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ONE-MODE INTERACTION WITH A FOUR-LEVEL ATOM IN A MOMENTUM EIGENSTATE

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We study the interaction between a four-level atom (ladder type) in a momentum eigenstate with a single mode cavity field in the presence of non-linearities of both the field and the intensity-dependent coupling. The constants of motion and the wave function for the atomic system have been obtained. Special attention is given to discuss some statistical aspects of the considered atomic system such as momentum increment, momentum diffusion and high-order squeezing. The influence of the Kerr-like medium and the intensity dependent coupling on the momentum increment and the high-order squeezing are investigated numerically. It is found that addition of these parameters has an important effect on both the momentum increment and the squeezing phenomenon.

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1. Introduction

The Hamiltonian of the Jaynes–Cummings model (JCM) [1] is the often preferred framework for describing the interaction of an atom with the electromagnetic field. In this model, the cavity field retains only one degree of freedom and the atom is a two-level system. In the rotating wave approximation (RWA), it allows exact solutions. This model can also be studied experimentally [2]. The two-photon process and its multiphoton counterparts are important because they can be used to study statistical properties of the optical field and may produce several non-classical effects [3]. On the other hand, there is a growing interest in nonlinear quantum dynamics [4]. It has been limited to the two-level atom [5] and extended to investigate the interaction of the three-level atom with the excited field with two-photon resonance transition [6]. Over the years, the JCM has been extended to include the atomic external effects due to quantization of atomic motion, where the centre-of-mass motion of an atom is cooled to extremely low temperature, so that vibrational motion is quantized [7 − 10]. By cosidering ultracoled atoms, Scully et al. [11] discovered that the quantum treatment of the centre-of-mass motion of an atom leads to a completely new kind of induced emission in the cavity. Recently, the motion has been treated quantum mechanically [12] for a single mode and for a two-mode interacting with a three-level atom.

The photon statistics of light emitted from a system comprising a single four-level atom strongly coupled to a single mode cavity field is studied [13]. The effect of the detuning on the collapse and revival phenomena is investigated for a four-level atom (ladder type) interacting with a single-mode cavity field, when the atom is initially prepared in coherent superposition of its upper and ground state [14]. The four-level system can be used to reproduce the dynamics of the simpler two-level atom, including the linewidth narrowing associated with reduced quadrature fluctuations [15 − 18]. Also, the interaction between a four-level atom and a three-mode field has been studied [19]. In this paper, we aim to study the problem of a moving four-level atom interacting with a single mode including all acceptable kinds of nonlinearities of both the field and the intensity-dependent coupling.

The observation of the phenomena of collapses and revivals shifted studies of this model from mere academic ones into the experimental realm, like the testing some other nonclassical effects that the model is capable of producing and the squeezing effects. The squeezed states of quantum systems have been an active area of interest for more than a decade [20]. By squeezing one usually means that the mean-square deviation of a certain quadrature of the field is smaller than its corresponding value when the field in the coherent state. The JCM has been investigated and it was found to produce squeezing effects in the one-photon and multi-photon interactions [21 − 26]. It has recently been generalized by Hong and Mandel [22], who introduced the concept of higher-order squeezing. Many systems, such as those producing resonance fluorescence, degenerate parametric down conversion, harmonic generation and the multiphoton JCM, have been analyzed for higher-order squeezing [22 − 28].

The plan of this paper is as follows. In Sec. 2, we introduce the problem and obtain the wave function of the system by using Schröodinger equation, when the input is a coherent field and the atom is initially in its upper state. In Sec. 3, we discuss some statistical aspects of the considered atomic system, such as momentum increment, momentum diffusion and high-order squeezing. In Sec. 4, we investigate numerically the influence of the Kerr-like medium and the intensity-dependent coupling on both the momentum increment and the high-order squeezing. The paper concludes with a discussion in Sec. 5.

