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EFFECTS OF DRIFT NEGATIVE ION PLASMAS ON ION-ACOUSTIC SOLITARY WAVES IN DRIFT COLD POSITIVE ION PLASMAS WITH ISOTHERMAL ELECTRONS

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The effects of higher-order non-linearity on ion-acoustic solitary waves in a collisionless isothermal electron plasma with cold positive and negative ions are investigated by using the pseudopotential method. The influence of negative ions on solitary waves as well as on the first- and second-order amplitudes, widths and phase velocities are discussed for the plasmas having He^{+},O^{-}, He^{+},Cl^{-} or H^{+},O^{-} ions with the variation of parameters. The results are depicted in figures. We discuss the range of the electrostatic potential function (ϕ) for which the pseudopotential function (ψ) is valid up to a desired degree of accuracy.

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1. *Introduction*

The important and interesting phenomenon of plasma in understanding the formation of ion-acoustic solitary waves has been studied by many authors [1 − 9] in a number of physical situations. They have found two types of solitary waves (compressive and rarefactive) in the cold-ion plasma. Korteweg–de Vries [10] made the first study of solitary waves. Zabusky–Kruskal, Washimi–Taniuti [11, 12], Buti [13] and many others are the pioneers in the field of study of ion-acoustic solitary waves in cold-ion, single- or two-temperature electron plasma. Propagation of ion-acoustic solitary waves in ionospheric plasma consisting of electrons and positive ions is important, but still it is more interesting in the plasma with the presence of negative ions in addition to that of electrons and positive ions. If negative ions are present in a plasma as a contamination, rarefactive and negative potential solitons are possible [14]. It is to be noted that negative ions are present in space and they can also be produced in the laboratory plasmas. Nakamura et al. [15], Ludwig et al. [16] and other authors experimentally observed the ion-acoustic soliton and double layers in negative ion plasma. But in the presence of a drift motion, ion-acoustic solitons and double layers have not yet been investigated in the laboratory so that we can say that the theory is inadequate to explain the experimental results. Ion-acoustic solitary waves in plasmas have been studied theoretically by a large number of physicists and confirmed experimentally with great reliability by Ichikawa and Watanabe in 1977, Ikezi in 1978, Tran in 1979, Nakamura in 1982 and Lonngren in 1983 (Refs. [17]). But they found only compressive ion-acoustic solitary waves in two-component unmagnetized plasma and no rarefactive solitary waves. Later on, many authors (Buti [13], Nishihara and Tajiri in 1981, Yadav and Sharma in 1990 and Mishra, Arora and Chhabra in 2002 (Refs. [18])) showed that both compressive and rarefactive solitary waves are formed in the presence of an ion component and two electron components with different electron temperatures. The propagation of ion-acoustic solitary waves in a multi-species plasma consisting of positive ions, electrons and negative ions have been investigated theoretically by Das and Tagare in 1975 [19], Watanabe in 1984 [20], Tagare in 1986 and Kalita and Devi in 1993 [Ref. [21]), and experimentally by Cooney, Gavin and Lonngren [22], etc. and their studies showed the rarefactive solitary waves owing to the presence of negative ions. Solitary waves in plasma were first experimentally observed by Ikezi et al. [23].

To remove the disagreement between the theoretically predicted values and the experimental results, researchers introduced various parameters, e.g. ion-temperature [24], two-temperature electrons [25], non-isothermality [26, 27], inhomogeneity [28] etc. The higher-order contributions of non-linear and dispersive terms to ion-acoustic solitary waves have also been found to be effective for removing the differences between the theoretical and experimental results on the solitary waves [29 – 31]. Negative ions in the plasma have a dominant role on the formation of both the ion-acoustic solitary waves (compressive and rarefactive) and double layers [14, 32 – 36]. In this work only solitary waves are discussed. Also drift motion of the ions plays an important role for the existence of ion-acoustic solitary waves in a plasma [37 – 43].

In general, negative ions are produced in the laboratory by electron attachment to neutral particles when an electronegative gas is admitted into electric gas discharges. In the F-region of the ionosphere, negative ions are produced by SF_6 vapours. Also in the ionosphere, negative ions of the type O^- and O_2^- have been observed. Recently, negative ions and relativistic effects have been considered by Das et al. and Chakraborty et al. in studies of ion-acoustic solitary waves in plasma. The results of these authors are more interesting than those of earlier work. More recently, the effect of negative ion with drift motion on solitary waves in a study of cold-ion plasma with single-temperature electrons by the pseudopotential method was found to be very important. Chattopadhyay et al. [44] found that drift motion of ions has a significant contribution on the excitation of ion-acoustic solitary waves in the presence of negative ions in plasma. We are therefore motivated here to study the propagation of ion-acoustic solitary waves in a plasma with cold positive and negative ions having drift motions.

In the present paper, we have treated the plasma as a fluid consisting of cold positive and negative ions and single-temperature electrons. We also consider the presence of both the drift velocity and the negative ions and study the propagation of ion-acoustic solitary waves using the pseudopotential method [45 − 47].

