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COLLECTIVE EFFECTS OF BOUND ELECTRONS, FREE ELECTRONS AND IONS IN WAVE-PLASMA INTERACTION

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Characteristics of propagation of small-amplitude waves in a cold plasma, assumed to be fluid-like, compressible, and a mixture of populations of weakly bound electrons, free electrons and ions, is studied. For an elliptically polarized wave, we have evaluated (i) the zeroth-harmonic magnetic moment, (ii) the damping factor and cut-off frequency and (iii) the collisional energy absorption. The relative roles of the frequency (ω_0) of the bound electrons, the characteristic frequency of free electrons (ω_f), the characteristic frequency of bound electrons (ω_b) and the resonance condition, where ω_0 equals the wave frequency ω, and the effects of the cut-off frequency, is studied. The cut-off frequency has a higher value; in the resonance case ($\omega^2 = \omega_0^2$) it increases when ω_0 is greater than ω_f, and decreases when ω_0 is less than ω_f. The field of magnetic-moment at this resonance is enhanced when ω_0 is large compared to ω_f. The exchange of energy between the wave field and the constituents of this plasma is considered.

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1. *Introduction*

In plasmas, some atoms remain neutral, but their valence electrons are weakly bound to their respective nuclei in partially ionized plasmas, for example in plasmas in the ionosphere, in the cosmic spaces, the solar chromosphere and photosphere, cool interstellar clouds, etc. Practical importance in modern research cannot be denied of plasmas containing a population of bound electrons because this model is more realistic than the models of free-electron plasma. Applicational possibility of this model also exists in laser-plasma interaction and in wave interaction with solid state plasmas.

In sodium and other alkali metals, the valence electrons are weakly bound, and the electron orbits are distorted by incident fields. So, considering these quasi-free charges as harmonically bound to their respective ionic cores, and ignoring the anharmonicity from interaction of weak Coulomb field of atomic cores and other electrons, the optical properties of atoms excited by strong electromagnetic radiation are determined [1].

Bonnedal and Wilhelmsson [2] investigated nonlinear effects in a model of partially ionized, collisionally damped, homogeneous magnetized plasma of infinite extent. This plasma is a mixture of a fully-ionized plasma of free electrons, and a partially ionized plasma of active molecules, whose the energy levels have inverted population, that is a maser system when the active molecules have a uniform drift velocity parallel to the magnetization direction. So, these authors replaced the molecules with an inverted-level population with a damped Lorentz oscillator model. The force acting on these molecules are the same as on the population of bound electrons.

The force of simple harmonic motion proportional to the field-induced displacement of electrons about their ionic cores, in addition to the Lorentz force, acts on the bound electrons only. Actually, the conservative central Coulomb potential of an atomic nucleus reduces to the centrifugal force of rotation of bound electrons about their nuclei. Its influence on polarization of the bound electrons is included in the electric displacement vector $\vec{D}$. Therefore, there exists in the displacement current in the Ampere-Maxwell equation which also contains the plasma current of the free charges. $\vec{D}$ moreover appears in Gauss's theorem. Hence, the concept of the polarization vector for bound electrons is introduced phenomenologically in the Maxwell equations, and the Lorentz theory of electrodynamics obtains a closed form of guiding equations avoiding the empirical state relations of the phenomenological theory of electrodynamics (see Bloembergen, Ref. [1]). Analytical definition of polarization, as the sum over all species of charges of the product of charge density and field-induced displacement, permits this type of mixing of the two classical theories.

The nonlinear distortion of orbits of bound electrons contributes to the optical properties of atoms excited by strong electromagnetic (EM) radiation. Electrons of neutral atoms are considered as harmonically bound to their respective ionic cores. This binding force originates from the centrifugal force acting on electrons in their orbital motions. The classical theory is valid when the excited non-oscillating part of the field of magnetic moment [3 – 5] is not so large that the gyro-radii of the charges due to this field are of the order of atomic dimensions.

Laser produced plasmas, consisting of multiply-ionised ions and highly charged heavy ions, accept many electrons in their high-lying loosely-bound orbitals. When the number of bound electrons becomes large, the line spectra may become very dense. The assumption that electrons are bound classically to their core ions seems valid in the study of this dynamics.

Recently, Chakraborty et al. [6] have considered the closed system of field equations of the continuum dynamics, including the collective behaviour of the bound electrons in plasma and developed a classical theory for understanding the dynamical behaviour of such a plasma. It has been found that the Poynting theorem for energy conservation and the Maxwell stress elements have some new terms. Subsequently, Chakraborty et al. [7] studied the response of this plasma to transverse waves. They investigated the dependence on temperature of the characteristics of their propagation through the medium, the related phase velocity and group velocity and the temperature-dependent total Thomson scattering cross section. They also discussed the Lagrangian and the Hamiltonian of a compressible plasma due to thermal motion, having bound electrons in the fluid-mixture approximation. The mathematical complexity increases considerably even in the study of the familiar particle dynamical theory for bound electrons, in presence of waves of infinitesimal amplitude, compared to the same for a free electrons plasma.

