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GENERAL THEORY OF HIGHER-ORDER CORRECTIONS TO SOLITARY WAVES IN A MULTICOMPONENT PLASMA

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A generalized theory on the combined effects of ion-temperature, negative ions and higher-order corrections on ion-acoustic solitary waves in a multicomponent plasma containing warm, relativistic positive ions and negative ions, two-temperature and non-isothermal electrons has been made using reductive perturbation method. The basic set of fluid equations for the warm ion-fluids have been reduced to the renormalized Korteweg–de-Vries equation for the first-order perturbed potential and then a renormalized linear inhomogeneous equation for the second-order perturbed potential. Steady-state solutions of the coupled equations have been derived and eventually the solution representing the potential associated with the ion-acoustic wave has been obtained. The soliton profiles have been displayed graphically under various conditions.

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1. Introduction

Over the last few decades, the plasma physicists have made a lot of investigations on various non-linear structures such as ion-acoustic solitary waves, shocks, double layers etc. in plasmas as these have been found to have relevance with regard to some experimental observations and astrophysical phenomena. Washimi and Taniuti [1] are the first to employ the reductive perturbation technique to derive K-dV equation for the study of ion-acoustic solitary waves in a cold collisionless plasma containing hot isothermal electrons. But if negative ions exist in the plasma, then the response of the plasma to disturbances gets remarkably modified owing to their heavy mass. The K-dV equation representing the non-linear ion-acoustic wave propagating in a collisionless plasma with one species of positive ions and one species of negative ions has been derived by Das and Tagare [2] in which the electrons are taken to be isothermal.

Jones et al. [3] has made both experimental and theoretical study on the propagation of ion-acoustic waves in a multicomponent plasma having ions and two-temperature electrons. They observe the speed of the ion-acoustic wave to be more strongly influenced by the low temperature electron component than by the high temperature electron component. Goswami and Buti [4] and others [5 − 8] have made extensive study on solitons and other aspects of ion-acoustic waves in two-temperature electron plasmas. However, more interesting results are obtained if there exist resonant electrons for which plasma exhibits non-isothermality. These electrons are non-Boltzmannian. Schamel [9, 10] for the first time has considered non-isothermality of electrons in a plasma and derived an expression for the electron-distribution completely different from the familiar expression $n_\mathrm{e} = \exp\{\phi\}$. He has also shown an ion-acoustic solitary wave in the lowest order to have a sech^4 profile in place of the usual sech^2 profile. Later on, other workers [11 − 13] have adopted Schamel's model and investigated the effects of non-isothermality of two-temperature electrons on solitary waves.

Studies on wave propagation in a relativistic plasma have been made by Tsytovich [14], Stenflo et al. [15] in astrophysical scenario and in laser-plasma interactions by Kaw et al. [16], Shukla et al. [17], Chakraborty et al. [18]. Moreover, non-linear effects on the excitation of ion-acoustic solitons in a relativistic plasma have been investigated by Das and Paul [19] and others [20 − 23] who have considered relativistic effects in investigating propagation of IAW under various plasma conditions.

In the experimental study involving the use of a double-plasma device on the properties of the ion-acoustic solitary wave, it is observed that the wave velocity is larger and the width is narrower than those predicted theoretically, i.e. by solution of the K-dV equation. In order to remove, or rather minimize this discrepancy, the effect of ion-temperature on the K-dV equation has been studied by several authors [24 − 27] without considering the higher-order corrections. On the other hand, Ichikawa et al. [28] and Kodama et al. [29] have investigated the contribution of higher-order terms to ion-acoustic solitary waves for cold ions using reductive perturbation method. Later on, with the use of a stationary solution of the Vlasov–Poisson equations with a Boltzmann distribution of electrons, Watanabe [30] has studied the combined effects of ion temperature and higher-order non-linearity on the ion-acoustic solitons. The combined effects have been found to cause an increase in the soliton-velocity and a decrease in the width which are compatible with the experimental observation. This work has further been done by Lai [31] where he has employed reductive perturbation method, and this treatment is analogous to that of Kodama and Taniuti but for cold ions [29]. Effect of higher-order non-linearity on ion-acoustic waves in multicomponent plasma with both non-isothermal and isothermal electrons and negative ions has been studied by Tagare and Reddy [32] without considering ion-temperature on the basis of reductive perturbation method. Again similar theoretical analysis but for a relativistic plasma containing cold ions and non-isothermal two-temperature electrons has been made by Roychowdhury et al. [33]. Furthermore, Das et al. [34] have investigated the contributions of higher-order non-linear and dispersive effects on ion-acoustic solitary waves in a plasma consisting of warm ions and two groups of non-isothermal electrons. In their analysis, they have applied the reductive perturbation method also to an integral form of the governing equations in terms of the pseudopotential.

In our paper, we have generalized the studies made by the authors [31 – 34] on the effect of ion-temperature and higher-order non-linearity on the ion-acoustic solitary wave in a multicomponent plasma, incorporating various parameters and using reductive perturbation technique. Besides this, we have displayed changes in the K-dV soliton profile (ϕ_1) due to higher-order non-linearity with various parameters viz. ion-temperature (σ), relativistic parameter (u_0/c), negative ion-concentration ($n_{\mathrm{j}0}$), electron-temperatures (T_{el}, T_{eh}) and non-isothermal parameters (b_{l}, b_{h}) separately. The most interesting point in our theoretical analysis is that our theory includes all works of the authors [31 – 34] as special cases. So our approach deserves to be considered as most general, that is more valuable and realistic.