2. *The model and the wave function*

In this paper, we consider a four-level atom in the ladder (Ξ) configuration, interacting with a single-mode cavity field. The four levels are respectively denoted by $|j\rangle$ ($j = 1, 2, 3, 4$) with energies ω_j and the field is of frequency Ω, and with annihilation (creation) operator $\hat{a}$ ($\hat{a}^\dagger$). The total Hamiltonian ($\hat{H}$) for the considered system in the RWA via m-photon is the sum of the atom-field Hamiltonian ($\hat{H}_{\rm AF}$) and the interaction Hamiltonian ($\hat{H}_{\rm IN}$)

$$\hat{H} = \hat{H}_{\rm AF} + \hat{H}_{\rm IN} \,, \tag{1}$$

where

$$\hat{H}_{\rm AF} = \frac{\hat{\vec{P}}^2}{2M} + \sum_{j=1}^{4} \omega_j \hat{\sigma}_{jj} + \Omega \hat{a}^\dagger \hat{a} \tag{2}$$

and

$$\hat{H}_{\rm IN} = g(\hat{n}) + \sum_{l=1}^{3} \lambda_l \left[\hat{R} \hat{\sigma}_{l,l+1} + \hat{R}^\dagger \hat{\sigma}_{l+1,l} \right] , \tag{3}$$

where $\hat{\vec{P}}$ is the centre-of-mass momentum operator, $\hat{\sigma}_{ij}$ are the lowering and raising operators between levels i and j defined by $\hat{\sigma}_{ij} = |i\rangle\langle j|$, $\hat{R} = \hat{a}^m f(\hat{n}) {\rm e}^{{\rm i}m\vec{k}\cdot\vec{r}}$, $f(\hat{n})$ is an arbitrary intensity dependent atom-field coupling [29]; $\vec{k}$ and $\vec{r}$ are the propagation and position vectors, respectively, $g(\hat{n})$ is the one-mode field nonlinearity, λ_1, λ_2 and λ_3 are the m-photon coupling constants corresponding to the atomic transitions $|1\rangle \to |2\rangle$, $|2\rangle \to |3\rangle$ and $|3\rangle \to |4\rangle$, respectively (see Fig. 1).

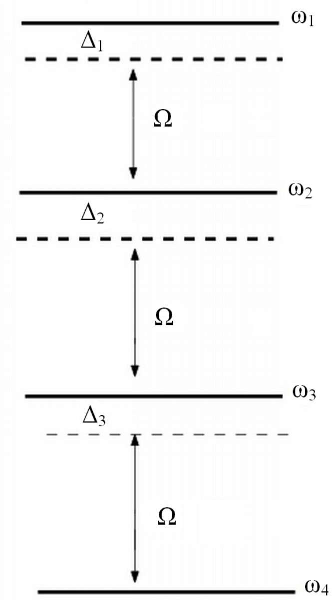

Fig. 1. Schematic representation for a four-level atom interacting with a single mode.

Notice that, when the centre of mass is not taken into account, then $g(n) = 0$, $m = 1$, $f(n) = const$ and $\lambda_2 = \lambda_3 = 0$, and we have the standard Jaynes–Cummings model. But if the centre of mass is taken into account, the two-level system [7] is obtained. Also, if one take $\lambda_3 = 0$, the three-level atom in the cascade type [30,31] is obtained.

The field operators $\hat{R}$, $\hat{R}^\dagger$ and the photon number operator $\hat{n} = \hat{a}^\dagger \hat{a}$ satisfy the following commutation relations

$$\big[\hat{R}, \hat{n}\big] = m\hat{R}, \qquad \big[\hat{R}^\dagger, \hat{n}\big] = -m\hat{R}^\dagger \tag{4}$$

and the atomic operators $\hat{\sigma}_{ij}$ satisfy the relation

$$\big[\hat{\sigma}_{ij}, \hat{\sigma}_{kl}\big] = \hat{\sigma}_{il}\delta_{jk} - \hat{\sigma}_{kj}\delta_{il}\,. \tag{5}$$

According to the Heisenberg equation and the previous relations, we have the following conservations of the atomic probability, excitation number and of atomic momentum of one mode

$$\sum_{j=1}^{4} \hat{\sigma}_{jj} = \hat{I}, \qquad \hat{a}^\dagger \hat{a} + m\big(2\hat{\sigma}_{11} + \hat{\sigma}_{22} - \hat{\sigma}_{44}\big) = \hat{N}_1, \text{ and } \hat{\vec{P}} + \vec{k}\hat{a}^\dagger \hat{a} = \hat{\vec{N}}_0\,. \tag{6}$$

