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LINEAR AND NONLINEAR OPTICAL PROPERTIES OF MODULATION QUANTUM WELL

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The electron energy levels and envelope wave functions in a double confinement quantum-barrier structure, are numerically calculated utilizing the transfer matrix method. Analytic forms of the linear and the nonlinear optical absorbtion coefficients and change in refractive indices due to intersubband optical transitions with an applied electric field are derived theoretically based on the density-matrix formalism.

1. *Introduction*

Quantum confinement of carriers in a semiconductor quantum well leads to the formation of discrete energy levels and the drastic change of optical absorption spectra [1]. One of the most remarkable properties of these quasi-two dimensional electronic systems is that the optical transitions between the size-quantized subbands are feasible. Recently, the linear intersubband optical absorption within the conduction band of GaAs quantum well has been studied experimentally without an electric field [2,3] and with an electric field [4]. This suggests a nonlinearity of the intersubband optical transitions in a quantum well. These linear and nonlinear size-quantized interband transitions are interesting, both from a fundamental and a practical point of view. They have the potential for device applications in far-infrared (wavelength $\approx$ 10 μm) laser amplifiers [5,6], photodetectors [6,7] and high-speed electro-optical-modulators [8].

In this paper, we consider interband optical processes in a quantum well (Fig. 1). A 10 nm thick GaAs well is bounded by two 4 nm thick Ga_7Al_3As barriers. We examine here only the resonant two-level system in the presence of an applied electric field.

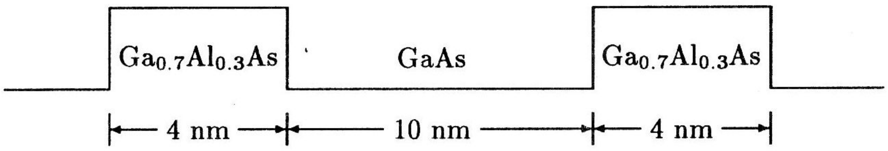

Fig. 1. Ga_7Al_3As–GaAs–Ga_7Al_3As *quantum well.*

The two levels correspond to the states at the minima of the first two subbands of the conduction band, and quasi-equilibrium conditions are assumed.

The two-level model, despite its simplicity, can be usefully applied to many complex systems. A good example is the band-gap resonant optical nonlinearity of semiconductors [9].

The formulation of linear and nonlinear intersubband optical-absorption coefficients and changes in refractive indices of the quantum well are discussed in Section 2. The density matrix and dipole operator for the system are used to derive the mathematical expressions for optical parameters. The obtained density matrix is based on the single particle formalism taking into account a phenomenological intrasubband relaxation.

In our calculations, we considered only the intrasubband transition. Also, we ignored the Coulomb interaction between an electron excited into the conduction and the hole in the valence band.

2. *Mathematical formulation of linear and nonlinear optical absorption*

The equation of one-electron density operator ϱ for a system with N subbands, including relaxation, has the form [10]:

$$\frac{\partial \varrho}{\partial t} = \frac{\mathrm{i}}{\hbar}[H_0 - M, E(t), \varrho] - \frac{1}{2}(\Gamma\varrho + \varrho\Gamma). \tag{1}$$

Here, H_0 is the Hamiltonian for the unperturbed system, M is the dipole operator for the transition, with the elements, $M_{lm} = \langle l|ex|m\rangle$ for the two states $\langle l$ and $m\rangle$

of H_0. $E(t)$ is the time dependent radiation electric field, whose polarization is assumed along the growth axis of the well. Γ is a diagonal matrix.

Equation (1) can be written as:

$$\frac{\partial \sigma}{\partial t} = \left[G + \frac{\mathrm{i}}{\hbar} E(t) M' \right] \sigma \tag{2}$$

where σ is a vector of dimension N^2

$$\sigma_{N(l-1)+m} = \varrho_{lm}, \qquad 1 \le l, m \le N. \tag{3}$$

For simplicity we assumed a single relaxation time constant for both longitudinal and transverse time constants. Hence, Γ is given by $\Gamma =$ identity matrix$\,/\,\tau$. G elements are:

$$G_{N(l-1)+i, N(m-1)+j} = \delta_{lm}\delta_{ij} \left[\frac{\mathrm{i}}{\hbar}(E_i - E_l) - \frac{1}{\tau} \right], \tag{4}$$

and M' is given by:

$$M'_{N(l-1)+i, N(m-1)+j} = \delta_{ij} M_{lm} - \delta_{ml} M_{ji}. \tag{5}$$

The elements of the initial value vector $\sigma(0)$ are all zeros except the elements given by

$$\sigma_{N(i-1)+i}(0) = 1 + \exp\left[(E_i - E_{\mathrm{F}})/KT\right] \qquad \text{where} \qquad 1 \le i \le N, \tag{6}$$

E_{F} is the Fermi level of the system.