2. *The basic equations and formulation of the problem*

We consider a collisionless unmagnetised plasma consisting of ions having finite temperature T_{i} together with negative ions and two types of non-isothermal electrons of which one is cold and the other is hot having temperatures T_{el} and T_{eh}, respectively. It is assumed that $T_{\mathrm{i}} \ll T_{\mathrm{el}}, T_{\mathrm{eh}}$ for which Landau damping is ignored. We also assume the motion of the positive ions and the negative ions to be weakly relativistic. The basic non-dimensional fluid equations which govern the dynamics of such a plasma are

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial u_{\alpha r}}{\partial t} + u_\alpha \frac{\partial u_{\alpha r}}{\partial x} + \frac{\sigma_\alpha}{n_\alpha Q_\alpha}\frac{\partial p_\alpha}{\partial x} = -q_\alpha \frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_{\alpha r}}{\partial x} = 0\,, \tag{3}$$

where $\alpha = \mathrm{i(j)}$ stands for positive (negative) ions and $q_{\mathrm{i}} = 1$, $q_{\mathrm{j}} = -1$.

The equations (1) – (3) are supplemented with Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{el}} + n_{\mathrm{eh}} + n_{\mathrm{j}} - n_{\mathrm{i}}\,, \tag{4}$$

where

$$Q_\alpha = \frac{m_\alpha}{m_{\mathrm{i}}}\,, \qquad \sigma_\alpha = \frac{T_\alpha}{T_{\mathrm{eff}}}\,,$$

$$T_{\rm eff} = \frac{T_{\rm el}T_{\rm eh}}{\mu T_{\rm eh} + \nu T_{\rm el}}\,, \qquad u_{\alpha r} = \frac{u_\alpha}{\sqrt{1-u_\alpha^2/c^2}} \approx u_\alpha\left(1+\frac{1}{2}\frac{u_\alpha^2}{c^2}\right).$$

Here n_α, $n_{\rm el}$, $n_{\rm eh}$ are, respectively, the number densities of the ions, low-temperature electrons and high-temperature electrons, u_α is the ion-fluid velocity, p_α is the ion-fluid pressure, ϕ is the electrostatic potential. Q is the ratio of the negative ion-mass to the positive ion-mass, i.e., $Q_{\rm j} = Q$. μ and ν denote the unperturbed number densities of the low-temperature and high-temperature electron components, i.e., $\mu = n_{\rm el}^{(0)}$ and $\nu = n_{\rm eh}^{(0)}$ such that $\mu + \nu = 1$. $T_{\rm eff}$ is the effective temperature of the plasma. The number densities n_α, $n_{\rm el}$ and $n_{\rm eh}$ have been normalized by n_0 where n_0 is the equilibrium number density of electrons, the ion velocities u_α have been normalized by $\lambda_{\rm De}\omega_{\rm pi} = \sqrt{k_{\rm B}T_{\rm eff}/m_{\rm i}}$, the ion-pressures p_α have been normalized by $n_0 k_{\rm B} T_{\rm eff}$ and the electrostatic potential ϕ has been normalized by $k_{\rm B}T_{\rm eff}/e$. $k_{\rm B}$ is the Boltzmann constant. x and t have been made dimensionless by the electron Debye length $\lambda_{\rm De} = \sqrt{(4\pi e^2 n_0)/(k_{\rm B}T_{\rm eff})}$ and the reciprocal of plasma frequency $\omega_{\rm pi}^{-1} = \sqrt{m_{\rm i}/(4\pi n_0 e^2)}$.

Due to non-isothermality of the two groups of electrons, the number densities of low-temperature and high-temperature electrons can be written as assumed by Schamel [9,10]

$$\begin{aligned}
n_{\rm el} &= \mu\left[1+\frac{\phi}{y}-\frac{4}{3}b_{\rm l}\left(\frac{\phi}{y}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\phi}{y}\right)^{2}-\frac{8}{15}b_{\rm l}^{(1)}\left(\frac{\phi}{y}\right)^{\frac{5}{2}}+\frac{1}{6}\left(\frac{\phi}{y}\right)^{3}-\dots\right],\\
n_{\rm eh} &= \nu\left[1+\frac{\beta\phi}{y}-\frac{4}{3}b_{\rm h}\left(\frac{\beta\phi}{y}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\beta\phi}{y}\right)^{2}-\frac{8}{15}b_{\rm h}^{(1)}\left(\frac{\beta\phi}{y}\right)^{\frac{5}{2}}\right.\\
&\qquad \left.+\frac{1}{6}\left(\frac{\beta\phi}{y}\right)^{3}-\dots\right],
\end{aligned} \tag{5}$$

where $y = \mu + \nu\beta$, $\beta = T_{\rm el}/T_{\rm eh}$, $b_{\rm l} = (1-\beta_{\rm l})/\pi^{1/2}$, $b_{\rm h} = (1-\beta_{\rm h})/\pi^{1/2}$, $\beta_{\rm l} = T_{\rm el}/T_{\rm elt}$, $\beta_{\rm h} = T_{\rm eh}/T_{\rm eht}$, $b_{\rm l}^{(1)} = (1-\beta_{\rm l}^2)/\pi^{1/2}$ and $b_{\rm h}^{(1)} = (1-\beta_{\rm h}^2)/\pi^{1/2}$. $T_{\rm elt}$ and $T_{\rm eht}$ denote the temperatures of the trapped electrons in the low and high temperature groups of electrons and $T_{\rm el}$ and $T_{\rm eh}$ are the same for free electrons.

