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LETTER TO THE EDITOR

# IS THERE ANOTHER SCALE IN THE FRACTIONAL QUANTUM HALL EFFECT?

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We reanalyse theoretical considerations and experimental data, in an attempt to decide whether there is another scale in the fractional quantum Hall effect problem, in addition to the magnetic scale defined by the magnetic lenght $a_c$ or the cyclotron energy $\hbar\omega_c$. We then discuss possible implications of a new scale on the formulation of a theoretical model of the fractional quantum Hall effect.

Since its discovery, the fractional quantum Hall effect (FQHE)[1)] has attracted considerable attention. The effect occurs in two-dimensional electron systems (realized at certain semiconductor interfaces), with applied magnetic fields so strong that only a fraction of the lowest Landau level is occupied. The questions to be answered are the same as for the integer quantum Hall effect (IQHE), namely, why the Hall conductance is very accurately quantized in multiples (IQ HE) or rational fractions (FQHE) of the fundamental quantity $e^2/h$, in spite of the irregularities present in real samples, and why the conductance plateaus occur over a finite interval of the magnetic field $B$. The answers are now rather well understood. An additional and most intriguing question in the FQHE is why the effect exists at all. In the IQHE, there is a large energy gap $\hbar\omega_c$ between Landau levels even in the idealized picture of non-interacting electrons and no irregularities, which is a good starting point for understanding the question why the states with an integer number of Landau levels filled are particularly stable. There is no similar simple argument in the FQHE. Obviously, the electron-electron interaction must be taken into account if any non-trivial property is to be obtained, and even then it is not clear at first sight why certain filling fractions (notably $v = 1/m$, where $m = 1, 3, 5 \ldots$) are energetically favoured.

Much of the present understanding of the FQHE is based on strongly correlated many-body wavefunctions proposed by Laughlin[2]. Their simplest form is

$$\psi_m = \prod_{j<k}^{J} (z_l - z_k)^m \ \exp\left\{ -\frac{1}{4} \sum_{l}^{J} |z_l|^2 \right\} \tag{1}$$

where $z_i = x_i + \mathrm{i}y_i$ is the coordinate of the $i$-th electron and the magnetic length $a_c$ has been set equal to one. These wavefunctions describe a circular droplet of $J$ electrons with a constant density, corresponding to a filling fraction $\nu = 1/m$. It can be easily seen that they consist entirely of one-electron states from the lowest Landau level, $n = 0$. This approximation has been justified by the fact that at large fields the energy gap between Landau levels is much larger than the effects we are looking for. Calculations[2] have indicated that Laughlin's wavefunctions are indeed more stable than other states at the same filling factor, and that there is an energy gap which makes them incompressible. (Various modifications have been proposed in order to explain the FQHE at other filling factors, or to treat other geometries. We shall not consider them here.) Laughlin's wavefunctions are the eigenfunctions of the short-range interaction of the type[3]

$$V(r) = \sum_m V_m \, \nabla^{2m} \delta^3(\boldsymbol{r}) \tag{2}$$

More generally, it seems that in order to have a finite energy gap to excited states, the interaction potential must be singular enough. Numerical diagonalisation of small systems shows that the true ground state is close to the corresponding Laughlin's function for any such potential, and, in particular, for the Coulomb potential. These calculations are, however, limited to the lowest Landau level and a small number of particles, so that the generality of their results must be regarded with some caution. Anyway, it is widely accepted that a theory which includes states only from the lowest Landau level and assumes a strong repulsive interaction describes the FQHE correctly. Equation (1) is believed to describe the qualitative properties correctly, such as the existence of the energy gap. In the following we point out that there are certain difficulties with this approach. It has recently been pointed out[4.5] that it is inconsistent to assume at the same time that the potential is extremely short-ranged and that only the states from the lowest Landau level are involved. We shall first list some general properties of short-range potentials in quantum mechanics, both with and without the magnetic field, then re-examine the FQHE in the light of these and, finally, confront the experimental data with the theory.

An overview of the properties of $\delta$-function potentials in quantum mechanics has recently been given by Gosdzinsky and Tarrach[6]. Using the regularisation procedure, it can be shown that attractive $\delta$-potentials in two and three spatial dimensions can give an interacting theory if the coupling constant (potential strenght) approaches zero in an appropriate way as the range of the potential is made to vanish. In two dimensions, there is a bound state, in three, the potential only scatters.

On the other hand, repulsive potentials in more than one dimension and attractive in more than three dimensions do not lead to interaction.