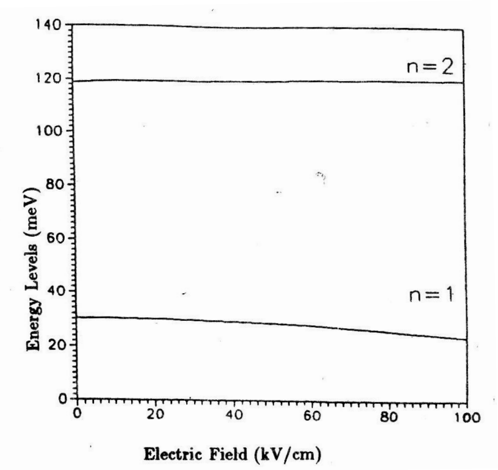

Fig. 2. The ground state ($n = 1$) and the first excited state ($n = 2$) energy levels, versus applied electric field.

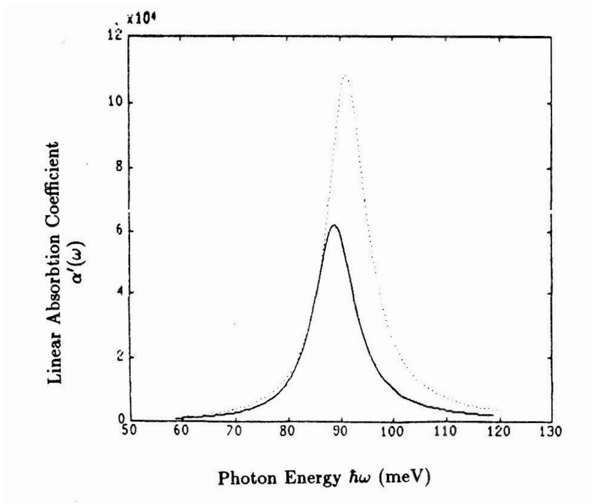

Fig. 3. Linear absorption coefficient, $\alpha'(\omega)$, versus photon energy, $\hbar w$, with $F = 0$ (solid line) and $F = 50$ kV/cm (dotted line).

The time dependence of the higher order terms $\sigma(t)$ is approximated by:

$$\sigma^{(n)}(T) = \sigma^{(n)}(\omega)e^{-\mathrm{i}\omega t} + \sigma^{(n)}(-\omega)e^{\mathrm{i}\omega t} \qquad n \text{ odd} \tag{7a}$$

$$= \sigma^{(n)}(0) \qquad n \text{ even.} \tag{7b}$$

Thus, we have neglected all harmonic higher than fundamental in the odd order terms of the expansion while retaining only the DC harmonic in the even order terms. The radiation electric field strength is assumed

$$E(t) = 2E_0\cos(\omega t) = E_0\left[e^{-\mathrm{i}\omega t} + e^{\mathrm{i}\omega t}\right]. \tag{8}$$

Using equations (7) and (8) and the equation (2), we get the following general relation for the even and odd terms:

$$\sigma^{(2)}(0) = S_0^{(n)}\sigma(0) \qquad n = 0, 1, 2, \ldots \tag{9}$$

$$\sigma^{(2n+1)}(\pm\omega) = S_\pm S_0^n(0) \qquad n = 0, 1, 2, \ldots \tag{10}$$

where

$$S_0 = -\left(\frac{E_0}{\hbar}\right)^2 G^{-1}M'\left[(G+\mathrm{i}\omega)^{-1} + (G-\mathrm{i}\omega)^{-1}\right]M' \tag{11}$$

and

$$S_\pm = -\frac{\mathrm{i}E_0}{\hbar}(G\pm\mathrm{i}\omega)^{-1}M'. \tag{12}$$

In this system, we considered:

$$E_0 = \sqrt{\frac{I}{2\varepsilon_0 n_r C}}$$

where I is the radiation field intensity, ε_0 is the permittivity in free space, n_r is the refractive index of GaAs. Thus the density vector $\sigma(t)$ is approximated by:

$$\sigma(t) = \sigma(\omega) e^{-\mathrm{i}\omega t} + \sigma(-\omega) e^{\mathrm{i}\omega t}, \tag{13}$$

where $\sigma(\omega)$ is the spectral density vector.