We formulate and discuss a non-relativistic, classical, closed systems of field equations of plasma regarded as a mixture of populations of bound electrons, free electrons, neutral particles and neutralising ions in the presence of applied wave fields. Particle dynamics of bound electron in the presence of applied wave fields, in the classical limit, exists and gives useful results for the Larmour precession effect, in the scattering theory of light of Rayleigh and Thomson, etc.

In the present paper, we discuss the wave-plasma interaction in a plasma considering the collective effect of bound electrons, free electrons and ions. The characteristics of propagation of small-amplitude waves in the plasma are investigated. It is seen that the characteristics of the wave propagation of the medium are modified and become complicated, compared to those in free-electron plasma. The cut-off frequency is changed and the resonant modes are different. The expressions for the zero-frequency magnetic moment generation have been derived which are found to be different from the those in work of earlier authors.

2. *Basic equations*

For the study of very small amplitude wave processes, in the presence of collective effects of bound electrons, free electrons and ions, the plasma is assumed to be cold, homogeneous, unmagnetized and collisionally damped. Electrons are assumed to be mobile and the ions to provide static charge-neutralising background. The closed system of linearized field equations are

$$m\frac{\partial \vec{u}_{\rm b}}{\partial t} = -m\omega_0^2\vec{\varepsilon}_{\rm b\perp} - e\vec{E} - m\nu_{\rm b}\vec{u}_{\rm b}, \tag{1}$$

$$
\begin{aligned}
m\frac{\partial \vec{u}_{\rm f}}{\partial t} &= -e\vec{E} - m\nu_{\rm f}\vec{u}_{\rm f}, &(2)\\
\vec{\nabla}\times\vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, &(3)\\
\vec{\nabla}\times\vec{B} &= \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{4\pi e}{c}\big(n_{0\rm b}\vec{u}_{\rm b} + n_{0\rm f}\vec{u}_{\rm f}\big), &(4)\\
\vec{\nabla}\cdot\vec{E} &= 4\pi\big(\rho_{\rm b} + \rho_{\rm f}\big), &(5)\\
\vec{\nabla}\cdot\vec{B} &= 0. &(6)
\end{aligned}
$$

Here ω_0 is the natural frequency of orbital motion of bound electrons, $\vec{\varepsilon}_{\rm b\perp}$ is the first-order component of the displacement which is perpendicular to the direction of wave propagation, $\vec{u}_{\rm b}$ and $\vec{u}_{\rm f}$ are the first-order velocity of bound electrons and free electrons, $\nu_{\rm b}$, $\nu_{\rm f}$ are the collision frequency of bound and free electrons, respectively, $\vec{E}$, $\vec{B}$, $\rho_{\rm b}$, $\rho_{\rm f}$ are the electric field, the magnetic field, charge density of bound and free electrons, respectively, while e, m, $n_{0\rm b}$ and $n_{0\rm f}$ are the electron charge and mass, the equilibrium number density of the bound electrons and the number density of the free electrons, respectively.

The elliptically polarized wave field (purely transverse) is

$$\vec{E} = \big(E_1 e^{{\rm i}\theta},\, -{\rm i}E_2 e^{-{\rm i}\theta^*},\, 0\big), \tag{7}$$

where $\theta = kz - \omega_1 t$ and $\theta^* = kz - \omega_1^* t$, k is the wave number, and ω_1 is the complex wave frequency. We write

$$\omega_1 = \omega - {\rm i}\gamma \quad \text{and} \quad \omega_1^* = \omega + {\rm i}\gamma,$$

where ω is the real wave frequency and γ is the real damping term.

3. Dispersion relation

Taking curl of Eq. (3) and substituting the result in (4), and using (5), we get

$$4\pi\big(\vec{\nabla}\rho_{\rm b} + \vec{\nabla}\rho_{\rm f}\big) - \nabla^2\vec{E} = -\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} + \frac{4\pi e}{c^2}\big(n_{0\rm b}\dot{\vec{u}}_{\rm b} + n_{0\rm f}\dot{\vec{u}}_{\rm f}\big). \tag{8}$$

Since the plasma is homogeneous, for purely transverse waves we put $\vec{\nabla}\rho_{\rm b} = 0$ and $\vec{\nabla}\rho_{\rm f} = 0$ and obtain

$$\big(k^2c^2 - \omega_1^2\big)\vec{E} = 4\pi e\big(n_{0b}\dot{\vec{u}}_{\rm b} + n_{0\rm f}\dot{\vec{u}}_{\rm f}\big)\,. \tag{9}$$

The equations of motion, (1) and (2), give

$$\big(\omega_0^2 - \omega_1^2 - {\rm i}\omega_1\nu_{\rm b}\big)\varepsilon_{{\rm b}x} \;=\; \frac{eE_1}{m}\,{\rm e}^{{\rm i}(kz-\omega t+{\rm i}\gamma t)}, \tag{10}$$

$$\big(-\omega_1^2 - \mathrm{i}\omega_1\nu_\mathrm{f}\big)\varepsilon_{\mathrm{f}x} \quad = \quad \frac{eE_1}{m}\,\mathrm{e}^{\mathrm{i}(kz-\omega t+\mathrm{i}\gamma t)}. \tag{11}$$