Finding the wave function $|\Psi(t)\rangle$ at any time t for the considered atomic system is now straightforward. Let us write down the time dependent Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle\,,$$

and assume the wave function at time $t > 0$ in the closed form

$$|\Psi(t)\rangle = \sum_n q_n \left[\sum_{j=1}^{4} A_j(n+(j-1)m, t)\mathrm{e}^{-\mathrm{i}\gamma_j t}|\vec{P}_0 - (j-1)m\vec{k}, n+(j-1)m, j\rangle\right], \tag{7}$$

where the coefficients A_j are the probability amplitudes, $|\vec{P}_0\rangle$ is the initial momentum eigenstate, $|j\rangle$ denotes the jth level atom, n is the photon of the field and

$$\gamma_j = \frac{\big(\vec{P}_0 - (j-1)m\vec{k}\big)^2}{2M} + \omega_j + \Omega\big(n + (j-1)m\big)\,. \tag{8}$$

By using the Schrödinger equation and the following properties

$$\begin{aligned}
\mathrm{e}^{\pm \mathrm{i}m\vec{k}\cdot\vec{r}}|\vec{P}_0\rangle = |\vec{P}_0 \mp m\vec{k}\rangle\,, \qquad & \hat{\vec{P}}_0|\vec{P}_0\rangle = \vec{P}_0|\vec{P}_0\rangle\,, \\
\hat{a}^m f(\hat{n})|n\rangle \quad = \quad & \sqrt{\frac{n!}{(n-m)!}}f(n)|n-m\rangle\,, \\
f(\hat{n})\hat{a}^{\dagger m}|n\rangle \quad = \quad & \sqrt{\frac{(n+m)!}{n!}}f(n+m)|n+m\rangle\,, \\
\sigma_{ab}|b\rangle \quad = \quad & |a\rangle\,,
\end{aligned} \tag{9}$$

we obtain the following system of differential equations:

$$\begin{aligned}
\mathrm{i}\dot{A}_1(n,t) &= V_1A_1(n,t) + W_1A_2(n+m,t)\mathrm{e}^{-\mathrm{i}\Delta_1 t}, \\
\mathrm{i}\dot{A}_2(n+m,t) &= V_2A_2(n+m,t) + W_1A_1(n,t)\mathrm{e}^{\mathrm{i}\Delta_1 t} + W_2A_3(n+2m,t)\mathrm{e}^{-\mathrm{i}\Delta_2 t}, \\
\mathrm{i}\dot{A}_3(n+2m,t) &= V_3A_3(n+2m,t) + W_2A_2(n+m,t)\mathrm{e}^{\mathrm{i}\Delta_2 t} \\
&\quad + W_3A_4(n+3m,t)\mathrm{e}^{-\mathrm{i}\Delta_3 t}, \\
\mathrm{i}\dot{A}_4(n+3m,t) &= V_4A_4(n+3m,t) + W_3A_3(n+2m,t)\mathrm{e}^{\mathrm{i}\Delta_3 t},
\end{aligned} \tag{10}$$

where

$$V_j = g\big(n+(j-1)m\big), \qquad W_l = \lambda_l\sqrt{\frac{(n+lm)!}{(n+(l-1)m)!}}f(n+lm), \tag{11}$$

and the detuning parameters are

$$\begin{aligned}
\Delta_1 &= \omega_2-\omega_1+m\Omega-\frac{m\vec{k}\cdot\vec{P}_0}{M}+\frac{m^2k^2}{2M}, \\
\Delta_2 &= \omega_3-\omega_2+m\Omega-\frac{m\vec{k}\cdot\vec{P}_0}{M}+\frac{3m^2k^2}{2M}, \\
\Delta_3 &= \omega_4-\omega_3+m\Omega-\frac{m\vec{k}\cdot\vec{P}_0}{M}+\frac{5m^2k^2}{2M}.
\end{aligned} \tag{12}$$

The previous detuning parameters are with recoil energy proportional to $k^2/(2M)$ of the atom and the Doppler shift is proportional to $\vec{k}\cdot\vec{P}_0/M$.

