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CREEP RUPTURE IN FIBER BUNDLE MODEL

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We study the creep rupture of fiber composites subjected to uniaxial constant load within the framework of the fiber bundle model. Our numerical studies show that in the primary creep, the deformation rate exhibits a power law relaxation followed by a power law acceleration in the tertiary creep. We also investigate the behavior of the failure time and the avalanche size distribution. A good agreement is found between experiments and the model prediction, particularly for deformation power-laws scaling in the primary-creep regime and in the tertiary-creep neighbourhood of rupture.

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1. Introduction

Fracture of heterogeneous materials has recently received much technological and industrial interest and has been widely studied in statistical physics. It covers a wide range of phenomena, like material science, rock mechanics and earth sciences. Many authors [1 − 9] are studying this fracture phenomenon through different models and through direct experiences.

An important class of approaches to the fracture problem are the fiber bundle

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models (FBM) which have been the subject of intense research during the last several years [10 − 13]. These models are being classified into two groups: static and time dependent or dynamic. The static version of FBM simulates the failure of materials by quasistatic loading. On the other hand, the dynamic FBM simulates failure by creep rupture. Usually a constant load is maintained on the system and the fibers break by fatigue after a period of time. Once the fibers begin to fail, one can choose among two load transfer rules. In the global load sharing (GLS) rule, the load of a broken fiber is equally shared with all intact fibers in the whole system. This model is known as the democratic fiber bundle and assumes long-range interaction among the fibers which makes it a mean-field approximation that can be solved analytically [14, 15]. The other rule is the local load sharing (LLS), which the terminal load of the failed fiber is given equally to all intact neighbors. This case assumes short-range interaction among the fibers.

In the present paper, we study the creep rupture of fiber bundles, which consists in submitting a sample to a constant load untill the failure time. Creep-tests are widely used by engineers in order to estimate the sample lifetime as a function of the applied stress. In the last years, many scientists have studied the creep rupture phenomena through direct experiments (16 − 18) as well by different models (19 − 21).

The rest of the paper is organized as follows. In the next section, we describe the FBM model under the GLS rule. In Section 3, we present and discuss the numerical results of the Langevin dynamic simulation. Finally, Section 4 is devoted to conclusions.

2. *The model*

The fiber bundle model (FBM) consists of N fibers arranged as a square lattice of size $L \times L$, where N fibers in parallel is subjected to an external force F. The geometrical structure of the model is illustrated in Fig. 1.

The applied force produces a local force f_i on each fiber. F is democratically distributed in the system, $F = \sum_{i=1}^{N} f_i$. The local force f_i on the i^{th} fiber produces a local strain ε_i. The force and displacement are linked by the Hooke's law: $f_i = k\varepsilon_i$,

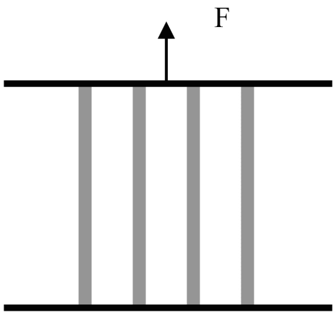

Fig. 1. A schematic illustration of the fiber bundle model.

where k is the stiffness, which is assumed to be the same for all fibers. In order to simulate the dynamics of fibers, we use the Langevin equation [22, 23]. This approach is characterized by the use of a stochastic differential equation which denotes all ingredients necessary to study the creep test in fiber bundle model such as thermal noise and frictional force. The dynamics of the system is completely determined by the Langevin equation

$$m_i \frac{\mathrm{d}^2 z_i}{\mathrm{d}t^2} = -m_i \gamma_i \dot{z}_i + f_i(z_i) + \vec{R}_i(t)\,,$$

where z_i, m_i and γ_i denote the position, mass and viscous friction coefficient of the fiber, respectively. This equation is a stochastic differential equation in which two force terms have been added to Newton's second law to approximate the effects of neglected degrees of freedom. One term represents the frictional force, $\gamma_i z_i$, the other a random force, $\vec{R}$, which describes the thermal noise. It is usually modelled by a Gaussian white noise with zero time average, $\left\langle \vec{R}_i(t) \right\rangle = 0$, and autocorrelation function

$$\left\langle \vec{R}_i(t) \vec{R}_i(t') \right\rangle = 2 m_i \gamma_i k_{\mathrm{B}} T \delta(t - t')\,.$$

The angular brackets denote here an average and k_{B} is Boltzmann's constant. We impose that the system is strongly frictional which allows to neglect inertial effects. In this case the system is overdamped.