In order to derive the K-dV type equation describing the propagation of nonlinear ion-acoustic waves in our multicomponent plasma from the basic system of fluid equations (1) – (9), we expand the densities, the fluid velocities, the fluid pressures and the electrostatic potential around the unperturbed state by a smallness parameter ε as

$$\begin{aligned}
n_\alpha &= n_{\alpha 0} + \varepsilon n_{\alpha 1} + \varepsilon^{3/2} n_{\alpha 2} + \varepsilon^2 n_{\alpha 3} + \varepsilon^3 n_{\alpha 4} + \cdots,\\
u_\alpha &= u_{\alpha 0} + \varepsilon u_{\alpha 1} + \varepsilon^{3/2} u_{\alpha 2} + \varepsilon^2 u_{\alpha 3} + \varepsilon^3 u_{\alpha 4} + \cdots,\\
p_\alpha &= 1 + \varepsilon p_{\alpha 1} + \varepsilon^{3/2} p_{\alpha 2} + \varepsilon^2 p_{\alpha 3} + \varepsilon^3 p_{\alpha 4} + \cdots,\\
\phi &= \varepsilon \phi_1 + \varepsilon^{3/2}\phi_2 + \varepsilon^2\phi_3 + \varepsilon^3\phi_4 + \cdots.
\end{aligned} \tag{6}$$

To derive the modified K-dV equation, we employ the stretched variables

$$\xi = \varepsilon^{1/4}(x - vt)\,, \qquad \tau = \varepsilon^{3/4}t\,, \tag{7}$$

where v is an unknown velocity to be determined. Substituting Eqs. (6) and (7) in Eqs. (1)–(5) and equating the coefficients of various powers of ε, we have the following equations. The lowest power of ε yields

$$n_{\alpha 1} = \frac{-q_\alpha \phi_1}{\gamma_{\alpha 0}\lambda_{\alpha 0}}\,, \qquad u_{\alpha 1} = \frac{-q_\alpha n_{\alpha 0}\phi_1}{\gamma_{\alpha 0}\lambda_{\alpha 0}}\,, \qquad p_{\alpha 1} = \frac{-3q_\alpha n_{\alpha 0}\phi_1}{\lambda_{\alpha 0}}\,. \tag{8}$$

Poisson's equation yields the linear dispersion relation

$$1 + \sum_{\alpha=\mathrm{i,j}} \frac{n_{\alpha 0}^2}{\gamma_{\alpha 0}\lambda_{\alpha 0}Q_\alpha} = 0\,, \tag{9}$$

where

$$\gamma_{\alpha 0} = 1 + \frac{3}{2}\left(\frac{u_{\alpha 0}}{c}\right)^2, \quad \lambda_{\alpha 0} = \frac{3\sigma_\alpha}{Q_\alpha} - n_{\alpha 0}v_{\alpha 0}^2\,, \quad v_{\alpha 0} = v - u_{\alpha 0}\,.$$

Equating the coefficients of $\varepsilon^{3/2}$, we obtain the following equations

$$\begin{aligned}
\frac{\partial n_{\alpha 1}}{\partial \tau} - v_{\alpha 0}\frac{\partial n_{\alpha 2}}{\partial \xi} + n_{\alpha 0}\frac{\partial u_{\alpha 2}}{\partial \xi} &= 0\,, \\
n_{\alpha 0}\gamma_{\alpha 0}\frac{\partial u_{\alpha 1}}{\partial \tau} - n_{\alpha 0}v_{\alpha 0}\gamma_{\alpha 0}\frac{\partial u_{\alpha 2}}{\partial \xi} + \frac{\sigma_\alpha}{Q_\alpha}\frac{\partial p_{\alpha 2}}{\partial \xi} &= \frac{-q_\alpha n_{\alpha 0}}{Q_\alpha}\frac{\partial \phi_2}{\partial \xi}\,, \\
\frac{\partial p_{\alpha 1}}{\partial \tau} - v_{\alpha 0}\frac{\partial p_{\alpha 2}}{\partial \xi} + 3\gamma_{\alpha 0}\frac{\partial u_{\alpha 2}}{\partial \xi} &= 0\,, \\
\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 - \frac{4}{3}\left[\frac{\mu b_l + \nu b_h \beta^{3/2}}{y^{3/2}}\right]\phi_1^{3/2} &+ \sum_{\alpha=\mathrm{i,j}} -q_\alpha n_{\alpha 2}\,.
\end{aligned} \tag{10}$$

Eliminating the second-order perturbed quantities from Eqs. (10) and using the expressions for the first-order perturbed quantities given by Eq. (8), we get the modified K-dV equation as

$$A_1\frac{\partial \phi_1}{\partial \tau} + A_2\phi_1^{1/2}\frac{\partial \phi_1}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3} = 0\,, \tag{11}$$

where

$$A_1 = \sum_\alpha \frac{v_{\alpha 0}n_{\alpha 0}^3}{\gamma_{\alpha 0}\lambda_{\alpha 0}^2 Q_\alpha} \quad \text{and} \quad A_2 = \frac{\mu b_\mathrm{l} + \nu b_\mathrm{h}\beta^{3/2}}{y^{3/2}}\,.$$

It is to be mentioned here that Eq. (11) is the generalized form of the K-dV equations obtained by various authors [30–32] under different conditions.

The second-order quantities $n_{\alpha 2}$, $u_{\alpha 2}$ etc. can be expressed in general forms in terms of ϕ_1 and ϕ_2 as

$$\begin{aligned}
u_{\alpha 2} &= \frac{-q_\alpha n_{\alpha 0}}{Q_\alpha \gamma_{\alpha 0}\lambda_{\alpha 0}}\phi_2 - \frac{2q_\alpha n_{\alpha 0}\lambda'_{\alpha 0}}{3\gamma_{\alpha 0}\lambda_{\alpha 0}^2}\frac{A_2}{A_1}\phi_1^{3/2} - \frac{q_\alpha n_{\alpha 0}\lambda'_{\alpha 0}}{2A_1\gamma_{\alpha 0}\lambda_{\alpha 0}^2}\frac{\partial^2\phi_1}{\partial\xi^2}\,, \\
n_{\alpha 2} &= \frac{-q_\alpha n_{\alpha 0}^2}{Q_\alpha \gamma_{\alpha 0}\lambda_{\alpha 0}}\phi_2 - \frac{4q_\alpha v_{\alpha 0} n_{\alpha 0}^3}{3Q_\alpha\gamma_{\alpha 0}\lambda_{\alpha 0}^2}\frac{A_2}{A_1}\phi_1^{3/2} - \frac{q_\alpha v_{\alpha 0} n_{\alpha 0}^3}{Q_\alpha A_1\gamma_{\alpha 0}\lambda_{\alpha 0}^2}\frac{\partial^2\phi_1}{\partial\xi^2}\,, \\
p_{\alpha 2} &= \frac{-3q_\alpha n_{\alpha 0}}{Q_\alpha \lambda_{\alpha 0}}\phi_2 - \frac{4q_\alpha v_{\alpha 0} n_{\alpha 0}^2}{Q_\alpha\lambda_{\alpha 0}^2}\frac{A_2}{A_1}\phi_1^{3/2} - \frac{3q_\alpha v_{\alpha 0} n_{\alpha 0}^2}{Q_\alpha A_1\lambda_{\alpha 0}^2}\frac{\partial^2\phi_1}{\partial\xi^2}\,,
\end{aligned} \tag{12}$$