Finally we get

$$\ddot{\varepsilon}_{\mathrm{b}x} \quad = \quad -\frac{eE_1}{m}\frac{(\omega^2-\gamma^2-2\mathrm{i}\gamma\omega)\,\mathrm{e}^{\mathrm{i}(kz-\omega t+\mathrm{i}\gamma t)}}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)+\mathrm{i}(2\gamma-\nu_\mathrm{b})\omega}, \tag{12}$$

$$\ddot{\varepsilon}_{\mathrm{f}x} \quad = \quad -\frac{eE_1}{m}\frac{(\omega^2-\gamma^2-2\mathrm{i}\gamma\omega)\,\mathrm{e}^{\mathrm{i}(kz-\omega t+\mathrm{i}\gamma t)}}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)+\mathrm{i}(2\gamma-\nu_\mathrm{f})\omega}. \tag{13}$$

Substituting Eqs. (12) and (13) in (10), we get, for the complex wave frequency ω and the real wave number k, the relation

$$\begin{aligned} k^2c^2 - \omega^2 + \gamma^2 + 2\mathrm{i}\gamma\omega \\ = \frac{\omega_\mathrm{b}^2\left\{(\omega^2-\gamma^2)(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)-2\gamma\omega^2(2\gamma-\nu_\mathrm{b})\right\}}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{b})^2} \\ -\frac{\mathrm{i}\omega_\mathrm{b}^2\left\{\omega(\omega^2-\gamma^2)(2\gamma-\nu_\mathrm{b})+2\gamma\omega(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)\right\}}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{b})^2} \\ +\frac{\omega_\mathrm{f}^2\left\{(\omega^2-\gamma^2)(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)-2\gamma\omega^2(2\gamma-\nu_\mathrm{f})\right\}}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)^2+\omega^2(2\gamma-\nu_f)^2} \\ -\frac{\mathrm{i}\omega_\mathrm{f}^2\left\{\omega(\omega^2-\gamma^2)(2\gamma-\nu_\mathrm{f})-2\gamma\omega(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)\right\}}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{f})^2}, \end{aligned} \tag{14}$$

where $\omega_\mathrm{b}^2 = (4\pi e^2 n_{0\mathrm{b}}/m)$, $\omega_\mathrm{f}^2 = (4\pi e^2 n_{0\mathrm{f}}/m)$. So, ω_b and ω_f are the characteristic frequencies of the bound electrons and free electrons. The real and imaginary parts of Eq. (14) approximately give

$$\begin{aligned} k^2c^2-\omega^2+\gamma^2 \quad &= \quad \frac{\omega_\mathrm{b}^2\left\{(\omega^2-\gamma^2)(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)-2\gamma\omega^2(2\gamma-\nu_\mathrm{b})\right\}}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{b})^2} \\ &\quad +\frac{\omega_\mathrm{f}^2\left\{(\omega^2-\gamma^2)(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)-2\gamma\omega^2(2\gamma-\nu_\mathrm{f})\right\}}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{f})^2} \end{aligned} \tag{15}$$

$$\begin{aligned} 2\gamma\omega \quad &= \quad -\Bigg[\frac{\omega_\mathrm{b}^2\left\{\omega(\omega^2-\gamma^2)(2\gamma-\nu_\mathrm{b})+2\gamma\omega(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)\right\}}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{b})^2} \\ &\quad +\frac{\omega_\mathrm{f}^2\left\{\omega(\omega^2-\gamma^2)(2\gamma-\nu_\mathrm{f})+2\gamma\omega(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)\right\}}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{f})^2}\Bigg] \end{aligned} \tag{16}$$

for the dispersion relation between the real wave frequency ω and the wave number k, and the relation for finding the decay constant γ. Since these relations are complicated, we consider some special cases which are easy to explain and understand.

Case I: $\omega^2 = \omega_0^2$ (resonance)

The dispersion relation and the equation for γ then become

$$\begin{aligned} k^2c^2 - \omega_0^2 + \gamma^2 &= \frac{\omega_\mathrm{b}^2 \left\{(\omega_0^2 - \gamma^2)(\gamma^2 - \nu_\mathrm{b}\gamma) - 2\gamma\omega_0^2(2\gamma - \nu_\mathrm{b})\right\}}{(\gamma^2 - \nu_\mathrm{b}\gamma)^2 + \omega_0^2(2\gamma - \nu_\mathrm{b})^2} \\ &\quad + \frac{\omega_\mathrm{f}^2 \left\{(\omega_0^2 - \gamma^2)(\gamma^2 - \omega_0^2 - \nu_\mathrm{f}\gamma) - 2\gamma\omega_0^2(2\gamma - \nu_\mathrm{f})\right\}}{(\gamma^2 - \omega_0^2 - \nu_\mathrm{f}\gamma)^2 + \omega_0^2(2\gamma - \nu_\mathrm{f})^2}, \qquad (17) \\ \pm 2\gamma\omega_0 &= -\Bigg[\frac{\omega_\mathrm{b}^2 \left\{\pm\omega_0(\omega_0^2 - \gamma^2)(2\gamma - \nu_\mathrm{b}) \pm 2\gamma\omega_0(\gamma^2 - \nu_\mathrm{b}\gamma)\right\}}{(\gamma^2 - \nu_\mathrm{b}\gamma)^2 + \omega_0^2(2\gamma - \nu_\mathrm{b})^2} \\ &\quad + \frac{\omega_\mathrm{f}^2 \left\{\pm\omega_0(\omega_0^2 - \gamma^2)(2\gamma - \nu_\mathrm{f}) \pm 2\gamma\omega_0(\gamma^2 - \omega_0^2 - \nu_\mathrm{f}\gamma)\right\}}{(\gamma^2 - \omega_0^2 - \nu_\mathrm{f}\gamma)^2 + \omega_0^2(2\gamma - \nu_\mathrm{f})^2}\Bigg]. \qquad (18) \end{aligned}$$