Initially, each fiber has a length z_0. When we apply a load $f_0 = k \min \delta_c(i)$, where $\delta_c(i)$ is a different random elongation taken from an uniform distribution, the fiber dilates and its length becomes $z = z_0 + \delta$, where δ is the fiber elongation.

If δ exceeds the elongation threshold value δ_c, the fiber is removed and its load is transferred equally to all intact ones. We consider only the case of global load sharing (GLS) for the redistribution of load following a fiber failure.

In numerical simulation, the cycle of complete breakdown of the material is performed many times in order to average out the effect of fluctuation.

3. *Numerical results*

Creep is the progressive deformation of a material under constant load at a given temperature. Three creep regimes are usually observed. The primary creep regime corresponds to a decay of the strain rate following the application of the constant stress, which can often be described by the so-called Andrade's law; the secondary regime describes a quasi-constant deformation rate, towards the tertiary creep regime in which the strain rate accelerates up to rupture. Figure 2 shows the three regimes of creep.

At time $t = 0$, a constant load $F = N f_0$ is applied to an initially undamaged system made of a very large number N of parallel elastic fibers. At all times, F is shared democratically among all $\rho(t)N$ surviving fibers, where $\rho(t)$ is the fraction of unbroken fibers at time t. Then the local force on each of the intact fibers is $f_i(t) = N f_0/\rho(t)$.

Figure 3 shows the behavior of the ratio $\rho(t)$, which defines the fraction of unbroken fibers, for different values of the load. The system fails rapidly as the applied load is increased.

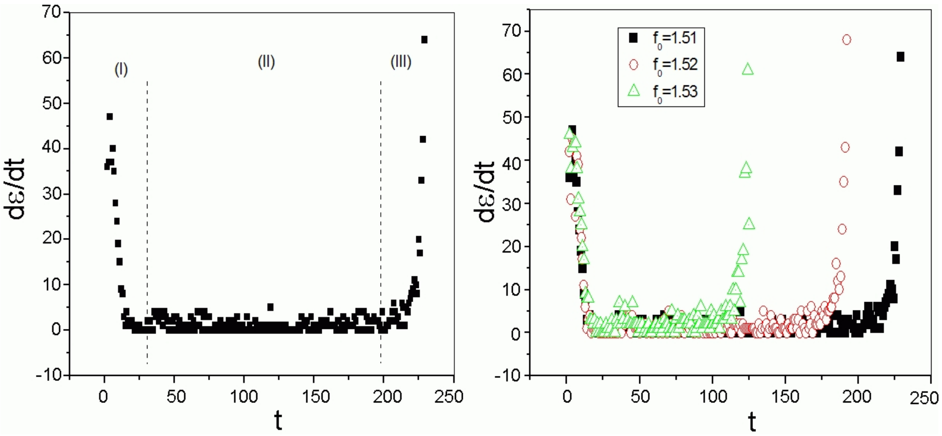

Fig. 2. Creep deformation rate.

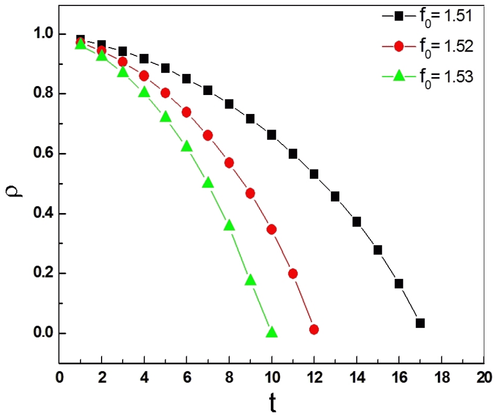

Fig. 3. Behavior of the fraction of unbroken fibers for three different forces and for system size $L = 128$.