where $\lambda'_{\alpha 0} = 3\sigma_\alpha/Q_\alpha + n_{\alpha 0}v_{\alpha 0}^2$.

To the next higher-order, we have

$$\begin{aligned}
&n_{\alpha 0}\frac{\partial u_{\alpha 3}}{\partial\xi} - v_{\alpha 0}\frac{\partial n_{\alpha 3}}{\partial\xi} + \frac{\partial n_{\alpha 2}}{\partial\tau} + \frac{\partial}{\partial\xi}(n_{\alpha 1}u_{\alpha 1}) = 0\,, \\
&\frac{q_\alpha n_{\alpha 0}}{Q_\alpha}\frac{\partial\phi_3}{\partial\xi} - v_{\alpha 0}n_{\alpha 0}\gamma_{\alpha 0}\frac{\partial u_{\alpha 3}}{\partial\xi} + \frac{\sigma_\alpha}{Q_\alpha}\frac{\partial p_{\alpha 3}}{\partial\xi} = v_{\alpha 0}\gamma_{\alpha 0}n_{\alpha 1}\frac{\partial u_{\alpha 1}}{\partial\xi} \\
&\quad -\gamma_{\alpha 0}n_{\alpha 0}u_{\alpha 1}\frac{\partial u_{\alpha 1}}{\partial\xi} + \frac{3}{c^2}n_{\alpha 0}u_{\alpha 0}v_{\alpha 0}u_{\alpha 1}\frac{\partial u_{\alpha 1}}{\partial\xi} - \frac{q_\alpha n_{\alpha 1}}{Q_\alpha}\frac{\partial\phi_1}{\partial\xi} - n_{\alpha 0}\gamma_{\alpha 0}\frac{\partial u_{\alpha 2}}{\partial\tau}\,, \\
&\quad 3\gamma_{\alpha 0}\frac{\partial u_{\alpha 3}}{\partial\xi} - v_{\alpha 0}\frac{\partial p_{\alpha 3}}{\partial\xi} = -u_{\alpha 1}\frac{\partial p_{\alpha 1}}{\partial\xi} - 3\gamma_{\alpha 0}p_{\alpha 1}\frac{\partial u_{\alpha 1}}{\partial\xi} - \frac{\partial p_{\alpha 2}}{\partial\tau}
\end{aligned} \tag{13}$$

and

$$\frac{\partial^3\phi_2}{\partial\xi^3} = \frac{\partial\phi_3}{\partial\xi} + \left(\frac{\mu+\nu\beta^2}{y^2}\right)\phi_1\frac{\partial\phi_1}{\partial\xi} - q_\alpha\frac{\partial n_{\alpha 3}}{\partial\xi} - 2A_2\frac{\partial}{\partial\xi}\left(\phi_1^{1/2}\phi_2\right).$$

Eliminating the third-order perturbed quantities from Eqs. (13) and using Eqs. (12), we get

$$A_1\frac{\partial\phi_2}{\partial\tau} + A_2\frac{\partial}{\partial\xi}\left(\phi_1^{1/2}\phi_2\right) + \frac{1}{2}\frac{\partial^3\phi_2}{\partial\xi^3} = S(\phi_1)\,, \tag{14}$$

where the source term $S(\phi_1)$ is given by

$$\begin{aligned}
S(\phi_1) &= B\phi_1\frac{\partial\phi_1}{\partial\xi} + C\phi_1^{1/2}\frac{\partial^3\phi_1}{\partial\xi^3} + D\phi_1^{-3/2}\left(\frac{\partial\phi_1}{\partial\xi}\right)^3 + E\phi_1^{-1/2}\frac{\partial}{\partial\xi}\left(\frac{\partial\phi_1}{\partial\xi}\right)^2 \\
&\quad + F\frac{\partial^5\phi_1}{\partial\xi^5}\,.
\end{aligned} \tag{15}$$

The coefficients A_1, A_2 occurring in Eqs. (11) and (14) have been defined earlier and the defining expressions for the coefficients B, C, D, E and F occurring in Eq. (15) are given in the Appendix.

3. *The stationary solutions*

In order to solve Eqs. (11) and (14), we employ the renormalisation method developed by Kodama and Taniuti [29] according to which Eqs. (11) and (14) get modified to

$$A_1 \frac{\partial \bar{\phi}_1}{\partial \tau} + A_2 \bar{\phi}_1^{1/2} \frac{\partial \bar{\phi}_1}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \bar{\phi}_1}{\partial \xi^3} + \delta\lambda \frac{\partial \bar{\phi}_1}{\partial \xi} = 0 \tag{16}$$

and

$$A_1 \frac{\partial \bar{\phi}_2}{\partial \tau} + A_2 \frac{\partial}{\partial \xi} \left(\bar{\phi}_1^{1/2} \bar{\phi}_2 \right) + \frac{1}{2} \frac{\partial^3}{\partial \xi^3} (\bar{\phi}_2) + \delta\lambda \frac{\partial \bar{\phi}_2}{\partial \xi} = S(\bar{\phi}_1) + \delta\lambda \frac{\partial \bar{\phi}_1}{\partial \xi} \, . \tag{17}$$

The parameter $\delta\lambda$ has been introduced in Eqs. (16) and (17) for the cancellation of the resonant term occurring in $S(\bar{\phi}_1)$ by the term $\delta\lambda \partial \bar{\phi}_1 / \partial \xi$ in Eq. (17).