The outline of this paper is as follows. In Section 2, an exact analytical form of the pseudopotential has been derived from a set of basic non-linear fluid equations, without any approximation. In the presence of negative ions, both compressive and rarefactive solitary waves are found, but only the compressive solitary waves are discussed. Section 3 contains the conditions for the existence of the solitary wave solution from which the phase velocity is obtained. Also in this section critical density of negative ion (n_{jc}) is found. In Section 4, first- and second-order amplitudes and widths are discussed. Section 5 gives the discussion of the entire problem. Concluding remarks are given in Section 6.

2. *Basic fluid equation and pseudopotential approach*

To study the propagation of ion-acoustic solitary waves of small but finite amplitude, we consider a collisionless, unmagnetized plasma consisting of isothermal electrons and cold positive and negative ions with uniform stream velocities. A similar propagation of ion-acoustic solitary waves appears also for warm magnetized plasma [48]. The basic set of fluid equations in dimensionless form for such a plasma are

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - \sum_\alpha Z_\alpha n_\alpha\,, \tag{3}$$

where $n_e = \exp\{\phi\}$, the subscript $\alpha = i$ is for positive ions and $\alpha = j$ for negative ions, n_α, u_α are the number densities and velocities of the ions, respectively. $Z_\alpha = 1$ for $\alpha = i$ and $Z_\alpha = -Z$ for $\alpha = j$, $Q_\alpha = (m_j/m_i)$ and $Q_\alpha = 1$ for $\alpha = i$ and $Q_\alpha = Q$ for $\alpha = j$. n_0 is the unperturbed electron density and ϕ denotes the electrostatic potential.

To obtain the solitary wave solutions, we introduce the single independent variable η defined by the relation

$$\eta = x - Vt\,. \tag{4}$$

Here V is the velocity of the solitary wave. We further assume that the basic equations are supplemented by the following boundary conditions

$$u_\alpha \to u_{\alpha 0}\,, \quad n_\alpha \to n_{\alpha 0} \;\text{ and }\; \phi \to 0 \;\text{ as }\; |x| \to \infty\,. \tag{5}$$

The charge neutrality condition of the plasma is

$$\sum_\alpha Z_\alpha n_{\alpha 0} = 1\,. \tag{6}$$

Using (4) in Eqs. (1)–(3), we get

$$-V\frac{\mathrm{d}n_\alpha}{\mathrm{d}\eta} + \frac{\mathrm{d}}{\mathrm{d}\eta}(n_\alpha u_\alpha) = 0\,, \tag{7}$$

$$-V\frac{\mathrm{d}u_\alpha}{\mathrm{d}\eta} + u_\alpha\frac{\mathrm{d}u_\alpha}{\mathrm{d}\eta} = -\frac{Z_\alpha}{Q_\alpha}\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\,, \tag{8}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = n_e - \sum_\alpha Z_\alpha n_\alpha\,. \tag{9}$$

Integrating Eq. (7) by using the boundary conditions (5), we get

$$n_\alpha = n_{\alpha 0}\frac{u_{\alpha 0} - V}{u_\alpha - V}\;. \tag{10}$$

Integrating Eq. (8) and using the boundary conditions (5) and Eq. (10), we get

$$n_\alpha = \frac{n_{\alpha 0}}{\sqrt{1 - \dfrac{2Z_\alpha\phi}{Q_\alpha(V - u_{\alpha 0})^2}}}\;, \tag{11}$$

where we have taken the negative branch of the density so as to satisfy the boundary conditions (5).

A similar correspondence is also used by Baboolal et al. [49, 50] to numerically select the correct branch of density in their investigation. In addition, requiring the densities to be real imposes the following restrictions on ϕ

$$-\frac{1}{2Z}Q(V - u_{j0})^2 < \phi < \frac{1}{2}(V - u_{i0})^2\,. \tag{12}$$

In the absence of negative ions without drift, the condition (12) is similar to the condition obtained by Sagdeev [51] for non-drifting cold positive ions only.

From Eq. (9) after putting the values of n_e and n_α we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = \mathrm{e}^\phi - \sum_\alpha \frac{Z_\alpha n_{\alpha 0}}{\sqrt{1 - \dfrac{2Z_\alpha\phi}{Q_\alpha(V - u_{\alpha 0})^2}}}\;. \tag{13}$$

Equation (13) can be written in the pseudopotential form

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{\partial\psi}{\partial\phi}\,. \tag{14}$$

Now, in the usual way, Eq. (14) can be integrated formally to yield "the energy law"

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 + \psi(\phi) = 0\,, \tag{15}$$

where

$$\psi(\phi) = (1-\mathrm{e}^{\phi}) + \sum_{\alpha} Z_\alpha Q_\alpha n_{\alpha 0}(V-u_{\alpha 0})^2\left[1-\sqrt{1-\frac{2Z_\alpha\phi}{Q_\alpha(V-u_{\alpha 0})^2}}\,\right]. \tag{16}$$

Here $\psi(\phi)$ is the Sagdeev pseudopotential function which actually gives the behaviour of solitary waves and is a moderately complicated non-linear function of ϕ. This pseudopotential function $\psi(\phi)$ contains three terms; the first is due to electrons, the second is due to drifting positive ions and the third is due to drifting negative ions. In the absence of negative ions, Eq. (16) is reduced to cold-ion solution as was obtained by Sagdeev [51].