The collision frequency of both types of electrons and the damping factor γ are small compared to the incident frequency (ω) and the natural frequency (ω_0) of bound electrons. So, Eqs. (17) and (18) yield

$$k^2c^2 = \omega_0^2 - \omega_\mathrm{f}^2 - \omega_\mathrm{b}^2\delta, \qquad (19)$$

$$\gamma = \frac{1}{2}\big(\gamma_\mathrm{b} + \omega_\mathrm{f}^2\nu_\mathrm{f}/\omega_0^2\big), \qquad (20)$$

where

$$\delta = \frac{1}{4}\big(\frac{\omega_0^2\nu_\mathrm{b}}{\omega_\mathrm{f}^2\nu_\mathrm{f}} + 3\big)\big(\frac{\omega_0^2\nu_\mathrm{b}}{\omega_\mathrm{f}^2\nu_\mathrm{f}} + 1\big). \qquad (21)$$

Eq. (19) can be solved with the help of Eqs. (20) and (21).

The modified cut-off frequency is calculated from Eq. (19) by putting $k = 0$ and $\omega^2 = \omega_0^2 = \omega_{c_1}^2$. Thus, we find that

$$\omega_{c_1}^2 = \omega_\mathrm{f}^2 + \omega_\mathrm{b}^2\delta \qquad (22)$$

or

$$\omega_{c_1}^2 = \omega_\mathrm{c}^2 + \omega_\mathrm{b}^2\big(\delta - 1\big), \qquad (23)$$

where ω_c is the cut-off frequency when the bound electrons are also replaced by free electrons. So,

$$\omega_\mathrm{c}^2 = \frac{4\pi(n_{0\mathrm{b}} + n_{0\mathrm{f}})}{m}\, e^2 = \omega_\mathrm{b}^2 + \omega_\mathrm{f}^2$$

when $\nu_\mathrm{b} \approx \nu_\mathrm{f}$, $\omega_\mathrm{f} > \omega_0$, and we find that

$$\delta < 1. \qquad (24a)$$

So, ω_{c_1} is decreased due to the presence of the bound electrons. When $\nu_\mathrm{b} \approx \nu_\mathrm{f}$, $\omega_0 > \omega_\mathrm{f}$, we have

$$\omega_0^2\nu_\mathrm{b}/\omega_\mathrm{f}^2\nu_\mathrm{f} > 1, \qquad \delta > 1. \qquad (24b)$$

So, ω_{c_1} is then increased due to the presence of the bound electrons.

Hence the cut off frequency (which is also the incident field frequency) depends on plasma frequency of electrons or the characteristic frequency of free electrons and natural frequency of bound electrons, but it is independent of the number density of bound electrons.

Case II: $\omega^2 \neq \omega_0^2$, $\nu_b \approx 0$, $\nu_f \approx 0$, $\gamma \ll \omega, \omega_b$

In this case the dispersion relation reduces to

$$k^2c^2 - \omega^2 = \frac{\omega_b^2\omega^2}{\omega_0^2 - \omega^2} - \omega_f^2 \tag{25}$$

and the cut-off frequency ω_{c_2} is given by

$$\omega_{c_2}^2 = \omega_f^2 - \frac{\omega_b^2\omega_{c_2}^2}{\omega^2 - \omega_{c_2}^2}. \tag{26}$$

Solving this quadratic equation for $\omega_{c_2}^2$ we obtain

$$\omega_{c_2}^2 = \frac{1}{2}\left(\omega_0^2 + \omega_b^2 + \omega_f^2\right) \pm \sqrt{\frac{1}{4}\left(\omega_0^2 + \omega_b^2 + \omega_f^2\right) - \omega_0^2\omega_f^2}. \tag{27}$$

The minus sign is to be neglected to bring the relation $\omega^2 = \omega_f^2$ when $\omega_0 = 0$ and $n_{0b} = 0$. So, approximately,

$$\omega_{c_2}^2 \approx \omega_0^2 + \omega_b^2 + \omega_f^2 - \frac{\omega_0^2\omega_f^2}{\omega_0^2 + \omega_b^2 + \omega_f^2}. \tag{28}$$

It can be written as

$$\omega_{c_2}^2 = \omega_c^2 + \frac{\omega_0^2\left(\omega_0^2 + \omega_b^2\right)}{\omega_0^2 + \omega_b^2 + \omega_f^2}, \tag{29}$$

where ω_c^2 $(= \omega_b^2 + \omega_f^2)$ is the cut-off frequency of plasma having the free electron number density $(= n_{0b} + n_{0f})$. When ω_0 and ω_b are large compared to ω, we obtain

$$\omega_{c_2}^2 \approx \omega_c^2 + \omega_0^2. \tag{30}$$

In the reverse case, when $\omega_f^2 > \omega_0^2, \omega_b^2$, we get

$$\omega_{c_2}^2 = \omega_c^2 + \frac{\omega_0^2\left(\omega_0^2 + \omega_b^2\right)}{\omega_f^2}. \tag{31}$$