Let us have the stationary solution by defining a new variable η as

$$\eta = \xi - (\lambda + \delta\lambda / A_1) \, \tau \, . \tag{18}$$

With this transformation, Eqs. (16) and (17) turn out to be

$$\frac{\mathrm{d}^3 \bar{\phi}_1}{\mathrm{d}\eta^3} + \frac{4}{3} A_2 \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\bar{\phi}_1 \right)^{3/2} - 2 A_1 \lambda \frac{\mathrm{d}\bar{\phi}_1}{\mathrm{d}\eta} = 0 \tag{19}$$

and

$$\frac{\mathrm{d}^3 \bar{\phi}_2}{\mathrm{d}\eta^3} + 2 A_2 \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\bar{\phi}_1^{1/2} \bar{\phi}_2 \right) - 2 A_1 \lambda \frac{\mathrm{d}\bar{\phi}_2}{\mathrm{d}\eta} = 2 \left[S(\bar{\phi}_1) + \delta\lambda \frac{\mathrm{d}\bar{\phi}_1}{\mathrm{d}\eta} \right] . \tag{20}$$

If Eqs. (19) and (20) be integrated under the boundary conditions

$$\bar{\phi}_1 = \bar{\phi}_2 = \frac{\mathrm{d}\bar{\phi}_1}{\mathrm{d}\eta} = \frac{\mathrm{d}\bar{\phi}_2}{\mathrm{d}\eta} = \frac{\mathrm{d}^2 \bar{\phi}_1}{\mathrm{d}\eta^2} = \frac{\mathrm{d}^2 \bar{\phi}_2}{\mathrm{d}\eta^2} = 0$$

as $\eta \to \infty$, then we readily obtain

$$\frac{\mathrm{d}^2 \bar{\phi}_1}{\mathrm{d}\eta^2} + \frac{4}{3} A_2 \bar{\phi}_1^{3/2} - 2 A_1 \lambda \phi_1 = 0 \tag{21}$$

and

$$\frac{\mathrm{d}^2 \bar{\phi}_2}{\mathrm{d}\eta^2} + 2 \left(A_2 \bar{\phi}_1^{1/2} - A_1 \lambda \right) \bar{\phi}_2 = 2 \int\limits_{-\infty}^{\eta} \left[S(\bar{\phi}_1) + \delta\lambda \frac{\mathrm{d}\bar{\phi}_1}{\mathrm{d}\eta} \right] \mathrm{d}\eta \, . \tag{22}$$

The solitary wave solution of Eq. (21) comes out to be

$$\bar{\phi}_1 = \phi_0 \mathrm{sech}^4 (\eta / d) \, , \tag{23}$$

where the amplitude (ϕ_0) and the width (d) of the solitary wave are given by

$$\phi_0 = \frac{225}{64}\left(\frac{A_1}{A_2}\right)\lambda^2 \quad \text{and} \quad d = \left(\frac{8}{\lambda A_1}\right)^{1/2}. \tag{24}$$

Now

$$\begin{aligned} 2\int\limits_{-\infty}^{\eta} &\left[S(\bar{\phi}_1) + \delta\lambda \frac{\mathrm{d}\bar{\phi}_1}{\mathrm{d}\eta}\right]\mathrm{d}\eta \\ &= \left[-\big(30C + 16D + 40E\big)\frac{\phi_0^{3/2}}{d^2} + \big(B\phi_0^2 + \frac{1680F\phi_0}{d^4}\big)\right]\mathrm{sech}^8(\eta/d) \\ &\quad + \left[-\frac{2080}{d^4}F\phi_0 + \frac{64}{3d^2}\left(C + D + 2E\right)\phi_0^{3/2}\right]\mathrm{sech}^6(\eta/d) \\ &\quad + \frac{1}{4}\left[\frac{4\phi_0\delta\lambda}{d} + \frac{1024F\phi_0}{d^5}\right]\mathrm{sech}^4(\eta/d)\,. \end{aligned} \tag{25}$$

For the cancellation of the secular terms in $S(\bar{\phi}_1)$, we have to put the last term in the right-hand side of Eq. (25) equal to zero. Thus we get

$$\delta\lambda = -4\lambda^2 A_1^2 F\,. \tag{26}$$

This relation is consistent with the equations (2.29) of Ref. [32] and (21) of Ref. [31].

If the value of the integral, as is given by Eq. (25), be substituted in Eq. (22), then we have

$$\begin{aligned} \frac{\mathrm{d}^2\bar{\phi}_2}{\mathrm{d}\eta^2} + 2\left(A_2\bar{\phi}_1^{1/2} - A_1\lambda\right)\bar{\phi}_2 = &\left[-\frac{30C}{d^2}\phi_0^{3/2} - \frac{16D}{d^2}\phi_0^{3/2}\right. \\ &\left. -\frac{40E}{d^2}\phi_0^{3/2} + B\phi_0^2 + \frac{1680F}{d^4}\phi_0\right]\mathrm{sech}^8(\eta/d) \\ &+ \left[-\frac{2080F}{d^4}\phi_0 + \frac{64}{3d^2}\left(C + D + 2E\right)\phi_0^{3/2}\right]\mathrm{sech}^6(\eta/d)\,. \end{aligned} \tag{27}$$