In the presence of negative ions, both compressive and rarefactive solitary waves are found. However, in the present case we impose an extra condition $\psi[\phi = (1/2)(V-u_{i0})^2] > 0$ for the existence of compressive solitary waves in order to keep ion density (n_α) real and thus prevent wave breaking. The above condition gives

$$\mathrm{e}^{M^2/2} < 1 + n_{i0}M^2 + Qn_{j0}(V-u_{j0})^2\left[1-\sqrt{1+\frac{Z}{Q}\left(\frac{M}{V-u_{j0}}\right)^2}\,\right], \tag{17}$$

where $M = (V-u_{i0})$. The relation (17) fixes the upper limit for M in the presence of negative ions. In the absence of negative ions, this condition reduces exactly to Sagdeev's condition for $n_{i0} \to 1$ [51].

3. *Solitary-wave solution*

In order to obtain the solitary-wave solutions from Eq. (15), the Sagdeev pseudopotential function $\psi(\phi)$ must satisfy the following conditions [51]

$$\left\{\begin{aligned} \psi(\phi) = \frac{\partial\psi(\phi)}{\partial\phi} &= 0, \quad \text{for} \quad \phi = 0\,, \\ \psi(\phi) &= 0, \quad \text{for} \quad \phi = \phi_m\,, \\ \psi(\phi) &< 0, \quad \text{for} \quad 0 < |\phi| < |\phi_m|\,, \end{aligned}\right. \tag{18}$$

where $|\phi_m|$ is the amplitude of the solitary waves. The condition for the existence of the solitary wave solution is

$$\left.\frac{\partial^2\psi}{\partial\phi^2}\right|_{\phi=0} < 0\,. \tag{19}$$

This inequality (19) gives

$$\frac{Z^2 n_{j0}}{Q(V-u_{j0})^2} + \frac{n_{i0}}{(V-u_{i0})^2} < 1\,. \tag{20}$$

In the absence of negative ions with non-drifting plasma, this inequality reduces to Sagdeev's condition for $n_{i0} \to 1$ [51]. Substituting M for $(V - u_{i0})$, we get from (20)

$$M > n_{i0}^{1/2}\left[1 - \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2}\right]^{-1/2}. \tag{21}$$

This gives the lower limit of M in the presence of negative ions. The phase velocity is given by the equation

$$\frac{Z^2 n_{j0}}{Q(V-u_{j0})^2} + \frac{n_{i0}}{(V-u_{i0})^2} - 1 = 0\,, \tag{22}$$

which gives

$$\begin{aligned} QV^4 - 2Q(u_{i0}+u_{j0})V^3 + (Qu_{i0}^2 + Qu_{j0}^2 + 4Qu_{i0}u_{j0} - Qn_{i0} - Z^2 n_{j0})V^2 \\ +2(Z^2 n_{j0}u_{i0} - Qu_{i0}u_{j0}^2 - Qu_{i0}^2 u_{j0} + Qn_{i0}u_{j0})V \\ +(Qu_{i0}^2 u_{j0}^2 - Qn_{i0}u_{j0}^2 - Z^2 n_{j0}u_{i0}^2) \quad = \quad 0\,. \end{aligned}$$

It is quadratic in V^2 and will give two values of V^2. This equation is the modified form of equations of Refs. [24] and [52]. When drift velocities are absent, we get the phase velocity (V) as $V = \sqrt{n_{i0} + (Z^2 n_{j0}/Q)}$ which exactly reduces to results of Refs. [24] and [52] for $Z = 1$. Again from Eq. (22) after using $V - u_{i0} = V - u_{j0} = \lambda$ (λ is assumed to be the phase velocity) and the charge neutrality condition (6) we finally get

$$\lambda^2 = (V - u_{\alpha 0})^2 = (1 + zn_{j0}) + \frac{z^2 n_{j0}}{Q}\,, \quad \text{where} \ \ \alpha = i, j\,. \tag{23}$$

Moreover the expression for critical negative ion concentration ($n_{j\mathrm{c}}$) is obtained from inequality (20) as

$$n_{j\mathrm{c}} = \frac{Q(V-u_{j0})^2[(V-u_{i0})^2 - 1]}{Z[Z(V-u_{i0})^2 + Q(V-u_{j0})^2]}\,.$$