In two dimensions, a magnetic field perpendicular to the plane closes the electron orbits, and the energy spectrum becomes discrete. The problem of a potential centre thus appears more similar to the perturbation calculation for bound states than to that for scattering. In the case of a $\delta$-function potential, one can proceed by taking a finite number $N$ of Landau levels instead of using a regulator[4)]. A potential centre at $z = 0$ leads to an energy shift of $m = 0$ states from the unperturbed values $\hbar\omega_c(n + 1/2)$. If the potential contains derivatives, such as the higher terms in Eq. (2), the states with the corresponding $m$ are affected. In all cases, the shifts for low $n$'s vanish as the number of Landau levels included goes to infinity. The only exception is the lowest state when the potential is attractive, which has a large negative energy shift, and can be made to converge to a bound state of a finite energy if the strenght of the potential approaches zero in a particular way when $N \to \infty$. These results are analogous to those in the case without the magnetic field, if the phrase "a scattering phase shift" is replaced by "an energy shift". In particular, there is no residual interaction when the potential is repulsive and $N \to \infty$.

In order to apply these results to real systems, we assume that the cutoff energy defined by $D = \hbar\omega_c N$ depends upon the material, but not upon the strenght of the magnetic field. In the following we divide all energies by $D$, making them proportional to "real" units, while the magnetic field becomes proportional to $1/N$. The most interesting quantity is the shift of the lowest Landau level in the limit of a very strong repulsive potential[4)]:

$$\Delta_m \approx \left[ N \sum_{j=1}^{N} \frac{1}{j} \begin{pmatrix} j + |m| \\ |m| \end{pmatrix} \right]^{-1} \tag{3}$$

This in a sense measures the strength of the effective potential felt by the electrons in the lowest Landau level. In the weak-field limit, the leading behaviour is $N^{-m-1}$, i.e. $B^{m+1}$, but higher terms become quickly important. For example:

$$\begin{aligned} \Delta_0 &\approx \left[N(\gamma + \ln N)\right]^{-1} \\ \Delta_1 &\approx \left[N(N + \gamma + \ln N)\right]^{-1} \\ \Delta_2 &\approx \left[N\left(\frac{N^2}{4} + \frac{7}{4}N + \gamma + \ln N\right)\right]^{-1} \end{aligned} \tag{4}$$

where $\gamma$ is Euler's constant. We have formulated our results in terms of an energy cutoff. By virtue of quantum-mechanical uncertainty relations, the latter is related to a spatial cutoff, which is more transparent physically. A large but finite energy cutoff is equivalent to a potential of a very short but finite range.

For long-range potentials, there is little connection, if any, between the problem of scattering on a fixed potential centre and the many-body problem with the interaction potential of the same form. This is not so in our case. In Refs. 4 and 5 it has been argued that in two-dimensional systems with an applied magnetic field the results concerning the effective strength of short-range interactions remain valid in the many-particle case, too. The most interesting consequence is that the effective interaction disappears if all Landau levels are taken into account. Including all Landau levels amounts to saying that no deviation from an ideal two-dimensionality is observed at any arbitrarily high energy. This vanishing of the effective interaction is in sharp contrast to what is obtained when Laughlin's wavefunctions (which only contain states from the lowest Landau level) and a short-range interaction are used to describe the FQHE. A way out of this apparent contradiction is to argue that the prescription "Laughlin's wavefunctions plus a $\delta$-function interaction" must not be taken literally, in spite of its appealing simplicity, and that the true finite-range interaction would give similar results, while being much less sensitive upon the inclusion of higher Landau levels. Thus the most likely chain of arguments is that the starting model for the FQHE problem should be "an ideal two-dimensional electron gas with an (approximately) Coulomb interaction". This model maps with great accuracy onto "lowest Landau level states with a short-range interaction", which is in turn diagonalised by Laughlin's wavefunctions at filling factors $\nu = 1/m$.

However, difficulties become evident even with this interpretation under further analysis. A crucial property of the model of "an ideal two-dimensional electron gas with a Coulomb interaction" is that it has only one scale, the magnetic scale defined by $a_c$ or $\hbar\omega_c$. The repulsive Coulomb interaction defines no scale of its own, which can be seen from the fact that the Rutherford scattering is classical. The average distance between electrons is also proportional to the magnetic length, because the FQHE occurs at constant values of the filling factor. A consequence of the single scale is that the strength of the electron-electron interaction (and hence of the magnitude of the FQHE gap) must vary as the average Coulomb repulsion, i.e. $e^2/a_c \sim B^{1/2}$, when the magnetic field $B$ is varied. This argument is general, and does not depend upon the use of Laughlin's wavefunctions or any other approximate approach. One is thus led to the conclusion that the experimental gap must scale with $B^{1/2}$, unless some physics beyond the model of "an ideal two-dimensional electron gas with a Coulomb interaction" is relevant. Before making further theoretical considerations, we look at experimental results.