For solving the inhomogeneous equation (27), we introduce a new variable μ defined as

$$\mu = \tanh(\eta/d)\,. \tag{28}$$

With this transformation, Eq. (27) becomes

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\left[(1-\mu^2)\frac{\mathrm{d}\bar{\phi}_2}{\mathrm{d}\mu}\right] + \left(30 - \frac{16}{1-\mu^2}\right)\bar{\phi}_2(\mu) = \tau(\mu)\,, \tag{29}$$

where $\tau(\mu) = G(1-\mu^2)^3 + H(1-\mu^2)^2$; G and H have been defined in the Appendix. It is important to note that this expression for $\tau(\mu)$ reduces to those given by Eq. (2.33) in Ref. [32] and Eq. (25) in Ref. [31]. Equation (29) has two independent solutions in terms of associated Legendre polynomials given by

$$P_5^4(\mu) = 945\mu(1-\mu^2)^2 \tag{30}$$

and

$$\begin{aligned} Q_5^4(\mu) \quad = \quad & \frac{945}{2}\mu(1-\mu^2)^2 \ln\frac{1+\mu}{1-\mu} - 384(1-\mu^2)^2 + 975\mu^2(1-\mu^2) + 630\mu^4 \\ & + 264\frac{\mu^6}{1-\mu^2} + 48\frac{\mu^8}{(1-\mu^2)^2}\,, \end{aligned} \tag{31}$$

which are the same as those given by Eqs. (2.32) and (2.35) in the paper by Tagare and Reddy [32]. Using the familiar method of variation of parameters, the particular solution of Eq. (29) can be written as

$$\bar{\phi}_{2P}(\mu) = \psi_1(\mu)P_5^4(\mu) + \psi_2(\mu)Q_5^4(\mu)\,, \tag{32}$$

where the functions $\psi_1(\mu)$ and $\psi_2(\mu)$ are given by

$$\psi_1(\mu) = -\int \frac{\tau(\mu)Q_5^4(\mu)}{945\times 384}\,\mathrm{d}\mu \quad \text{and} \quad \psi_2(\mu) = \int \frac{\tau(\mu)P_5^4(\mu)}{945\times 384}\,\mathrm{d}\mu\,. \tag{33}$$

After detailed calculation, we have

$$\begin{aligned} \psi_1(\mu) = -\frac{1}{315\times 384}\Bigg[& -\left(\frac{407G}{4} + \frac{193H}{2}\right)\mu + \left(\frac{3335G}{12} + \frac{3555H}{15}\right)\mu^3 \\ & - \left(\frac{843G}{2} + \frac{1344H}{5}\right)\mu^5 + \left(\frac{693G}{2} + \frac{882H}{6}\right)\mu^7 - \left(\frac{595G}{4} + \frac{378H}{12}\right)\mu^9 \\ & + \frac{105G}{4}\mu^{11} - \left(\frac{105G}{8}(1-\mu^2)^6 + \frac{63H}{4}(1-\mu^2)^5\right)\ln\frac{1+\mu}{1-\mu}\Bigg], \\ \psi_2(\mu) = -\frac{1}{384}\, & \left[\frac{G}{12}(1-\mu^2)^6 + \frac{H}{10}(1-\mu^2)^5\right]. \end{aligned} \tag{34}$$

Here the constants of integration have been put equal to zero because of the boundary conditions. On substituting Eq. (34) in (32), we find the particular solution to come out in terms of old variables as

$$\bar{\phi}_{2P}(\eta) = \frac{G}{6}\left[\mathrm{sech}^4(\eta/d) - \frac{1}{2}\mathrm{sech}^6(\eta/d)\right] + \frac{H}{10}\mathrm{sech}^4(\eta/d)\,. \tag{35}$$

The complementary solution of Eq. (29) is given by

$$\phi_{2c}(\mu) = C_1 P_5^4(\mu) + C_2 Q_5^4(\mu)\,. \tag{36}$$

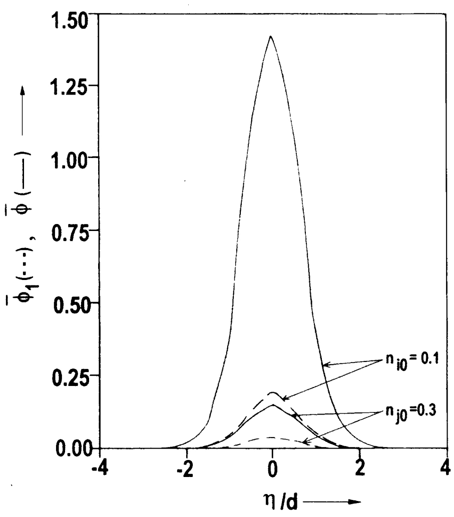
Here the first term is the secular term which can be eliminated by renormalisation of the amplitude. Again, $C_2 = 0$ because of the boundary condition for $\bar{\phi}_2(\eta)$ as $|\eta| \to \infty$. Consequently, only the particular solution given by Eq. (35) contributes. Thus, the stationary solution for the potential for the ion-acoustic wave is given by

$$\bar{\phi}(\eta) = \bar{\phi}_1(\eta) + \bar{\phi}_2(\eta)\,, \tag{37}$$

where $\bar{\phi}_1(\eta)$ and $\bar{\phi}_2(\eta)$ are given by Eqs. (23) and (35), respectively.

4. Results and discussion

In Figs. 1 − 6, the profiles of the K-dV soliton, i.e. the first-order soliton and the second-order soliton have been displayed under diverse situations. In these figures, $\bar{\phi}_1$ and $\bar{\phi}$, respectively, denote the first-order and the second-order perturbed potential which have been represented by dashed curves and solid curves. The plots are very effective in revealing various momentous aspects related to the higher-order non-linearity.