Now we discuss the behaviour of Sagdeev pseudopotential function in the vicinity of the critical negative ion density. Actually, $n_{j\text{c}}$ vanishes for $V - u_{i0} = 1$ or $V - u_{j0} = 0$. When $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $Q = 4$ and $Z = 1$, then $n_{j\text{c}} = 0.17812$. If we take $n_{jo} = 0.05 < n_{j\text{c}} = 0.17812$ with $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $Q = 4$ and $Z = 1$, then $\psi(\phi) = 0$ for $\phi = 0.2135$, and in this case compressive solitary waves are formed as shown in Fig. 1. Secondly, if we take $n_{j0} = n_{j\text{c}} = 0.17812$ with $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $Q = 4$ and $Z = 1$, then $\psi(\phi) = 0$ for $\phi = 0.00135$ (up to 5 decimal places) and $\phi = -0.01295$ (up to 5 decimal places), and in this case compressive and rarefactive solitary waves are formed. In the third case, when $n_{j0} = 0.18 > n_{j\text{c}} = 0.17812$ with $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $Q = 4$ and $Z = 1$, then $\psi(\phi) = 0$ for $\phi = -0.01301$ (up to 5 decimal places), and in this case rarefactive solitary waves are formed.

4. *Amplitude and width of solitary-wave solutions*

To compare our results with perturbation theory [53] and the solution of the KdV equation, we expand $\psi(\phi)$ in power series of ϕ. Neglecting terms up to order $0(\phi^3)$, and noting that

$$\psi(\phi) = 0 = \frac{\partial \psi}{\partial \phi} \quad \text{at} \quad \phi = 0\,,$$

we can write from (14)

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{\partial \psi}{\partial \phi} = a\phi - b\phi^2\,,$$

where

$$a = -\left.\frac{\partial^2 \psi}{\partial \phi^2}\right|_{\phi=0} \quad \text{and} \quad b = \left.\frac{1}{2}\frac{\partial^3 \psi}{\partial \phi^3}\right|_{\phi=0}.$$

To compare our second-order result with the above mentioned case, we include a third-order term in the expansion of the pseudopotential function $\psi(\phi)$ in terms of ϕ. From equations (13) and (14), retaining terms of order $0(\phi^3)$, we obtain

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{\partial \psi}{\partial \phi} = a\phi - b\phi^2 + c\phi^3,$$

where a and b are identical with those given above and c is given by

$$c = -\left.\frac{1}{6}\frac{\partial^4 \psi}{\partial \phi^4}\right|_{\phi=0}.$$

From the solitary wave solution for small amplitude solitary wave, the first-order K-dV amplitude [44, 54] of the solitary wave is

$$\Phi_{01} = \frac{3a}{2b}\,, \tag{24}$$

and the first-order width [46,54] of the solitary wave is

$$D_1 = \frac{2}{\sqrt{a}}\,. \tag{25}$$

The second-order amplitude of the solitary wave solution is

$$\Phi_{02} = \frac{6a}{2b + \sqrt{4b^2 - 18ac}}\,, \tag{26}$$

and the second-order width [46] of solitary wave is

$$D_2 = \frac{2}{\sqrt{a}} \cosh^{-1} \sqrt{\frac{1.381b}{\sqrt{4b^2 - 18ac}} + 1.6905}\,. \tag{27}$$

From Eqs. (16) and (18), we have $\psi(\phi_m) = 0$ where $0 < |\phi| < |\phi_m|$ and $|\phi_m|$ is the amplitude of the solitary waves. The above condition gives

$$1 - \mathrm{e}^{\phi_m} + n_{i0}(V - u_{i0})^2 \left[1 - \sqrt{1 - \frac{2\phi_m}{(V - u_{i0})^2}} \right]$$
$$+ Q n_{j0}(V - u_{j0})^2 \left[1 - \sqrt{1 + \frac{2Z\phi_m}{Q(V - u_{j0})^2}} \right] = 0\,.$$

This is a non-linear equation in ϕ_m. Thus the Sagdeev pseudopotential $\psi(\phi)$ gives a fully non-linear amplitude ϕ_m showing non-linear curves only (compressive for $\phi_m > 0$ and rarefactive for $\phi_m < 0$). In our paper, after truncation of the Sagdeev pseudopotential up to third order, the first- and second-order amplitudes and widths are obtained from the K-dV theory.

5. *Discussion*

In this paper, we study the effect of positive and negative ion drifts on the existence of solitary waves and the first- as well as the second-order amplitudes and widths of the waves using the Sagdeev's pseudopotential approach. The energy condition $\phi_0 < \frac{1}{2}M^2$ is used, where $M = V - u_{i0}$ and ϕ_0 is the amplitude for the well established compressive solitary wave solution. We take for simplicity $\xi = V - u_{i0}$ and $\zeta = V - u_{j0}$ as shown in Figs. 1 to 7. In the presence of negative ions and for single-electron-temperature plasma, the solitary waves are formed when the following conditions are fulfilled:

(i) $n_{j0} < n_{j\mathrm{c}}$ (compressive waves),

(ii) $n_{j0} = n_{j\mathrm{c}}$ (compressive and rarefactive waves) and

(iii) $n_{j0} > n_{j\mathrm{c}}$ (rarefactive waves),

where n_{j0} is the initial negative ion concentration and $n_{j\mathrm{c}}$ is the critical negative ion concentration.