The energy gap $\Delta$ has been determined experimentally by measuring the thermal activation behaviour of the diagonal resistivity at temperatures below 1 K, with the strength of the magnetic field corresponding to the centres of the FQHE plateaus[7–11]. In Fig. 1 we show a log-log plot of the magnitudes of the gap vs. the magnetic field. A theory which takes into account only the lowest Landau level implies the electron-hole symmetry within the Landau level, and hence the equivalence of the FQHE states at filling factors 1/3 and 2/3, 1/5 and 4/5, 2/5, 3/5, etc. While in the following we argue that the restriction to the lowest Landau level does not correctly give the absolute values of the energy gaps and their scaling with the magnetic field, we expect that at any value of the field the equivalence still holds to a good approximation. This means that the gap of the 1/3 state is very similar to what the gap of the 2/3 state at the same field would be (which has not been verified experimentally, because there is no way to vary the concentration of the electrons in the layer by such a large factor). The reason for this similarity is that the electron-hole symmetry argument gives the same value of the gap for both states, and that the reduction by the higher Landau level is rather similar. We have connected the experimental points referring to the same sample and to the equivalent filling factors. Note that the states at 1/5 and at 2/5 are not equivalent, and, indeed, the experimental values of the energy gaps are widely apart. In Ref. 8 the data were interpreted by a $B^{1/2}$ dependence, but with a constant negative

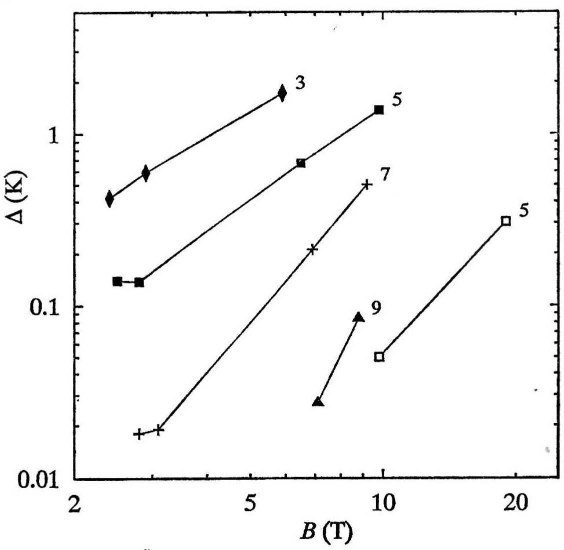


*Fig. 1. Experimental values of the FQHE gap as a function of the magnetic field, for several filling factors $p/q$. Diamonds: $q = 3$, and, in order from left to right, $p = 5, 4, 2$; full squares: $q = 5$, $p = 8, 7, 3, 2$; crosses: $q = 7$, $p = 10, 9, 4, 3$; triangles: $q = 9$, $p = 5, 4$ (all from Ref. 7); empty squares: $g = 5$, $p = 1$ for both points (Ref. 8).*

offset due to the disorder present in the sample. According to this interpretation, a (sample-dependent) threshold magnetic field should exist at which the FQHE gap becomes zero, and below which there is no FQHE. The agreement was poor. Our plot shows no sign of a threshold. The points lie on straight lines, with slopes depending upon the denominator of $\nu$. This corresponds to a power dependence upon $B$, with powers clearly larger than 1/2, which is reminiscent of our result for a single potential centre (4). It seems to us that this dependence is genuine, and not a consequence of the disorder, because the behaviour is more universal at low fields than at higher fields, where there is a large scatter of data. Thus we reject the interpretation in terms of a $B^{1/2}$ dependence shifted by disorder, and conclude that there must be another scale in the problem that depends upon the properties of the medium and is independent of $B$. In the limit $B \to 0$, the magnetic length becomes large, while other medium-dependent scales stay constant (the width of the electron layer, the small-distance unscreened portion of the Coulomb interaction, etc.; the new scale must result from some of these). In other words, this is the limit of the ideal two-dimensionality. The fact that in this limit no residual $B^{1/2}$ dependence of the gaps is observed leads to the conclusion that in a strictly two-dimensional electron gas a bare Coulomb interaction produces no FQHE at all.