In the presence of only a residual number density of bound electrons, we write $\omega_0^2 > \omega_b^2$ and obtain

$$\omega_{c_2}^2 = \omega_c^2 + \omega_0^4/\omega_f^2. \tag{32}$$

So, the cut-off frequency is increased in the presence of bound electrons, and it depends on the number density of both types of electrons and the natural frequency of bound electrons. But, for the resonance of Case I, it is independent of the number density of bound electrons.

4. *The wave-field-induced zero-frequency magnetic moment*

The expression for the magnetic moment per unit volume, as a function of the co-ordinates of position ρ and time t, is given by

$$\vec{\mu} = \frac{1}{2c} \sum_s \left(\vec{\varepsilon}_s \times \vec{j}_s \right), \tag{33}$$

where $\vec{\varepsilon}_s$ is the wave-induced displacement of charges from their average position in the field free state, $\vec{j}_s (= n_{s0} q_s \vec{u}_s)$ is the surface current density, n_{s0} is the electron number density, q_s is the charge per particle and $\vec{u}_s$ is the average velocity of the s^{th} species of charges. The induced magnetic field is

$$4\pi\vec{\mu} = \vec{H}^{\text{in}} = \frac{2\pi}{c} \sum_s \left(\vec{\varepsilon}_s \times \vec{j}_s \right). \tag{34}$$

For our model

$$\vec{H}^{\text{in}} = \frac{2\pi}{c} \left[\left(\vec{\varepsilon}_{\text{b}} \times \vec{j}_{\text{b}} \right) + \left(\vec{\varepsilon}_{\text{f}} \times \vec{j}_{\text{f}} \right) \right]. \tag{35}$$

From the equations of motion (1) and (2) we get

$$\left(-\omega_1^2 - \mathrm{i}\omega_1\nu_{\text{b}} + \omega_0^2 \right) \varepsilon_{\text{b}x} = -\frac{e}{m} E_1 \, \mathrm{e}^{\mathrm{i}\theta}, \tag{36}$$

$$\left(-\omega_1^2 - \mathrm{i}\omega_1\nu_{\text{f}} \right) \varepsilon_{\text{f}x} = -\frac{e}{m} E_1 \, \mathrm{e}^{\mathrm{i}\theta}, \tag{37}$$

$$\left(-{\omega_1^*}^2 - \mathrm{i}\omega_1^*\nu_{\text{b}} + \omega_0^2 \right) \varepsilon_{\text{b}y} = \frac{\mathrm{i}e}{m} E_2 \, \mathrm{e}^{-\mathrm{i}\theta^*}, \tag{38}$$

$$\left(-{\omega_1^*}^2 - \mathrm{i}\omega_1^*\nu_{\text{f}} \right) \varepsilon_{\text{b}y} = \frac{\mathrm{i}e}{m} E_2 \, \mathrm{e}^{-\mathrm{i}\theta^*}, \tag{39}$$

where $\theta = kz - \omega t + \mathrm{i}\gamma t$ and $\theta^* = kz - \omega t - \mathrm{i}\gamma t.$

The real parts of the components of the displacements and current are

$$\varepsilon_{\text{b}x} = \frac{AeE_1 \, \mathrm{e}^{-\gamma t}}{m(A^2 + B^2)} \cos\theta_0, \qquad \varepsilon_{\text{b}y} = \frac{AeE_2 \, \mathrm{e}^{-\gamma t}}{m(A^2 + B^2)} \sin\theta_0, \tag{40}$$

$$\varepsilon_{\text{f}x} = \frac{CeE_1 \, \mathrm{e}^{-\gamma t}}{m(C^2 + D^2)} \cos\theta_0, \qquad \varepsilon_{\text{f}y} = \frac{CeE_2 \, \mathrm{e}^{-\gamma t}}{m(C^2 + D^2)} \sin\theta_0, \tag{41}$$

$$j_{\text{b}x} = \frac{Ae^2 n_{0\text{b}} E_1 \, \mathrm{e}^{-\gamma t}}{m(A^2 + B^2)} (\omega \sin\theta_0 - \gamma \cos\theta_0),$$

$$j_{\text{b}y} = \frac{Ae^2 n_{0\text{b}} E_2 \, \mathrm{e}^{-\gamma t}}{m(A^2 + B^2)} (\omega \cos\theta_0 - \gamma \sin\theta_0), \tag{42}$$

$$
\begin{aligned}
j_{\mathrm{f}x} &= \frac{Ce^2 n_{0\mathrm{f}} E_1 \,\mathrm{e}^{-\gamma t}}{m(C^2+D^2)}(\omega \sin\theta_0 - \gamma\cos\theta_0), \\
j_{\mathrm{f}y} &= \frac{Ce^2 n_{0\mathrm{f}} E_2 \,\mathrm{e}^{-\gamma t}}{m(C^2+D^2)}(\omega \cos\theta_0 - \gamma\sin\theta_0),
\end{aligned}
\tag{43}
$$

where

$$
\theta_0 = kz - \omega t, \qquad A = \omega_0^2 - \omega^2 + \gamma^2 - \nu_{\mathrm{b}}\gamma,
$$
$$
B = \omega(2\gamma - \nu_{\mathrm{b}}), \qquad C = \gamma^2 - \nu_{\mathrm{f}}\gamma - \omega^2 \qquad \text{and} \qquad D = \omega(2\gamma - \nu_{\mathrm{f}}).
$$