Figure 1 shows the typical Sagdeev pseudopotential curves ($\psi(\phi)$ vs. ϕ) of compressive solitary waves ($\phi > 0$) for a chosen set of parameters ($n_{j0} = 0.05$, $Q = 4$, $Z = 1$), and various values of $V - u_{\alpha 0}$, where $\alpha = i$ for positive ions and $\alpha = j$ for negative ions. It is also observed that as the values of $(V - u_{\alpha 0})$ (that is $\xi = V - u_{i0} = 1.1$, 1.2, 1.4 and $\zeta = V - u_{j0} = 1.3, 1.3, 1.7$) increase for some fixed values of Z, n_{j0} and Q, amplitude of the solitary waves also increases. This property is also true in the absence of negative ions ($n_{j0} = 0$) as is shown in Fig. 2. The effect of negative ions is clearly observed when comparing Figs. 1 and 2. The amplitude of the solitary waves in the presence of negative ions is smaller than the amplitude of the waves when negative ions are not present.

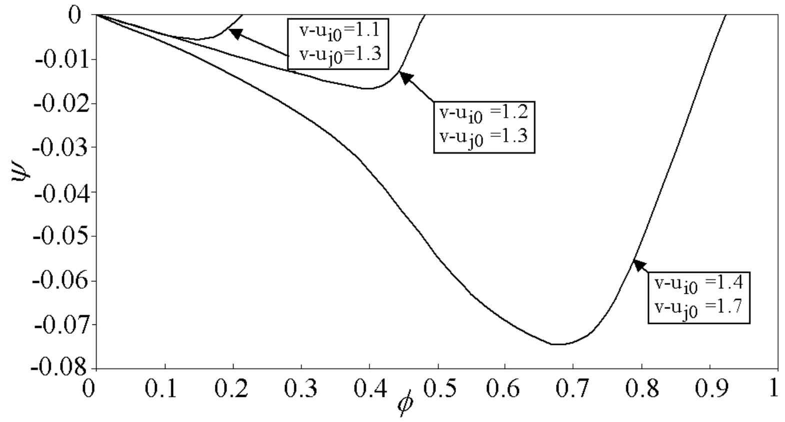

Fig. 1. Solitary wave profiles (ψ) versus electrostatic potential (ϕ) with the variation of velocities including positive and negative ion drift ($V - u_{\alpha 0}$) for constant mass ratio $Q = 4$, negative ion concentration $n_{j0} = 0.05$ and $Z = 1$.

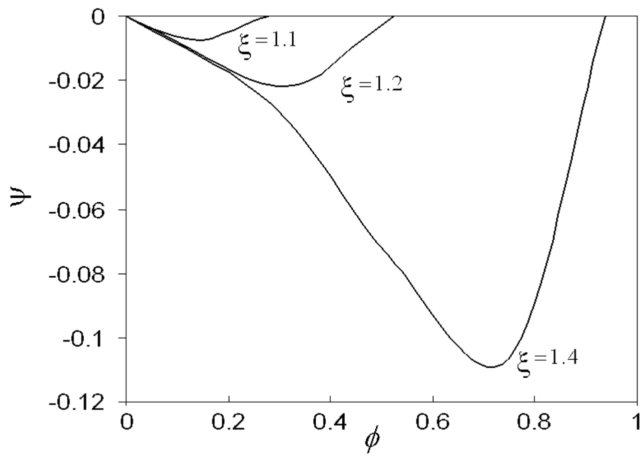

Fig. 2. Solitary wave profiles (ψ) vs. electrostatic potential (ϕ) with the variation of positive ion velocity including drift ($V - u_{\alpha 0}$) for constant positive ion concentration (n_{i0}) only.

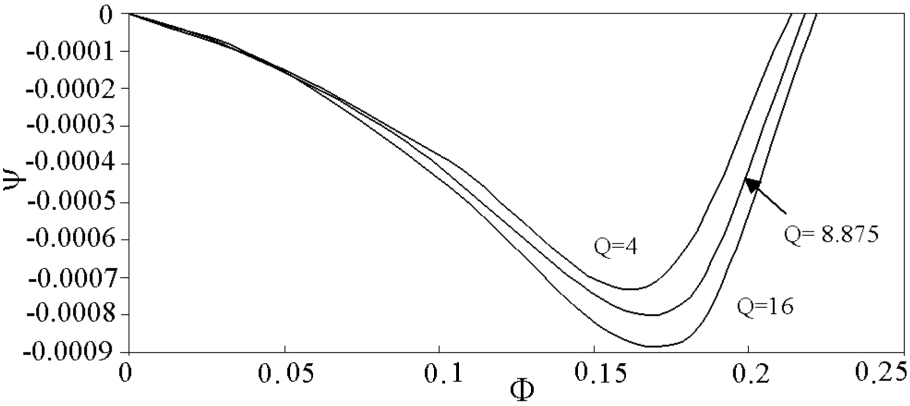

Fig. 3. Solitary wave profiles (ψ) vs. electrostatic potential (ϕ) with the variation of mass ratio (Q_α) for constant positive and negative ion velocities including drift ($V - u_{\alpha 0}$) with negative ion concentration (n_{j0}).