This very surprising statement runs contrary to the general belief, and we must consider whether it is in contradiction with any firmly established theoretical results. We think that this is not the case. The calculations which give a definite numerical prediction for the FQHE gaps are based either on numerical diagonalisation within the lowest Landau level or upon Laughlin's wave functions, which also imply the restriction to the lowest Landau level. This is an artificial restriction of the Hilbert space of physical states. The conclusion reached in the preceding paragraph suggests that the gaps would vanish if higher Landau levels were included, which is at present impossible to verify numerically. (The inclusion of one or several higher Landau levels cannot give conclusive results.) There are other calculations, e.g. based on the Landau-Ginsburg approach, which do not make the restriction to the lowest Landau level. To our knowledge, these calculations have proved that the symmetry of the ground-state wave function at rational filling factors coincides with that of the corresponding Laughlin's functions, but they have not been successful in calculating the FQHE gap, and even the argument that the state is incompressible (i.e. that the gap is finite) depends on unproved additional assumptions.

We therefore suggest that, in real systems, the FQHE depends upon the existence of another scale. Judged from the results for a single potential centre, the energy scale is large compared with the characteristic energy $\hbar\omega_c$ of the problem, i.e. the spatial scale is small compared with $a_c$. The origin of this scale must lie beyond the usual assumptions, which are: (a) strict two-dimensionality, (b) Coulomb interaction, (c) translational invariance, i.e. no impurities. A possible candidate is the screening of the Coulomb interaction in real semiconductor devices, which modifies the assumption (b). The effective interaction varies from the bare Coulomb form $r^{-1}$ at distances smaller than, say the interatomic separation, to the screened one, $(r\varepsilon)^{-1}$, where $\varepsilon$ is the dielectric constant of the surrounding medium at large distances. Another possibility is that the new scale is associated with relaxing of assumption (a), i.e. that the motion of the electron in the third dimension becomes important. The characteristic length is the width of the potential well which binds the electron gas to the interface, and the corresponding characteristic energy is that of the first excited state of the perpendicular motion. Taking into account this degree of freedom invalidates the assumption of the perfect two-dimensionality, but only on an energy scale which is much larger than the characteristic energies of the system, i.e. the FQHE gaps. This modification of the model has deep consequences, because some properties that depend upon strict two-dimensionality, such as the possibility of performing the transformation of electrons into "anyons", particles with arbitrary quantum statistics, become only approximative. The mechanism which allows the high-energy properties to become relevant can be visualised in the following way. Quantum fluctuations can bring two electrons so close one to the other that the energy of the repulsive Coulomb interaction equals that of the first excited state of the perpendicular motion. Then the electrons can exchange their positions by one passing "above" the other, i.e. in the "high-energy physics" of the system, the trajectories by which the electrons exchange their positions avoiding each other on one or the other side are no longer topologically inequivalent, in contrast to the requirement of the theories which perform a transformation to anyons. The Coulomb interaction is essential, but only its short-range part is selected by this mechanism.

In order to clarify the origin of the proposed new scale, an improved theoretical treatment of various models is necessary. Should such a treatment show that the standard model of two-dimensional electrons with a Coulomb interaction would give the energy gaps in the FQHE even when the higher Landau levels would be taken into account, the unusual low-field dependence of the gaps would remain to be explained. We do not think that this interpretation is probable, although there is still the possibility that a new scale is induced by the disorder in the sample, which causes a profound change of the dependence of the gaps upon the magnetic field. We consider it more probable that the new scale is due to a modification of the other two assumptions of the original model. This could be either the dielectric screening of the electron-electron interaction, which should be possible to prove theoretically, or if it turns out that a two-dimensional theory is not sufficient, the fact that the electrons in real samples are bound to a plane only by a finite potential well. This latter possibility seems the most appealing.

To conclude, the analysis of experimental data makes us believe that there is another scale in the FQHE problem. A comparison with theoretical calculations on the effect of short-range potentials suggests that the scale corresponds to an energy large compared with, say, the FQHE gap, or to a length small compared with the magnetic length. The most likely mechanism to generate this scale is the three-dimensionality of the real system, which becomes evident at high energies. The low-energy consequence of this is the opening of finite FQHE gaps. At present, we are not able to propose a full theoretical treatment of the problem.

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## POSTOJI LI DODATNA SKALA U FRAKCIONALNOM KVANTNOM HALLOVOM EFEKTU?

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U nastojanju da se utvrdi postoji li u frakcionalnom kvantnom Hallovom efektu dodatna skala, pored magnetske skale određene magnetskom dužinom $a_c$ ili ciklotronskom energijom $\hbar\omega_c$, preispitani su neki postojeći teorijski rezultati i eksperimentalni podaci. Zatim se razmatraju moguće posljedice nove skale na formuliranje teorijskog modela frakcionalnog kvantnog Hallovog efekta.