To solve the system (10), we assume that $A_4(n+3m,t) = \exp\{\mathrm{i}\mu t\}$. Substituting into Eq. (10), we find that μ satisfies the following fourth-order equation

$$\mu^4 + x_1\mu^3 + x_2\mu^2 + x_3\mu + x_4 = 0\,, \tag{13}$$

where

$$\begin{aligned}
x_1 &= -\Delta_1-\Delta_2-\Delta_3+V_1+\zeta_1\,, \\
x_2 &= -\zeta_1(\Delta_1+\Delta_2+\Delta_3)+\zeta_2+V_1\zeta_1-W_1^2\,, \\
x_3 &= -\zeta_2(\Delta_1+\Delta_2+\Delta_3)+\zeta_3+V_1\zeta_2-\eta_1W_1^2\,, \\
x_4 &= -\zeta_3(\Delta_1+\Delta_2+\Delta_3)+V_1\zeta_3-\eta_2W_1^2\,,
\end{aligned} \tag{14}$$

with

$$\zeta_1 = -2\Delta_3-\Delta_2+V_2+V_3+V_4\,,$$

$$
\begin{aligned}
\zeta_2 &= \eta_1(-\Delta_3 - \Delta_2 + V_2) + \eta_2 + W_2^2 \,, \\
\zeta_3 &= \eta_2(-\Delta_3 - \Delta_2 + V_2) + V_4 W_2^2 \,, \\
\eta_1 &= V_3 + V_4 - \Delta_3 \,, \\
\eta_2 &= -V_4\Delta_3 + V_3 V_4 - W_3^2 \,.
\end{aligned} \tag{15}
$$

Now, to obtain the solution of the Schrödiger equation corresponding to the Hamiltonian (1), we can write $A_4(n+3m,t) = \sum_{j=1}^{4} C_j \exp\{\mathrm{i}\mu_j t\}$, and insert it into Eq. (13). When the atom is initially in the excited state, i.e. $A_1(n,0) = 1$ and $A_2(n+m,0) = A_3(n+2m,0) = A_4(n+3m,0) = 0$, we obtain the following solutions of the coupled system

$$
\begin{aligned}
A_1(n,t) &= -\sum_{j=1}^{4} \frac{1}{W_1 W_2 W_3}\big(\mu_j^3 + \zeta_1\mu_j^2 + \zeta_2\mu_j + \zeta_3\big) C_j \mathrm{e}^{\mathrm{i}(\mu_j - \Delta_1 - \Delta_2 - \Delta_3)t}, \\
A_2(n+m,t) &= \sum_{j=1}^{4} \frac{1}{W_2 W_3}\big(\mu_j^2 + \eta_1\mu_j + \eta_2\big) C_j \mathrm{e}^{\mathrm{i}(\mu_j - \Delta_2 - \Delta_3)t}, \\
A_3(n+2m,t) &= -\sum_{j=1}^{4} \frac{1}{W_2 W_3}\big(\mu_j + V_4\big) C_j \mathrm{e}^{\mathrm{i}(\mu_j - \Delta_3)t}, \\
A_4(n+3m,t) &= \sum_{j=1}^{4} C_j \mathrm{e}^{\mathrm{i}\mu_j t},
\end{aligned} \tag{16}
$$

where

$$
C_j = -\frac{W_1 W_2 W_3}{\mu_{jk}\mu_{jp}\mu_{jq}}, \quad k \neq p \neq q \neq j \quad \text{and} \quad \mu_{jk} = \mu_j - \mu_k \,. \tag{17}
$$

Having obtained the wave function $|\Psi(t)\rangle$, we are in a position to discuss any property related to the atom and the field. In what follows, we discuss some statistical aspects such as the momentum increment, the momentum diffusion and the high-order squeezing when the field is assumed to be initially in a coherent state.