Figure 3 shows the Sagdeev pseudopotential curves ($\psi(\phi)$ vs. ϕ) of compressive solitary waves ($\phi > 0$) for some particular values of ($V - u_{\alpha 0}$), Z and n_{j0} ($\xi = V - u_{i0} = 1.1$, $\zeta = V - u_{j0} = 1.3$, $n_{j0} = 0.05$, $Z = 1$) for various mass ratios $Q_\alpha = (m_j/m_i)$. It is seen from this figure that the amplitude increases with Q_α, but the increase remains quantitatively much smaller than the increase w.r.t. M_α ($= V - u_{\alpha 0}$).

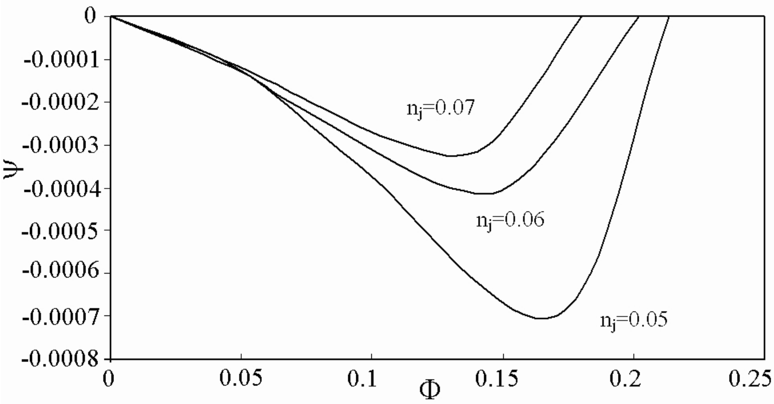

Fig. 4. Solitary wave profiles (ψ) vs. electrostatic potential (ϕ) with the variation of negative ion concentration (n_{j0}) for constant positive and negative ion velocities including drift ($V - u_{\alpha 0}$) and mass ratio (Q_α).

Figure 4 shows the Sagdeev pseudopotential curves ($\psi\,(\phi)$ vs. ϕ) for compressive solitary waves ($\phi > 0$) with a chosen set of parameters ($\xi = V - u_{i0} = 1.1$, $\zeta = V - u_{j0} = 1.3$, $Q = 4$ and $Z = 1$) for various negative-ion concentrations n_{j0}. The most pronounced effect arises due to the presence of negative ions. As the initial negative ion concentration n_{j0} increases ($n_{j0} = 0.05, 0.06, 0.07$), the amplitude of the solitary waves decreases. Howecer, the amplitude increases in the absence of negative ions ($n_{j0} = 0$) for a non-drifting positive ion plasma [29, 31, 53].

The compressive solutions are also allowed when $\phi < Q_\alpha(V - u_{\alpha 0})^2/(2Z_\alpha)$, which is similar to the condition obtained by Ghosh et al. [55] for a drifting positive-ion plasma. Now the necessary condition for the compressive solitary waves is $\psi(\phi = Q_\alpha(V - u_{\alpha 0})^2/(2Z_\alpha)) > 0$ for which $V < 1.6$, when negative ions are absent and for a non-drifting positive ion plasma in the region $\phi > 0$. For this type of solution ($\phi > 0$), Eq. (11) corresponds to $n_\alpha > n_{\alpha 0}$, i.e. the ion density is larger than the unperturbed ion density.

For the small-amplitude solitary wave solution we have from (24) and (26) that the first-order solution (Φ_{01}) as well as second-order solution (Φ_{02}), the amplitudes change with the parameters as follows.

Case 1: When $n_{j0} < n_{jc}$:

(i) Both Φ_{01} and Φ_{02} decrease for a constant mass ratio (Q) and constant values of the velocities including drift ($V - u_{\alpha 0}$) when the negative ion concentration ($n_{j0} < n_{jc}$) increases. Also, it is seen that as ($V - u_{\alpha 0}$) (that is $\xi = V - u_{io} = 1.1$, 1.2 and $\zeta = V - u_{j0} = 1.3$, 1.3) increase, Φ_{01} and Φ_{02} are also increasing for some fixed value of mass ratio (Q). This is shown in Fig. 5.