Now we study the avalanche size in this mean field model. We define the avalanche size distribution as the number of fibers that break between two steps of time. Here the avalanche size can be interpreted as the rate of breaking ($\mathrm{d}\rho/\mathrm{d}t$). In the case of GLS, the avalanche size distribution $n(s)$ follows a universal power law: $n(s) \sim s^{-\alpha}$, with an exponent $\alpha = 5/2$. In Fig. 4 we plot the avalanche size distribution. This simulation result has been confirmed by analytical means in the case of global load sharing [10, 14, 15].

Now we explore in Fig. 5 the behaviour of the deformation rate in the primary creep regime, The deformation rate shows a rapid decrease, which can be described by the Andrade's law $d\varepsilon/dt = 1/t^p$ with an exponent p equal to 1 (Fig. 5). In the secondary creep, a steady-state is observed over an important part of the total creep time (Fig. 2), and then followed by an increasing creep rate (tertiary creep regime) culminating in fracture. In Fig. 6 we plot the creep deformation rate in the tertiary creep regime. The acceleration of the deformation rate before failure is well fitted by a power-law singularity $d\varepsilon/dt = 1/(t_f - t)^{p'}$ with

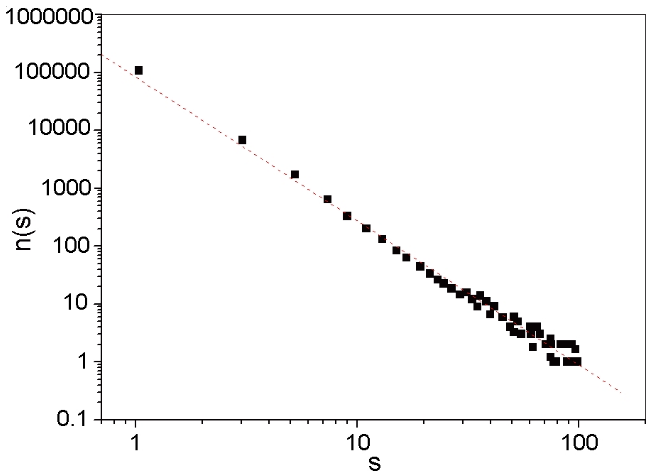

Fig. 4. Avalanche size distributions. The dashed line corresponds to the power law $s^{-5/2}$.

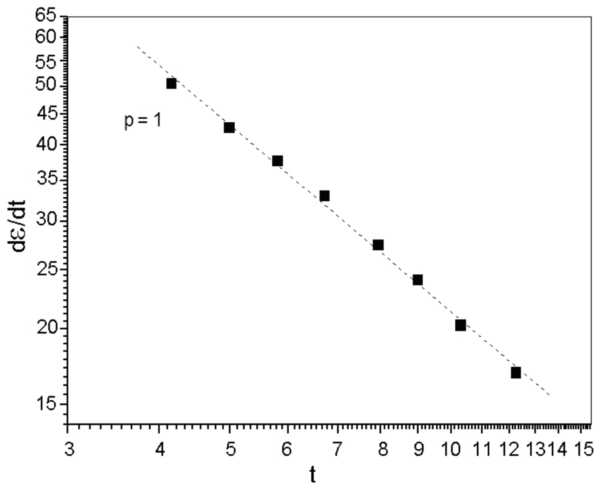

Fig. 5. Creep deformation rate in the primary regime.

an exponent p' equal to 1 (Fig. 6). Here t_f is the failure time. The key ingredients leading to these results are the choice of the fiber bundle model (FBM) under global load sharing rule and the Langevin equation which make the system thermally activated and also frictional. With these ingredients, the model reproduces the Andrade relaxation law in the primary creep regime followed by a power-law singularity of the strain rate before failure with exponents $p = p' = 1$. The expression for the strain rate as a function of time was derived analytically [24]. This model reproduces a power-law singularity of the strain rate before failure $\mathrm{d}\epsilon/\mathrm{d}t = (t_f - t)^{-p'}$ with $p' = -1/2$ in the case of disorder distributions $p(\epsilon)$ defined in a finite interval.

In another numerical work [25] the strain rate $\mathrm{d}\epsilon/\mathrm{d}t$, at macroscopic rupture t_f, was found to have a power law divergence $\mathrm{d}\epsilon/\mathrm{d}t = (t_f - t)^{-\gamma}$, with an exponent γ which does not depend on the external load and the disorder distribution, but depends only on the stress exponent m governing the relaxation of broken fibers.