3. *Statistical aspects*

Once the wave function has been calculated, the expectation value of any dynamical operator $\hat{O}$ can be easily obtained through the formula $\langle\hat{O}\rangle = \langle\Psi(t)|\hat{O}|\Psi(t)\rangle$. Then for the generators $\hat{\sigma}_{jj}$ we obtain

$$
\langle\hat{\sigma}_{jj}\rangle = \sum_n P_n |A_i|^2, \tag{18}
$$

where $P_n = |q_n|^2$ stands for the initial photon distribution for a single mode in the coherent state, thus $P_n = \exp\{-\bar{n}\}\bar{n}^n/n!$, and $\bar{n}$ is the initial mean photon number of the coherent field.

The expectation values of the atomic momentum increment $\langle\Delta\vec{P}\rangle = \langle\vec{P}\rangle - \vec{P}_0$ and the momentum diffusion $\langle(\Delta\vec{P})^2\rangle = \langle\vec{P}^2\rangle - \langle\vec{P}\rangle^2$ can be calculated in the same manner,

$$\begin{aligned}\langle\Delta\vec{P}\rangle &= -m\vec{k}\big(\langle\hat{\sigma}_{22}\rangle + 2\langle\hat{\sigma}_{33}\rangle + 3\langle\hat{\sigma}_{44}\rangle\big)\,, \\ \langle(\Delta\vec{P})^2\rangle &= m^2k^2\big[\langle\hat{\sigma}_{22}\rangle + 4\langle\hat{\sigma}_{33}\rangle + 9\langle\hat{\sigma}_{44}\rangle - \big(\langle\hat{\sigma}_{22}\rangle + 2\langle\hat{\sigma}_{33}\rangle + 3\langle\hat{\sigma}_{44}\rangle\big)^2\big]\,.\end{aligned} \tag{19}$$

Also, the expectation value in the general form for the field operation $\hat{a}^{\dagger r}\hat{a}^s$ is in the form

$$\begin{aligned}\langle\hat{a}^{\dagger r}\hat{a}^s\rangle &= \sum_n q^*_{n+r}q_{n+s}\Bigg[\sum_{j=1}^{4} A^*_j(n+(j-1)m+r)A_j(n+(j-1)m+s) \\ &\quad\times\sqrt{\frac{(n+r+m(j-1))!(n+s+m(j-1))!}{(n+m(j-1))!}}\Bigg].\end{aligned} \tag{20}$$

Employing the previous calculations, we shall be able to investigate the high-order squeezing phenomenon. We discuss this phenomenon in terms of the field quadrature operators X and Y, which are defined as

$$X = \frac{1}{2}(\hat{a} + \hat{a}^\dagger) \quad \text{and} \quad Y = \frac{1}{2\mathrm{i}}(\hat{a} - \hat{a}^\dagger)\,. \tag{21}$$

These operators satisfy the commutation relation

$$\big[X, Y\big] = \frac{\mathrm{i}}{2}\,. \tag{22}$$

The commutation relation implies the uncertainly relation

$$(\Delta X)^2(\Delta Y)^2 \geq \frac{1}{4}|\langle[X,Y]\rangle|^2, \tag{23}$$

$$\text{or} \qquad (\Delta X)^2(\Delta Y)^2 \geq \left(\frac{1}{4}\right)^2. \tag{24}$$

The field is said to be squeezed to second order (or normal squeezed) [21] if

$$(\Delta X)^2 < \frac{1}{4} \quad \text{or} \quad (\Delta Y)^2 < \frac{1}{4}\,. \tag{25}$$

As discussed in Refs. [22,23], the higher-order moments of the field can exhibit a nonclassical behaviour called higher-order squeezing, that is, when the Nth order moment of the quadrature operator $(\Delta X)^N$ is smaller than its value in a completely coherent state of the field. That is when

$$(\Delta X)^N < (N-1)!!\left(\frac{1}{4}\right)^{N/2}. \tag{26}$$

The above condition is uniquely non-classical for the even moment. We consider here the fourth-order variance ($N = 4$) which can be written as

$$(\Delta X)^4 = \langle X^4\rangle - 4\langle X^3\rangle\langle X\rangle + 6\langle X^2\rangle\langle X\rangle^2 - 3\langle X\rangle^4, \tag{27}$$