The nonlinear optical absorption coefficient $\alpha(\omega)$ is defined by:

$$\alpha(\omega) = \frac{\omega}{V E_0} \sqrt{\frac{\mu}{\varepsilon_r}} \mathrm{Im}(\mathrm{Tr}[\sigma(\omega) M]), \tag{14}$$

where V is the volume of the system, μ is the permittivity and ε_r is the real part of permittivity.

3. *The change of refractive index*

The linear refractive index is given by [11]:

$$n_0(\omega) = \sqrt{1 + \mathrm{Re}\,\chi^{(1)}(-\omega; \omega)}, \tag{15}$$

where $\mathrm{Re}\,\chi^{(1)}(-\omega; \omega)$ is the real part of the first order susceptibility. In terms of the effective dielectric constant $\varepsilon(\omega)$ we can write the change in refractive index δn induced by the incident field as:

$$\begin{aligned} \delta n &= \sqrt{1 + \mathrm{Re}\,\overline{\chi}(-\omega; E)} - n_0(\omega), \\ \delta n &= \sqrt{\mathrm{Re}\,\varepsilon(\omega)} - n_0(\omega), \end{aligned} \tag{16}$$

where

$$\varepsilon(\omega) = 1 + \chi^{(1)}(-\omega; \omega) + \frac{3}{4} \chi^{(3)}(-\omega; \omega, -\omega; \omega) |E|^2. \tag{17}$$

The corrected susceptibility $\overline{\chi}(\omega; E)$ can be expressed in the simple form:

$$\overline{\chi}(\omega; E) = \frac{\chi^{(1)}(-\omega; \omega)}{1 + I_\omega / I_{\mathrm{sat}}}, \tag{18}$$

where I_{sat} is the saturation intensity, and it is defined as the steady-state intensity which reduces $\overline{\chi}(\omega; E)$ to one half of its value at low intensity.

4. Results and discussion

The structure considered in our calculations is a double-confinement rectangular quantum barrier (Fig. 1). We used electron effective mass $m_w^* = 0.067m_0$ and $m_b^* = 0.092m_0$, in the well and the barriers, respectivelly. V_b, the barrier height equals to 0.34 eV, the temperature $T = 77$ K and the Fermi level $E_{\mathrm{F}} = 25.38$ meV.

The energy values for the two levels, $n = 1$ (the ground state) and $n = 2$ (the first excited state), were calculated. As an example, see Fig. 2, which represents the energy levels for different values of the applied electric field. We note that the ground state ($n = 1$) lies very low in the well and shows a large energy shift with increasing of the applied electric field. The first excited state ($n = 2$) sits higher up in the well and is less affected by the electric field. A good agreement is found between our results for these two states and those obtained by Harwit et al. [12].

To study the electric field effect on the shift and broadening of energy levels, we plot in Fig. 3. the linear absorption coefficient $\alpha^{(1)}(\omega, I = 0)$ for electric fields $F = 0$ (solid line) and $F = 50$ kV/cm (dotted line). We notice that the peaks of the absorption coefficient are shifted upward both in energy and in magnitude. Upward shifts of energy can be explained by the quantum-confined Stark effect, which was observed by Harwit and Harris [12]. We see that the increase of the magnitude of the absorption peaks is partially due to the fact that overlap integral is not changed as much as electrons in states $n = 1$ and $n = 2$ are shifted to the same side of the well by the applied electric field, (see Fig. 2), and mainly because $E_{\mathrm{F}} - E_i$ is increased for a fixed Fermi level.