The zeroth-harmonic part of the induced magnetization is

$$
\begin{aligned}
H_z^{\mathrm{in}} = -\frac{eE_1E_2\,\mathrm{e}^{-2\gamma t}}{2mc}\Bigg[& \frac{\omega\omega_{\mathrm{b}}^2(\omega_0^2-\omega^2+\gamma^2-\nu_{\mathrm{b}}\gamma)^2}{\left\{(\omega_0^2-\omega^2+\gamma^2-\nu_{\mathrm{b}}\gamma)^2+\omega^2(2\gamma-\nu_{\mathrm{b}})^2\right\}^2} \\
& + \frac{\omega\omega_{\mathrm{f}}^2(\gamma^2-\omega^2-\nu_{\mathrm{f}}\gamma)^2}{\left\{(\gamma^2-\omega^2-\nu_{\mathrm{f}}\gamma)^2+\omega^2(2\gamma-\nu_{\mathrm{f}})^2\right\}^2} \\
& + \frac{\omega(\omega_{\mathrm{b}}^2+\omega_{\mathrm{f}}^2)(\omega_0^2-\omega^2+\gamma^2-\nu_{\mathrm{b}}\gamma)}{(\omega_0^2-\omega^2+\gamma^2-\nu_{\mathrm{b}}\gamma)^2+\omega^2(2\gamma-\nu_{\mathrm{b}})^2} \\
& \times \frac{\gamma^2-\omega^2-\nu_{\mathrm{f}}\gamma}{(\gamma^2-\omega^2-\nu_{\mathrm{f}}\gamma)^2-\omega^2(2\gamma-\nu_{\mathrm{f}})^2}\Bigg].
\end{aligned}
\tag{44}
$$

Case I: $\omega^2 = \omega_0^2$

For large ω compared to ν_{b}, ν_{f} and γ, we obtain

$$
\begin{aligned}
H_z^{\mathrm{in}} &= -\frac{eE_1E_2\,\mathrm{e}^{-2\gamma t}}{2mc}\Bigg\{\omega_{\mathrm{b}}^2\left[\frac{\gamma^2(\gamma-\nu_{\mathrm{b}})^2}{\omega_0^3(2\gamma-\nu_{\mathrm{b}})^4} - \frac{\gamma(\gamma-\nu_{\mathrm{b}})}{\omega_0^3(2\gamma-\nu_{\mathrm{b}})^2}\right] \\
&\quad + \omega_{\mathrm{f}}^2\left[\frac{1}{\omega^3} - \frac{\gamma(\gamma-\nu_{\mathrm{b}})}{\omega_0^3(2\gamma-\nu_{\mathrm{b}})^2}\right]\Bigg\}.
\end{aligned}
\tag{45}
$$

It can be written as

$$
H_z^{\mathrm{in}} = -\frac{eE_1E_2\,\mathrm{e}^{-2\gamma t}}{2mc}\left[\frac{\gamma(\gamma-\nu_{\mathrm{b}})\omega_{\mathrm{b}}^2}{\omega_0^3(2\gamma-\nu_{\mathrm{b}})^2} - \frac{\omega_{\mathrm{f}}^2}{\omega_0^3}\right]\left[\frac{\gamma(\gamma-\nu_{\mathrm{b}})}{(2\gamma-\nu_{\mathrm{b}})^2} - 1\right]. \tag{46}
$$

Following (20), we put $\gamma = \frac{1}{2}(\nu_{\mathrm{b}} + \omega_{\mathrm{f}}^2\nu_{\mathrm{f}}/\omega_0^2)$ in (45) and obtain

$$
H_z^{\mathrm{in}} = -\frac{eE_1E_2\mathrm{e}^{-2\gamma t}}{2mc}\frac{1}{4\omega_0^3}\left[3 + \frac{\omega_0^4\nu_{\mathrm{b}}^2}{\omega_{\mathrm{f}}^4\nu_{\mathrm{f}}^2}\right]\left[\frac{\omega_{\mathrm{b}}^2}{4}\left(1 - \frac{\omega_0^4\nu_{\mathrm{b}}^2}{\omega_{\mathrm{f}}^4\nu_{\mathrm{f}}^2}\right) - \omega_{\mathrm{f}}^2\right]. \tag{47}
$$

The numerical estimates for the zeroth-harmonic magnetic fields have been made for plasma induced by Nd-glass laser [wavelengths are given in μm, pulse lengths in nano seconds (ns) and intensities in W/cm^2]. Numerical results show (see Fig. 1) that for $\omega_\mathrm{f} > \omega_0$, the induced field is enhanced and it has higher value for lower values of ω_0 which is shown by dotted lines. When $\nu_\mathrm{b}^0 \approx \nu_\mathrm{f}^0$ and $\omega_0 > \omega_\mathrm{f}$, the induced field is enhanced and it becomes paramagnetic in nature which is shown by solid lines. Nature of the field depends on both $n_{0\mathrm{b}}$ and $n_{0\mathrm{f}}$. If $n_{0\mathrm{b}} > n_{0\mathrm{f}}$, then the induced field is diamagnetic. In the reverse case it is paramagnetic.