where

$$\begin{aligned}
\langle X^2\rangle &= \frac{1}{4}\left(\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle + 2\langle a^\dagger\hat{a}\rangle + 1\right), \\
\langle X^3\rangle &= \frac{1}{8}\left[\langle\hat{a}^3\rangle + \langle\hat{a}^{\dagger 3}\rangle + 3(\langle\hat{a}^\dagger\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\hat{a}\rangle) + 3(\langle\hat{a}^\dagger\rangle + \langle\hat{a}\rangle)\right], \\
\langle X^4\rangle &= \frac{1}{16}\left[\langle\hat{a}^4\rangle + \langle\hat{a}^{\dagger 4}\rangle + 4(\langle\hat{a}^\dagger\hat{a}^3\rangle + \langle\hat{a}^{\dagger 3}\hat{a}\rangle) + 6(\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle)\right. \\
&\quad \left. + 6\langle(a^\dagger\hat{a})^2\rangle + 6\langle a^\dagger\hat{a}\rangle + 3\right].
\end{aligned} \tag{28}$$

Clearly, from the condition (26), the fourth-order squeezing occurs when $(\Delta X)^4 < 3/16$, and by using Eq. (20) and specifying the exponents r, s, we get the expressions in Eq. (28).

4. *Results and conclusions*

In this section, we investigate the temporal behaviour for both the momentum increment $\vec{k}\cdot\langle\Delta\vec{P}\rangle$ and the fourth-order variance $(\Delta X)^4$ for the considered model when the atom is initially prepared in the upper state $|1\rangle$ and the field is in a coherent state. The evolution of the momentum increment and the high-order squeezing are plotted against the scaled time λt for fixed initial mean photon number $\bar{n} = 10$, in the exact resonance case ($\Delta_l = 0$). The interaction is considered to be a three-photon process ($m = 1$) and the coupling constants $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. In what follows, we study the effect of the intensity-dependent coupling and the Kerr medium [32 –34] ($g(\hat{n}) = \chi\hat{n}(\hat{n}-1)$; χ is related to the third-order nonlinear susceptibility) on the evolution of the previous phenomenon. The Kerr medium can be modeled as an anharmonic oscillator with frequency ω.

Figs. 2a, 2b and 2c show the influence of the Kerr medium on the time evolution of $\vec{k}\cdot\langle\Delta\vec{P}\rangle$ in the absence of the intensity-dependent coupling ($f(\hat{n}) = const$), that by taking $\chi = 0.0$, 0.3 and 0.6, respectively. On the other hand, the curves d, e and f show the effect of the intensity-dependent coupling ($f(\hat{n}) = \sqrt{\hat{n}}$, $\hat{n}$ and $1/\sqrt{\hat{n}}$) when $\chi = 0$. We notice that the momentum increment shows the collapse and