Recently, an experimental work on heterogeneous materials [26] proves that these power laws are also valid for acoustic emission emitted by microscopic failure events during the creep tests. The exponents p and p' are similar for the acoustic emission event rate and for the acoustic emission energy rate.

Now we explore in Fig. 7 the behavior of failure time t_f as a function of the load, which is determined when all fibers are broken. This failure time characterises the tertiary creep regime. Our numerical study shows that t_f has a power law divergence at f_c with a universal exponent $\beta = 1/2$. This power law can be written as $t_f \sim (f_0 - f_c)^{-\beta}$ and it has been verified previously [21, 26] in the case of $f_0 > f_c$.

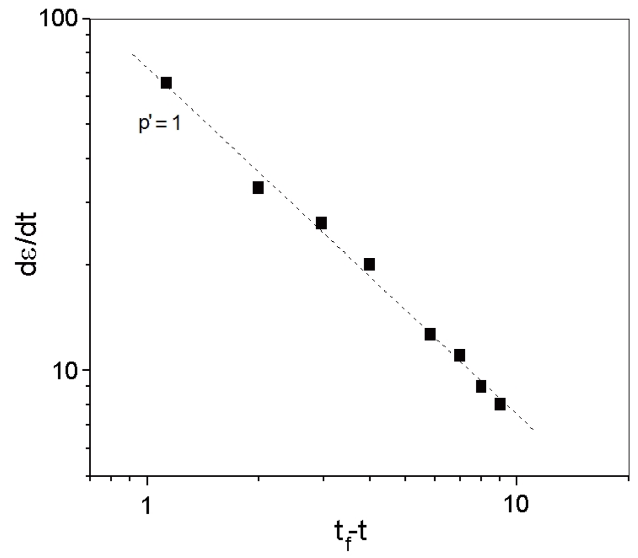

Fig. 6. Creep deformation rate in the tertiary regime. The dotted line corresponds to an exponent $p' = 1$.

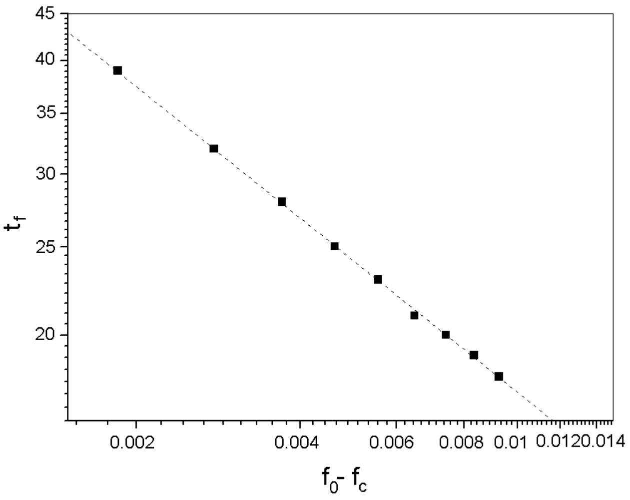

Fig. 7. The behavior of time to failure t_f. The dashed line correspond to a decay power 0.5.

4. Conclusion

In summary, we have studied the creep rupture of heterogeneous materials within the framework of fiber bundle model under the GLS rule. In our numerical study, we have showed that in the primary creep, the deformation rate exhibits a power law relaxation followed by a power law acceleration in the tertiary creep. We have also investigated the behavior of some properties of creep rupture like the failure time and the avalanche size distribution, which are in agreement with some recent work on the fiber bundle model.

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KLIZNI LOM U MODELU SNOPA VLAKANA

Proučavamo klizni lom vlaknastog kompozita opterećenog stalnom jednoosnom silom primjenom modela snopa vlakana. Naša numerička istraživanja pokazuju da deformacije u početnom klizanju slijede potencijalni zakon, a u tercijalnom ubrzanje s drugim potencijalnim zakonom. Ispitujemo također vrijeme loma i raspodjelu jakosti lavine. Nalazimo dobro slaganje eksperimentalnih podataka s predviđanjima modela, posebice za sumjerenost potencijalnog zakona u početnim kliznim uvjetima i blizu tercijalnog kliznog loma.