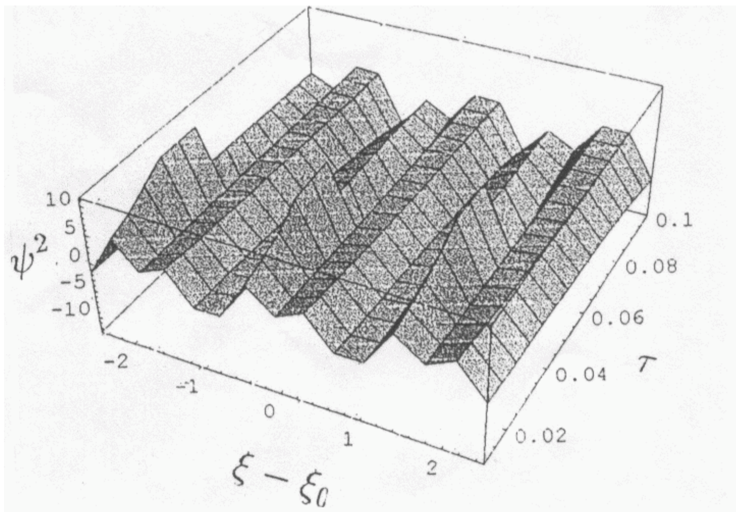

Fig. 7. The three-dimensional view for the same critical values as in Fig. 6.

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Appendix A

Here we give expressions for a_i, b_i, c_i, d_i, A and B

$$a_1 = [(1-\beta)\gamma + \gamma_1]\delta_3\,, \quad a_2 = \frac{1+3\beta}{2}\delta_3^2\,, \quad a_3 = \left[\frac{1}{2}\gamma + (1-\beta)\gamma_1\delta_3 + \gamma_2\delta_3\right]\delta_3\,,$$

$$b_2 = \frac{1+8\beta}{6}\delta_3^2\,, \quad b_3 = \left[\frac{1}{2}\gamma_1\delta_3 + (1-\beta)\gamma_2\delta_3^2 + \gamma_3\delta_3^2 + \frac{1+3\beta}{6}\gamma\right]\delta_3\,,$$

$$c_1 = (\omega_e\delta_2 + \mu\delta_1)\gamma\,, \quad c_2 = (1-\beta)\gamma\delta_2\delta_3\omega_e\,, \quad c_3 = \frac{1}{2}\gamma\delta_2\delta_3\omega_e\,, \quad c_4 = \frac{1+3\beta}{6}\gamma\delta_2\delta_3\omega_e\,,$$

$$d_1 = (\omega_e\delta_1 - \mu\delta_2)\gamma\,, \quad d_2 = (1-\beta)\gamma\delta_1\delta_3\omega_e\,, \quad d_3 = \frac{1}{2}\gamma\delta_1\delta_3\omega_e\,, \quad d_4 = \frac{1+3\beta}{6}\gamma\delta_1\delta_3\omega_e\,,$$

$$A = (1-\beta+k^2)2\mathrm{i}\omega v_g + 2\mathrm{i}\gamma k\cos\theta - \gamma(\omega_e\delta_2 + \mu\delta_1)\,,$$

$$B = \delta_3[-\omega^2 + i(\delta_1\sin\theta - \delta_2\cos\theta)k\gamma\omega_e(1-\beta)]\,.$$

Appendix B

Here we give expressions for P, Q, S and Θ_i

$$P = -\frac{(1-\beta_c+k^2)v_g^2 + 4\omega k_x v_g + \gamma_c}{2\omega(1-\beta_c+k^2)}\,,$$

$$Q = -\frac{[3\mathrm{i}\omega v_g + ((1-\beta_c)\gamma_c + \gamma_1)4\mathrm{i}k_x - 2(1-\beta_c)\gamma_c\delta_2\omega_e]\delta_3}{2\omega(1-\beta_c+k^2)}\,,$$

$$R = -\frac{[\mathrm{i}\omega v_g + ((1-\beta_c)\gamma_c + \gamma_1)2\mathrm{i}k_x - (1-\beta_c)\gamma_c\delta_2\omega_e]\delta_3}{2\omega(1-\beta_c+k^2)}\,,$$

$$S = \frac{[\omega^2\delta_3(\frac{1+3\beta_c}{2}) + 3k^2(\frac{1}{2}\gamma_c + (1-\beta_c)\gamma_1\delta_3 + \delta_3\gamma_2) - \mathrm{i}\frac{3}{2}\gamma_c\omega_e(k_x\delta_2 + k_y\delta_1)]\delta_3}{2\omega(1-\beta_c+k^2)}\,,$$

$$\Theta_1 = \frac{2K}{Q_1+R_1}[P_2(Q_2+R_2) + P_1(Q_1+R_1)]\,, \quad \Theta_2 = \frac{P_2(Q_1+R_1) - P_1(Q_2+R_2)}{Q_1+R_1}\,,$$

$$\Theta_3 = \frac{P_2K^2(Q_1+R_1)+(\Omega-P_1K^2)(Q_2+R_2)}{Q_1+R_1}\,, \quad \Theta_4 = \frac{(R_1^2+R_2^2)-(Q_1^2+Q_2^2)}{Q_1+R_1}K,$$

$$\Theta_5 = \frac{S_1(Q_2+R_2)-S_2(Q_1+R_1)}{Q_1+R_1}\,.$$

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MODULIRAN IONSKO-ZVUČNI VAL BLIZU KRITIČNOSTI – NOVA NELINEARNA SCHRÖDINGEROVA JEDNADŽBA U SUDARNOJ PLAZMI

Proučavamo modulaciju ionsko-zvučnih valova u tromoj sudarnoj plazmi za neke kritične vrijednosti parametara plazme, kuta širenja, sudarne učestalosti, temperature iona i postotka brzih čestica u plazmi. Pretpostavlja se da elektroni koji čine pozadinu nisu termalizirani. Našli smo da se pri tim kritičnim vrijednostima nelinearni ionsko-zvučni val dobro opisuje novim oblikom nelinearne Schrödingerove (NLS) jednadžbe koja sadrži više-redne i derivativne nelinearnosti. Izveli smo uvjet za stabilnost modulacija za neke kritične vrijednosti parametara plazme. Dobili smo solitonsko rješenje NLS jednadžbe koje se ponešto mijenja za različite za kritične vrijednosti.