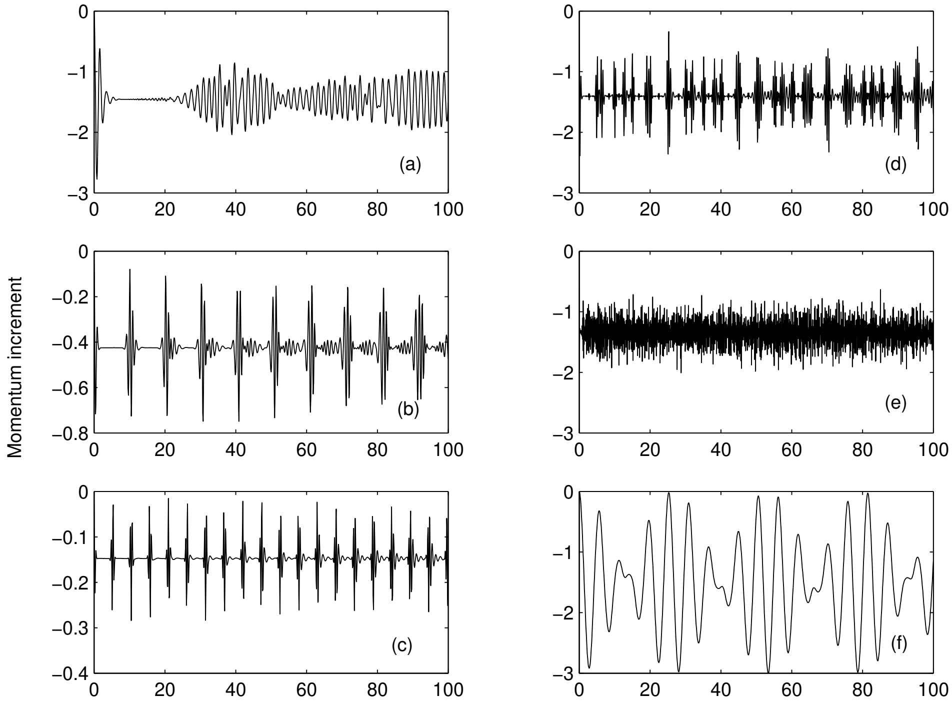

Fig. 2. The temporal behaviour of the momentum increment $\vec{k} \cdot \langle \Delta \vec{P} \rangle$ versus the scaled time λt when $f(\hat{n}) = const$, $\chi = 0$, 0.3 and 0.6 in Figs. a, b and c; $\chi = 0$, $f(\hat{n}) = \sqrt{\hat{n}}$, $\hat{n}$ and $1/\sqrt{\hat{n}}$ in Figs. d, e and f.

revival phenomena, where its behaviour is similar to the behaviour of the photon number $\langle \hat{a}^\dagger \hat{a} \rangle$ with a negative sign (see the constants of motion). Also, it behaves as a damped oscillator in the interval $(0 < \lambda t \leq 5)$ and completely damped in the interval $(5 < \lambda t \leq 25)$, while it oscillates at $(25 < \lambda t < 100)$ (see Fig. 2a). It is remarkable that the behaviour of the momentum increment in this case is similar to the case of a two-level atom [7]. On the other hand, when the Kerr medium takes place, taking different values of the Kerr medium parameter $\chi = 0.3$ and $\chi = 0.6$, one obtains the results shown in Figs. 2b and 2c. We see that the collapse and revival phenomenon occurs periodically, and this periodicity increases by increasing the Kerr-medium parameter.

Now, we turn our attention to investigate the effect of the intensity-dependent coupling $f(\hat{n})$ on the behaviour of the momentum increment in the absence of the Kerr medium, and take $f(\hat{n}) = \sqrt{\hat{n}}$, $\hat{n}$ and $1/\sqrt{\hat{n}}$ as shown in Figs. 2d, 2e and 2f, respectively. We observe that the oscillations increase in the presence of the intensity-dependent coupling, and a chaotic behaviour of the momentum increment occurs (see Figs. 2d and 2e). The case in which $f(\hat{n}) = 1/\sqrt{\hat{n}}$ (Fig. 2f) is quite interesting, where in this case the momentun increment oscillates periodically as the scaled time goes on. Formally, when we take $f(\hat{n}) = 1/\sqrt{\hat{n}}$, it leads to $W_l = \lambda_l$, with $\chi = 0$ and this implies that $A_i(t)$ are independent of n. In this case, the momentum increment depends on the function $\exp\{i\mu t\}$, which reflects the occurence of the periodic oscillations.