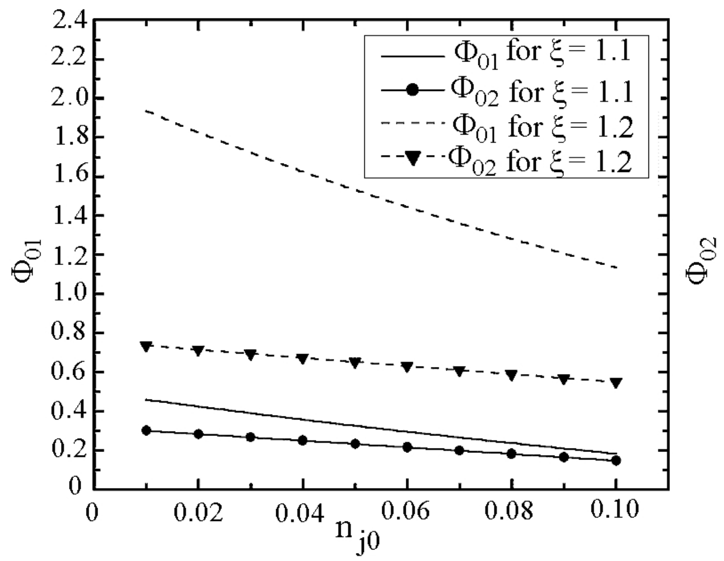

Fig. 5. Variation of the first- and second-order amplitudes (Φ_{01} and Φ_{02}) of solitary waves vs. negative ion concentration (n_{j0}) with the variation of positive and negative ion velocities including drift ($V - u_{\alpha 0}$) for constant mass ratio (Q_α).

(ii) Also, both Φ_{01} and Φ_{02} decrease for constant values of velocities ($V - u_{\alpha 0}$) including drift and a fixed mass ratio (Q) when the negative ion concentration (n_{j0}) increases. It should also be noted that Φ_{01} and Φ_{02} are increasing as the mass ratio

(Q) is increasing for three different ion pairs (He^+, O^-),(He^+,Cl^-) and (H^+,O^-) having mass ratios 4, 8.875 and 16, respectively, assuming the values of $(V - u_{\alpha 0})$ fixed.

Case 2: When $n_{j0} = n_{jc}$:

In this case, it is observed that the coefficient of nonlinearity in the K-dV equation vanishes [24] at the critical value of the negative ion concentration (n_{jc}), so the compressive as well as rarefactive solitary waves are formed for some model plasmas having (He^+,O^-), (He^+,Cl^-) and (H^+,O^-) ions.

Case 3: When $n_{j0} > n_{jc}$:

In this case the most important and interesting situation appears. It is discussed above that compressive solitary waves ($\phi > 0$) occur for $n_{j0} < n_{jc}$ in which particular amplitudes are found, and for $n_{j0} = n_{jc}$ the nonlinear coefficient in the K-dV equation vanishes. Therefore, to discuss the solitary-wave solution at the critical concentration of negative ions (n_{jc}), one must consider the higher-order nonlinearity. It is found again that for $n_{j0} = n_{jc}$ compressive and rarefactive solitary waves coexist, but for $n_{j0} > n_{jc}$, rarefactive solitary waves ($\phi < 0$) will occur with a definite amplitude, and this is the condition for transition from compressive to rarefactive solution.

Similarly to the above three cases (i.e. $n_{j0} < n_{jc}$, $n_{j0} = n_{jc}$ and $n_{j0} > n_{jc}$) the effects of the first- (D_1) and the second-order (D_2) widths are discussed below.

For $n_{j0} < n_{jc}$:

As negative ion concentration (n_{j0}) increases, D_1 and D_2 of solitary waves also increase. If the mass ratio (Q) is increased, then D_1 and D_2 decrease, while all other parameters remain constant. When $(V - u_{i0})$ increases, the widths D_1 and D_2 also decrease for some fixed value of $(V - u_{j0})$. This is shown in Fig. 6.

For $n_{j0} = n_{jc}$:

In this case D_1 and D_2 are much larger than in the previous case $n_{j0} < n_{jc}$ so that measurable widths are obtained.

For $n_{j0} > n_{jc}$:

In this case rarefactive solitary waves are formed from which D_1 and D_2 are calculated easily.

Figure 7 shows the phase velocity $\lambda = V - u_{\alpha 0}$ of the solitary waves versus the negative ion concentration (n_{j0}) for different values of the mass ratio (Q) of negative and positive ions. As n_{j0} increases, the phase velocity $\lambda = V - u_{\alpha 0}$ increases for some particular values of Q and a fixed value of Z. Again when Q increases (that is $Q = 4$, 8.875, 16), the phase velocity (λ) decreases for some fixed value of the negative ion concentration (n_{j0}) and a fixed value of Z.