The values of the total absorption coefficients, α, for incident photons of energy $\hbar\omega$, for three different values of the applied electric field, $F = 0, 50$ and 100 kV/cm,

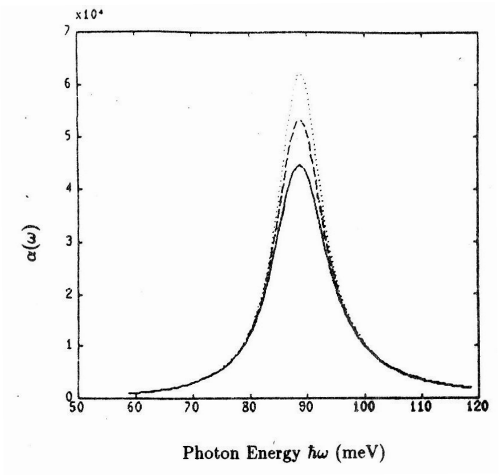

Fig. 4. Total absorption coefficient, $\alpha(\omega)$, versus photon energy, $\hbar w$, at $F = 0$. The three peaks stand for three different optical intensities: (i) $I = 0$ (dotted line), (ii) $I = 0.5$ $\mathrm{MW/cm^2}$ (dashed line) and (iii) $I = 1.0$ $\mathrm{MW/cm^2}$ (solid line).

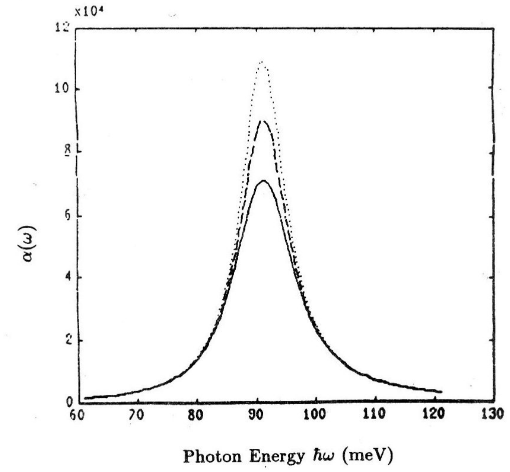

Fig. 5. Same as figure 4, except here $F = 50$ kV/cm.

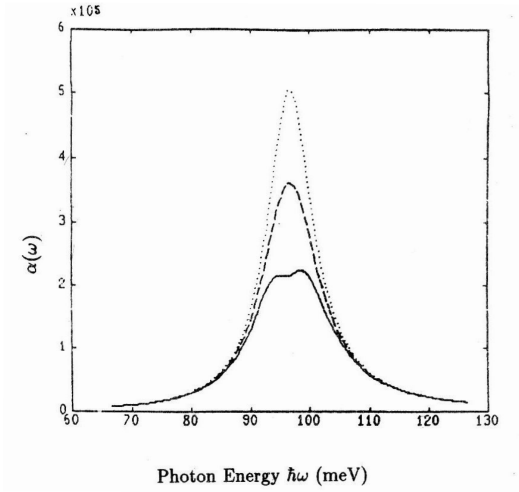

Fig. 6. Same as figure 4, except $F = 100$ kV/cm.

are shown in Figs. 4–6. The three peaks in each graph correspond to three different values of optical intensities, $I = 0, 0.55$ and 1.0 MW/cm^2.

The peaks of all the absorption spectra are mainly due to the intrasubband optical transition between the two levels in the GaAs well. In Fig. 6 the absorption coefficient reduces half its value of linear absorption (at the saturation intensity, $I = 1$ MW/cm^2) and for an electric field $F = 100$ kV/cm.

We notice that the magnitude of the peaks decreases, because of the decrease of the dipole moments with applied electric field. That is because the saturation intensity is proportional to the square of the dipole moment for the transition. A large saturation intensity is obtained with a larger applied field, with a fixed density of electrons in GaAs well.

The change of the refractive index, δn, with the incident photon energy $\hbar\omega$, for the three different values, $F = 0$, 50 and 100 kV/cm, are shown in Figs. 7, 8 and 9, respectively. Each curve corresponds to two different values of optical intensities $I = 0.5$ and 1.0 MW/cm^2.