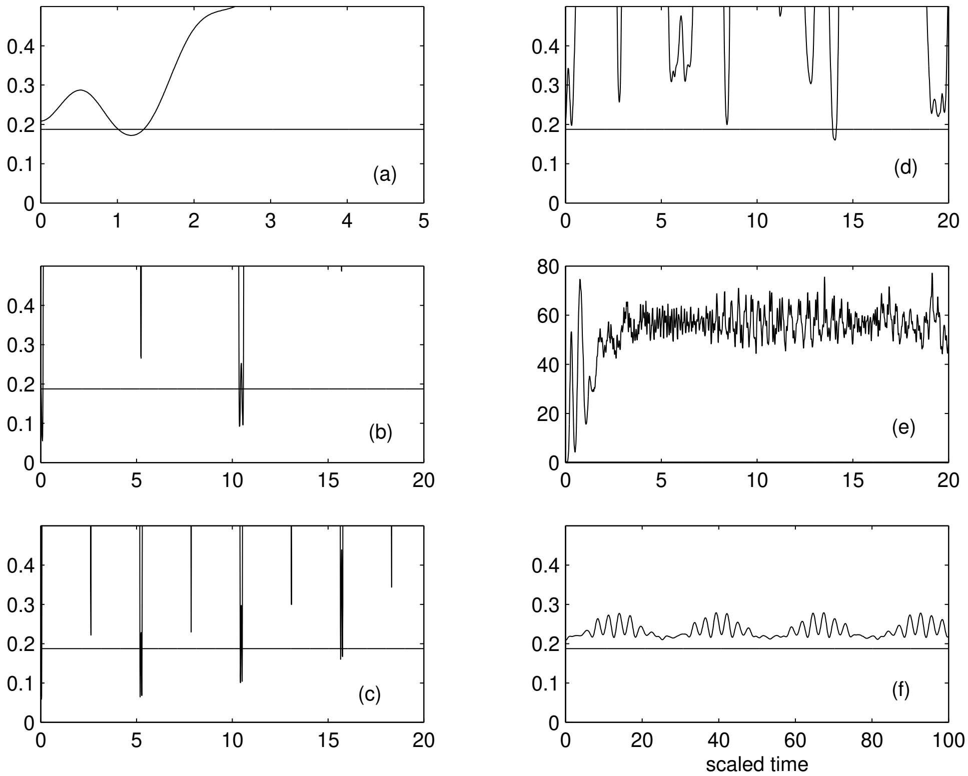

Fig. 3. The same as in Fig. 2 but for the high-order squeezing $(\Delta X)^4$.

In Fig. 3, we consider the time development of the fourth-order variance for the considered model. Numerical results for different values of the Kerr parameter and the intensity dependent coupling are the same as in Fig. 2. We observe that the fourth-order squeezing occurs in a short time in the cases $\chi = 0.0$ and $f(\hat{n}) = const$, as shown in Fig. 3a, while it occurs after a long time when the Kerr medium takes place as shown in Figs. 3b and 3c. Also, the squeezing occurs periodically, and this periodicity increases by increasing χ, while the amount of squeezing decreases with time. Clearly, the presence of the intensiy-dependent coupling with $\chi = 0$ leads to a decrease of the amount of squeezing in the case $f(\hat{n}) = \sqrt{\hat{n}}$ (see Fig. 3d), and when we put $f(\hat{n}) = \hat{n}$ and $1/\sqrt{\hat{n}}$, as shown in Figs. 3e and 3f, the squeezing disappears.

5. Conclusion

We have studied a four-level atom with a momentum eigenstate interacting with a one-mode cavity field. The nonlinearity Kerr medium and intensity-dependent coupling are taken into account. The constants of motion and the wave function are obtained. The momentum increment, the momentum diffusion and the fourth-order variance are calculated for the system. We investigate numerically the momentum increment and the high-order squeezing when the atom is prepared in the upper state. We notice that the Kerr-like medium and the intensity-dependent coupling have effects on the momentum increment and on the high-order squeezing phenomenon.

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MEĐUDJELOVANJE ČETIRI-RAZINSKOG ATOMA U IMPULSNOM SVOJSTVENOM STANJU S JEDNOMODNIM POLJEM

Proučavamo međudjelovanje četiri-razinskog atoma (poput ljestvi) u impulsnom svojstvenom stanju s jednomodnim poljem u rezonatoru, uz nelinearnosti polja i vezanja ovisnog o intenzitetu. Izveli smo stalnice gibanja valne funkcije atomskog sustava. Posebnu smo pažnju posvetili raspravi o statističkim odlikama razmatranog atomskog sustava, kao što su povećanje i difuzija impulsa i zbijanje višeg reda. Numerički smo istražili utjecaj Kerrovog sredstva i vezanja ovisnog o intenzitetu na povećanje impulsa i zbijanje višeg reda. Našli smo da dodavanje tih parametara ima snažan učinak na povećanje impulsa i pojavu zbijanja.