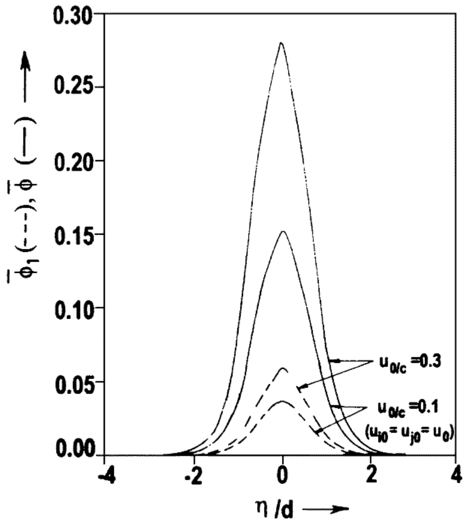
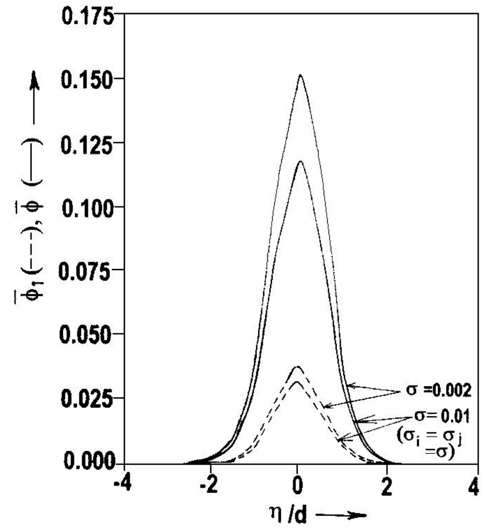


Fig. 1 (left). Effect of higher-order non-linearity and negative-ion concentration ($n_{\mathrm{j}0}$) on the soliton amplitudes when $u_0/c = 0.1$, $Q = 0.4$, $\sigma = 0.002$, $\mu = 0.15$, $\nu = 0.85$, $b_{\mathrm{l}} = 0.1$, $b_{\mathrm{h}} = 0.32$, $\lambda = 0.1$ and $\beta = 0.025$.

Fig. 2. Profiles of the K-dV soliton and higher-order soliton with u_0/c as parameter when $n_{\mathrm{j}0} = 0.3$, $Q = 0.4$, $\sigma = 0.002$, $\mu = 0.15$, $\nu = 0.85$, $b_{\mathrm{l}} = 0.1$, $b_{\mathrm{h}} = 0.32$, $\lambda = 0.1$ and $\beta = 0.025$.

In Fig. 1, we see that amplitude of the solitary wave, for a particular set of values of the plasma parameters, gets remarkably modified due to higher-order nonlinearity. Moreover, it is evident in this figure that the amplitude of the second-order soliton increases to a greater extent when the negative ion-concentration (n_{j0}) is reduced, though the amplitude of the first-order soliton increases simultaneously for the same cause. In Fig. 2, apart from the increase in the amplitude of the solitary wave because of the higher-order nonlinearity, we observe that amplitudes of both the first-order and the second-order soliton increase appreciably as the motion of the ions in the plasma is more relativistic, i.e. as u_0/c increases, though the rate of increase in the second case is greater.



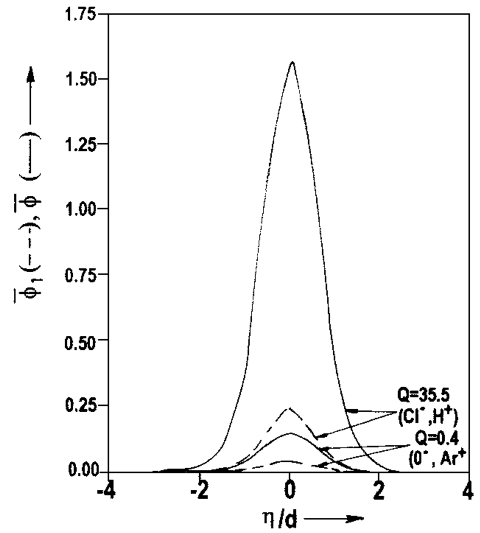


Fig. 3 (left). Effect of higher-order correction and ion temperature on the soliton-amplitudes when $n_{j0} = 0.3$, $u_0/c = 0.1$, $Q = 0.4$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.1$, $b_h = 0.32$, $\lambda = 0.1$ *and* $\beta = 0.025$.

Fig. 4. Profiles of the K-dV soliton and higher-order soliton with $Q(= m_j/m_i)$ *as parameter when* $n_{j0} = 0.3$, $u_0/c = 0.1$, $\sigma = 0.002$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.1$, $b_h = 0.32$, $\lambda = 0.1$ *and* $\beta = 0.025$.

Figure 3 shows that amplitudes of the first-order and the second-order soliton get reduced with the increase in the ion-temperature. In addition to it, we find that the rate of fall in the amplitude of the second-order soliton is greater due to increase in the ion-temperature. In Fig. 4, it is found that the increase in Q, which is the ratio of the negative ion-mass to the positive ion-mass, causes enhancement in amplitudes of the first-order and the second-order soliton. It is interesting to note here that Q has a greater influence on the amplitude of the second-order soliton compared to the first-order soliton. Figure 5 demonstrates dependence of the amplitudes of the first- and the second-order soliton on the values of μ and ν which denote the equilibrium number densities of the low-temperature and the high-temperature electron components in the plasma. Figure 6 indicates that amplitudes of both types of soliton are affected by the nonisothermality of electrons. Moreover, higher-order nonisothermality brings about a reduction of the amplitude of the first-order and the second-order soliton.