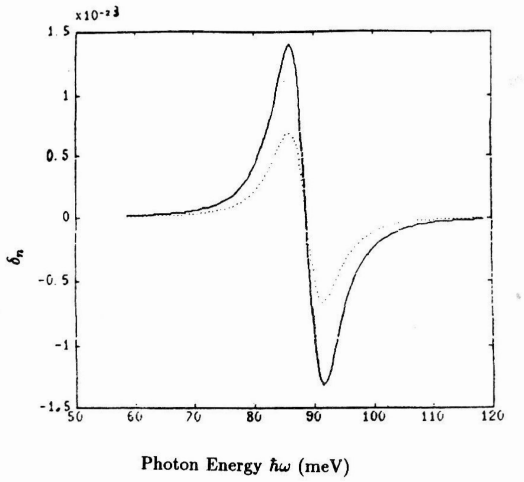

Fig. 7. The change of refractive index, δn, versus photon energy, $\hbar w$, with $F = 0$ and two different values of optical intensities: (i) $I = 0.5$ (dotted line) and (ii) $I = 1.0$ (solid line).

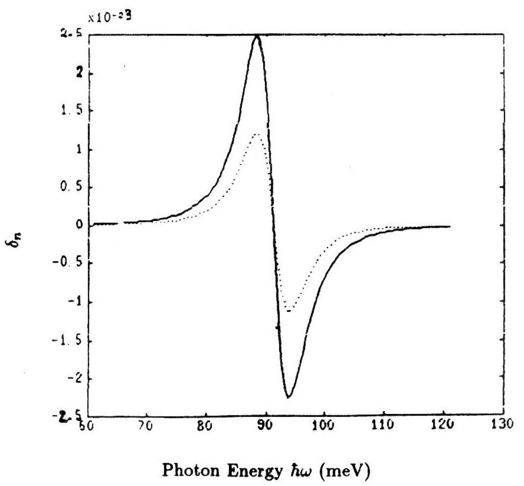

Fig. 8. Same as Fig. 7, but here $F = 50$ kV/cm.

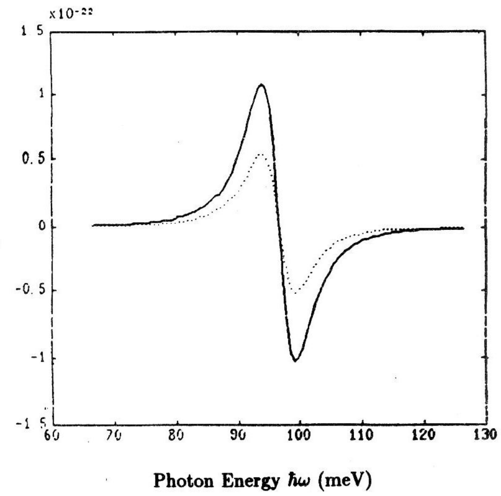

Fig. 9. Same as Fig. 7, but here $F = 100$ kV/cm.

It is clear from the last three figures, that the change of the refractive index increases as the electric field and optical intensity increase. Also, we see that the nonlinear refractive index is negative for field frequencies below resonance, and positive above resonance.

On the other hand, for the negative part of the refractive index change, we see that the amplitude of change in refractive indices decreases with the increase of I, (optical intensities), which is in agreement with equation (18), which represents the reduction of χ with the value of $I_{\rm sat}$.

5. *Conclusion*

We have obtained the electron energy levels and envelope wave functions, by numerical calculation utilizing the transfer matrix method. Our system has two quasi-eigenstates in the quantum well. The ground state lies very low and the first excited state is higher up in the well.

We have studied analytically the first and third-order nonlinear absorption coefficient, for two-level quantum system. It is shown that the magnitude of the peak absorption coefficient is reduced to one when the electric field equals to 100 kV/cm, and the intensity of radiation reaches 1.0 MW/cm^2.

Also, the change of the refractive index has been calculated for different values of the uniform electric field and for different radiation intensities. The results presented here are expected to be of use for high speed intersubband infrared photodetectors and electro-optical modulators.

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LINEARNA I NELINEARNA OPTIČKA SVOJSTVA MODULIRANE KVANTNE JAME

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Za konačno duboku kvantnu jamu određeni su numerički energetski nivoi elektrona i valne funkcije primjenom metode matrice prijelaza. Nađen je analitički oblik za koeficijente linearne i nelinearne optičke apsorpcije te za promjenu indeksa loma zbog optičkih prijelaza među podvrpcama uzrokovanih vanjskim električnim poljem. Račun je napravljen pomoću formalizma matrice gustoće.