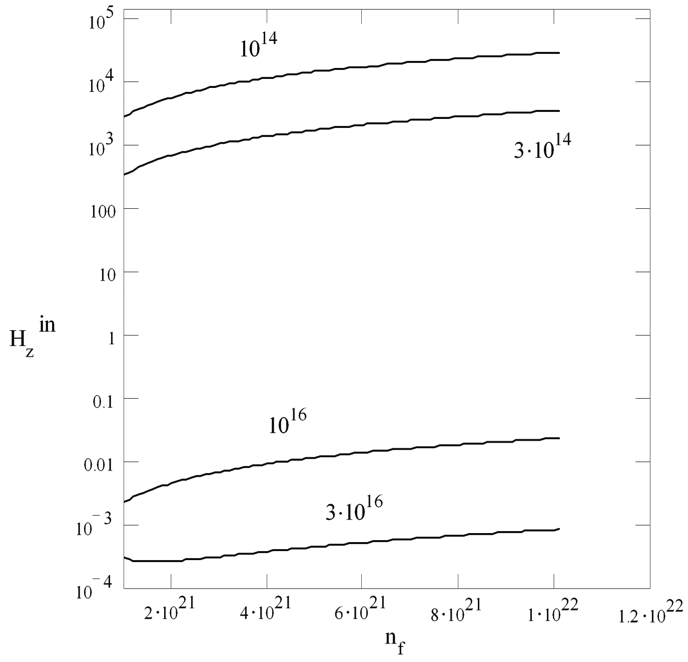

Fig. 1. Variation of the logarithms of the zeroth harmonic of the induced magnetic field in Gauss with number density of free electrons (n_f) for incident pulsed laser beam of pulse length $\tau = 5\,\mathrm{ns}$, wavelength $\lambda = 1.06\,\mu\mathrm{m}$ and beam intensity $I = 10^{15}$ W/cm^2, for four values of ω_0 (expressed in s^{-1}).

Case II: $\omega^2 \neq \omega_0^2$

For $(\omega_0, \omega) > (\nu_\mathrm{b}, \nu_\mathrm{f}, \gamma)$, Eq. (44) reduces to

$$\begin{aligned} H_z^{\mathrm{in}} &= -\frac{eE_1E_2\mathrm{e}^{-2\gamma t}}{2mc}\frac{(2\omega^2-\omega_0^2)}{\omega(\omega_0^2-\omega^2)}\left[\frac{\omega_\mathrm{b}^2}{(\omega_0^2-\omega^2)} - \frac{\omega_\mathrm{f}^2}{\omega_0^2}\right] \\ &\approx -\frac{eE_1E_2\,\mathrm{e}^{-2\gamma t}}{2mc\,\omega^3}\left(\omega_\mathrm{b}^2+\omega_\mathrm{f}^2\right) \quad \text{when} \quad \omega^2 \gg \omega_0^2. \end{aligned} \tag{48}$$

In this case the plasma is diamagnetic.

5. *Energy absorption*

The energy flux of the propagating modes in a dissipative plasma medium is the sum of the electromagnetic flux and the kinetic flux due to the correlated movement of particles with the applied wave field. The zero-frequency part of the kinetic flux is

$$\begin{aligned}\langle(\vec{E}\cdot\vec{j})\rangle &= -\frac{\gamma e^2\,\mathrm{e}^{-2\gamma t}}{8\pi}(E_1^2+E_2^2)\left[\frac{\omega_\mathrm{b}^2(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)}{(\omega_0^2-\omega^2+\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{b})^2}\right.\\ &\left.+\frac{\omega_\mathrm{f}^2(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)}{(\gamma^2-\omega^2-\nu_\mathrm{f}\gamma)^2+\omega^2(2\gamma-\nu_\mathrm{f})^2}\right].\end{aligned} \tag{49}$$

It represents the exchange of energy between the applied electromagnetic field and the plasma constituents.