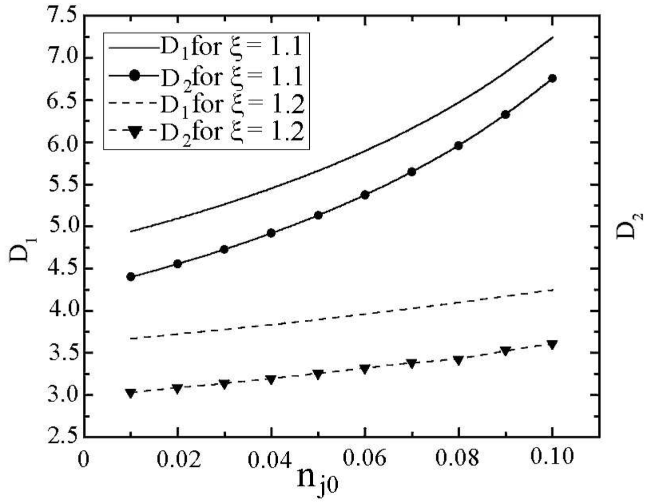

Fig. 6. Variation of first- and second-order widths (D_1 and D_2) of solitary waves vs. negative ion concetration (n_{j0}) with the variation of positive and negative ion velocity including drift ($V - u_{\alpha 0}$) for constant mass ratio (Q_α).

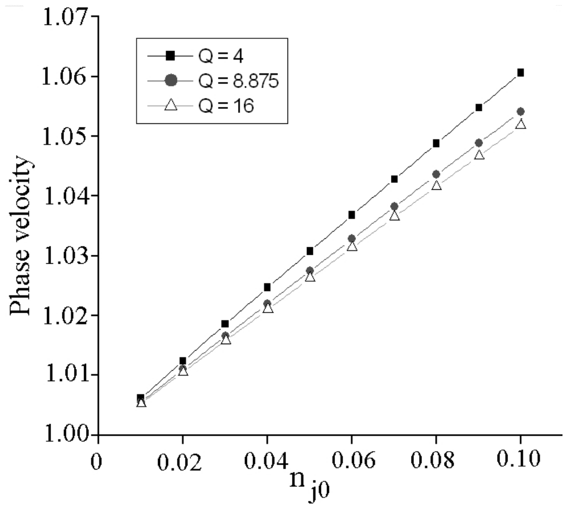

Fig. 7. Variation of phase velocity λ ($= V - u_{\alpha 0}$) of solitary waves against negative ion concentration (n_{j0}) for different values of Q (ratio of negative to positive ion masses) and a fixed value of z.

6. *Concluding remarks*

In this paper we investigate the effect of cold positive and negative ions in plasma with hot electrons on the existence of solitary waves, and on amplitudes, widths and phase velocity of solitary waves, using Sagdeev's pseudopotential approach. We have shown the Sagdeev pseudopotential curves ($\psi(\phi)$ vs. ϕ) and first- as well as second-order amplitudes, widths and the phase velocity of the waves. It is interesting to note from Eqs. (17) and (21) that the upper and lower limits of M $(= V - u_{i0})$ is found which is a modified form of Sagdeev's result. In the absence of negative ions, Eqs. (17) and (21) reduces to Sagdeev's condition. For a negative-ion plasma, Nakamura et al. [15], Ikezi et al. [23] and many other authors have observed experimentally the ion-acoustic solitary waves with cold positive ions and warm electrons. We are planning to study ion-acoustic solitary waves near the critical density of negative ions for a relativistic plasma [56] with electron drifts and using pseudopotential approach.

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Appendix. Some remarks

The Sagdeev pseudopotential function (ψ) is a moderately complicated nonlinear function of the electrostatic potential (ϕ) and is obtained by the Taylor's series expansion keeping only a few terms in the expansion of ψ. In the present study, the Sagdeev pseudopotential function (ψ) is convergent at least for the parameters used for the results, i.e. the method of expansion of $\psi(\phi)$ in powers of ϕ is valid for a specified range of the electrostatic potential (ϕ) given by

$$-\frac{Q}{2Z}(V - u_{j0})^2 < \phi < \frac{1}{2}(V - u_{i0})^2 \,,$$

from which the desired degree of accuracy of the roots of $\phi = \phi_s$ are obtained, where $s = 0$ and $s = m$ (correct up to four decimal places) of the equation $\psi(\phi) = 0$. ϕ_0 and ϕ_m are the minimum and maximum values of ϕ.

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UČINCI NEGATIVNO-IONSKE PLAZME NA IONSKO-ZVUČNE SOLITARNE VALOVE U HLADNOJ POSMIČNOJ POZITIVNO-IONSKOJ PLAZMI S IZOTERMIČKIM ELEKTRONIMA

Primjenom metode pseudopotencijala istražujemo učinke nelinearnosti višeg reda na ionsko-zvučne solitarne valove u bezsudarnoj izotermičkoj elektronskoj plazmi s hladnim pozitivnim i negativnim ionima. Raspravljamo utjecaj negativnih iona na solitarne valove te na amplitude prvog i drugog reda, širine i fazne brzine u plazmama koje sadrže ione (He^+, O^-), (He^+, Cl^-) ili (H^+, O^-), uz promjene parametara. Postignuti ishodi prikazuju se u crtežima. Raspravljamo također područje elektrostatskog potencijala (ϕ) za koje na željenu točnost vrijedi pseudopotencijalna funkcija (ψ).