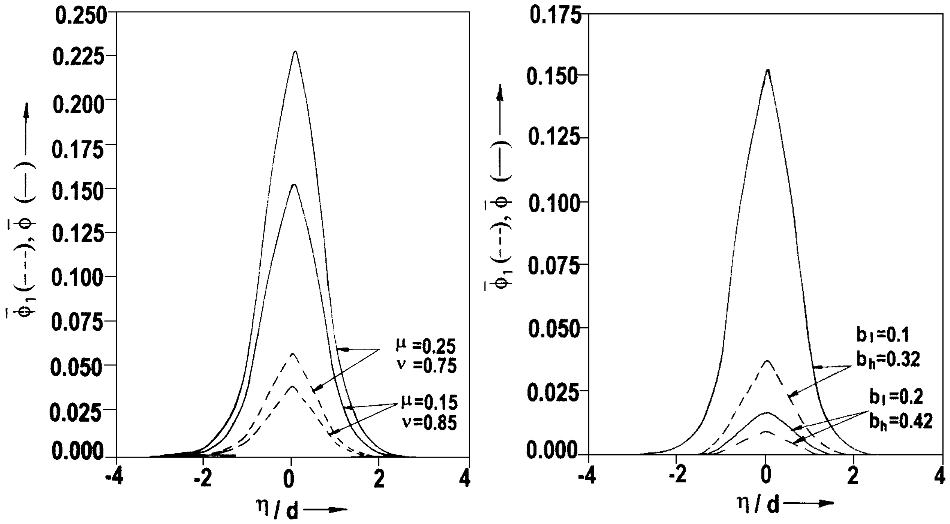


Fig. 5 (left). Effect of higher-order non-linearity and concentrations (unperturbed) of two-temperature electrons when $n_{j0} = 0.3$, $u_0/c = 0.1$, $Q = 0.4$, $\sigma = 0.002$, $b_l = 0.1$, $b_h = 0.32$, $\lambda = 0.1$ *and* $\beta = 0.025$.

Fig. 6. Profiles of the K-dV soliton and higher-order soliton with b_l, b_h *as parameters when* $n_{j0} = 0.3$, $u_0/c = 0.1$, $Q = 0.4$, $\sigma = 0.002$, $\mu = 0.15$, $\nu = 0.85$, $\lambda = 0.1$ *and* $\beta = 0.025$.

5. Concluding remarks

Various authors like C. S. Lai, S. G. Tagare et al. and A. Roychowdhury et al. [31−33] and others have extensively studied effects of ion-temperature, presence of negative ions, (weakly) relativistic ion-motion, two-temperature non-isothermal electrons and higher-order non-linearity on different aspects of solitary waves excited in collisionless unmagnetised multicomponent plasma. We have developed a general theory incorporating all parameters for a more realistic situation and have found that the results obtained by the authors mentioned above are the special cases of those we have obtained. Furthermore, this work may be extended for a magnetized multicomponent plasma considering collisional effects.

Appendix

$$
\begin{aligned}
B &= \frac{1}{2}\Bigg[\left(\frac{\mu+\nu\beta^2}{y^2}\right)+\left(\frac{A_2}{A_1}\right)^2\sum_{\alpha=\mathrm{i,j}}\left(\frac{\lambda'_{\alpha 0}n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^3_{\alpha 0}}-\frac{2n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^2_{\alpha 0}}+\frac{6\sigma_\alpha n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^3_{\alpha 0}}\right)\\
&\quad+\sum_{\alpha=\mathrm{i,j}}\left(\frac{-2q_\alpha n^3_{\alpha 0}}{Q^2_\alpha\gamma^2_{\alpha 0}\lambda^2_{\alpha 0}}-\frac{q_\alpha v^2_{\alpha 0}n^4_{\alpha 0}}{Q^2_\alpha\gamma^2_{\alpha 0}\lambda^3_{\alpha 0}}+\frac{q_\alpha\lambda''_{\alpha 0}v^2_{\alpha 0}n^4_{\alpha 0}}{Q^2_\alpha\gamma^3_{\alpha 0}\lambda^3_{\alpha 0}}+\frac{3q_\alpha\sigma_\alpha n^3_{\alpha 0}}{Q^2_\alpha\gamma^2_{\alpha 0}\lambda^3_{\alpha 0}}\right.\\
&\quad\left.+\frac{9q_\alpha\sigma_\alpha n^3_{\alpha 0}}{Q^2_\alpha\gamma_{\alpha 0}\lambda^3_{\alpha 0}}-\frac{q_\alpha n^3_{\alpha 0}}{Q^2_\alpha\gamma^2_{\alpha 0}\lambda^2_{\alpha 0}}\right)\Bigg],\\
C &= \frac{A_2}{2A_1}\sum_{\alpha=\mathrm{i,j}}\left(\frac{\lambda'_{\alpha 0}n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^3_{\alpha 0}}-\frac{2n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^2_{\alpha 0}}+\frac{6\sigma_\alpha n^3_{\alpha 0}}{Q_\alpha\gamma_{\alpha 0}\lambda^3_{\alpha 0}}\right),\\
D &= -\frac{C}{8},\quad E=\frac{C}{8},\quad F=\frac{C}{4A_2},\\
G &= 5\left(\frac{3B}{A_2}-C\right)\phi_0^{3/2},\quad H=-\frac{32C}{3}\phi_0^{3/2}.
\end{aligned}
$$

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OPĆA TEORIJA POPRAVKI VIŠEG REDA ZA SOLITONSKE VALOVE U VIŠEKOMPONENTNOJ PLAZMI

Primjenom reduktivne metode smetnje, razvili smo poopćenu teoriju složenih učinaka temperature iona, negativnih iona i popravaka višeg reda na ionsko-zvučne solitarne valove u višekomponentnoj plazmi koja sadrži tople relativističke pozitivne i negativne ione, te dvotemperaturne i neizotermičke elektrone. Osnovni skup jednadžbi fluida za tople ionske fluide svodi se na renormaliziranu Korteweg–de-Vries jednadžbu za potencijal smetnje prvog reda i zatim na renormaliziranu linearnu nehomogenu jednadžbu za potencijal smetnje drugog reda. Izveli smo rješenja vezanih jednadžbi i konačno rješenje koje predstavlja potencijal povezan s ionsko-zvučnim valom. Grafički predstavljamo solitonske oblike u različitim uvjetima.