Case I: Resonant absorption

When $\omega=\pm\omega_0$ and $\omega>(\nu_\mathrm{b},\nu_\mathrm{f},\gamma)$, Eq. (49) becomes

$$\langle(\vec{E}\cdot\vec{j})\rangle = -\frac{\gamma e^2\,\mathrm{e}^{-2\gamma t}}{8\pi}(E_1^2+E_2^2)\left[\frac{\omega_\mathrm{b}^2(\gamma^2-\nu_\mathrm{b}\gamma)}{(\gamma^2-\nu_\mathrm{b}\gamma)^2+\omega_0^2(2\gamma-\nu_\mathrm{b})^2}-\frac{\omega_\mathrm{f}^2}{\omega_0^2}\right]. \tag{50}$$

Case II: Non-resonant absorption

For $\omega\neq\omega_0$ and $(\omega,\omega_0)>(\nu_\mathrm{b},\nu_\mathrm{f}b,\gamma)$, we find that

$$\langle(\vec{E}\cdot\vec{j})\rangle = -\frac{\gamma e^2\,\mathrm{e}^{-2\gamma t}}{8\pi}(E_1^2+E_2^2)\left[\frac{\omega_\mathrm{b}^2}{\omega_0^2-\omega^2}-\frac{\omega_\mathrm{f}^2}{\omega^2}\right]. \tag{51}$$

6. *Discussion*

In the resonant case ($\omega^2=\omega_0^2$), the dispersion relation, Eq. (19), and the expression in Eq. (20) for the damping factor have been derived for some physically valid approximations. The damping factor depends on the collision frequency of both types of electrons (bound and free), the plasma frequency ω_f of free electrons, and ω_0, the natural frequency of bound electrons, but not on ω_b, the plasma frequency of bound electrons. The cut-off frequency of the medium is modified in the presence of bound electrons. It decreases when ω_f is large compared to ω_0. But in the reverse case, the cut-off frequency is increased in the presence of bound electrons compared to that in a plasma where bound electrons are also replaced by free electrons. In the non-resonant case, the cut-off frequency increases in the presence of classically bound electrons.

The zeroth-harmonic part of the generated field of magnetic moment, in the resonant case ($\omega^2=\omega_0^2$), is enhanced and is paramagnetic in nature when ω_0 (or ω)

is large compared to ω_f, the characteristic frequency of free electrons. In the reverse case, the induced field is comparatively small and the nature of the field depends on the number density of both types of electrons. If the number density of bound electrons is very small compared to that of free electrons, then the moment field is diamagnetic in nature. In the reverse case it is paramagnetic. The expressions for the DC part of the kinetic flux (the exchange of energy between the field and the plasma constituents), $(\vec{E} \cdot \vec{j})$, has been derived, both for the non-resonant and resonant interaction of a wave field with the plasma.

Steiger and Woods [3] found that for Nd-glass laser ($I = 10^{17}$ W/cm^2, and $N = 10^{21}$/cm^3, where I is the intensity of the laser beam and N is the plasma number density), the magnetic moment is of the order of 5.48×10^5 G for waves of circular polarization. Deschamps et al. [8] observed experimentally uniform magnetization ($\approx 10^{-2}$ G) in plasma using pulsed microwaves (1 MW) of frequency of 3000 MHz.

It is to be mentioned here that different kinds of mechanisms (e.g. thermo-electric process, radiation process, filamentation, resonance absorption, Weibel instability, dynamo effect, ion-acoustic turbulence etc.) for the generation of large- and small-scale toroidal magnetic field have been proposed by various authors [9–15]. Generation of induced magnetization arising out of nonlinear optical response of the plasma has been theoretically investigated by Chakraborty et al. [16, 17] and others [18, 19]. Stamper et al. [20] has reviewed the various applications of such magnetic field in laser-produced plasma. However, Sudan [21] showed that axial magnetic field of gigagauss strength may be produced owing to the electron currents driven by spatial gradients and temporal variations of the ponderomotive force. Recently, Bhattacharyya et al. [22] theoretically investigated simultaneous generation of toroidal and poloidal magnetic fields in an underdense regoin of a laser produced plasma considering two-fluid model and neglecting the effect of dissipation. They showed that toroidal fields decrease with increasing pulse lengths and increase rather slowly with an increase in laser wavelengths. But the poloidal fields are insensitive to the laser pulse lengths and they increase exponentially with the laser wavelengths. All above mentioned authors did not consider the effect of bound electrons for the generation of the field of magnetic moment in the plasma. In our future work, we shall study the generation of magnetic field due to inverse Faraday effect in plasma, taking the collective effects of bound electrons, free electrons, free ions from which we may get some new information regarding self-generated field of magnetic moment in the plasma.

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SKUPNI UČINCI VEZANIH I SLOBODNIH ELEKTRONA TE IONA U MEĐUDJELOVANJU VALOVA I PLAZME

Proučavamo značajke širenja valova male amplitude u hladnoj plazmi za koju pretpostavljamo da je fluidna, stišljiva i smjesa slabo vezanih i slobodnih elektrona te iona. Za eliptički polarizirane valove izveli smo (i) magnetski moment prvog harmonika, (ii) faktor gušenja i graničnu frekvenciju i (iii) sudarnu apsorpciju energije. Istražujemo relativnu ulogu frekvencije vezanih elektrona (ω_0), karakteristične frekvencije slobodnih elektrona (ω_{f}), karakteristične frekvencije vezanih elektrona (ω_{b}) i rezonantnog uvjeta kada je ω_0 jednako frekvenciji valova ω te učinaka granične frekvencije. Granična je frekvencija viša u slučaju rezonancije ($\omega^2 = \omega_0^2$), ona raste ako je ω_0 manji od ω_{f}, a smanjuje se ako je ω_0 veći od ω_{f}. Magnetski moment polja je na rezonanciji povećan ako je ω_0 veći od ω_{f}. Razmatramo i izmjenu energije između valnog polja i sastavnica plazme.