Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

SQUEEZING PROPERTIES OF THE q-DEFORMED JAYNES-CUMMINGS MODEL

JOSIP ČRNUGELJ, MLADEN MARTINIS and VESNA MIKUTA-MARTINIS

Ruđer Bošković Institute, POB 1016, 41000 Zagreb, Croatia

Received 19 November 1993

UDC 539.186

Original scientific paper

We study the squeezing properties of the Jaynes-Cummings model with the q--intensity-dependent coupling interacting with initially q-deformed coherent radiation. We observe strong squeezing for small positive and small negative q values. If $q \approx 1$ or > 1, squeezing is very weak.

1. *Introduction*

The Jaynes-Cummings model (JCM) [1] of a single two-level atom interacting with a single mode of the quantized radiation field in the dipole and rotating-wave approximations has been the subject of many recent investigations [2] in laser physics and quantum optics. It has been observed [3] that the long-time behaviour of the model is very sensitive to the statistical properties of the radiation field in the initial state, revealing a number of unexpected pure quantum features of the model [2-4]. For example, the mean photon number exhibits quasi-periodic collapses and revivals. The dynamics predicted by the model has been supported in experiments with Rydberg atoms in high-Q microwave cavities [5].

Recently, there have been several generalizations of the JC Hamiltonian in which the interaction between the atom and the radiation field is no longer linear in the field variables. These are the JCM with the intensity-dependent coupling [6-9] and the multiphoton generalization of the JCM [10-12].

The squeezing properties of the JCM have been investigated by many authors [8,10,11,13,14], in particular after it has been possible to produce a squeezed electromagnetic field in the laboratory [15]. States containing a large amount of squeezing can be obtained from the JCM with the intensity-dependent coupling and with the multiphoton-transition mechanism. Theoretically, it has been recognized that squeezing states can be constructed as generalized coherent states using group-theoretical methods [16]. One of such generalizations is the conjecture [17] that quantum-group coherent states [18,19] are natural candidates for describing squeezing states of matter.

The aim of the present paper is to investigate the squeezing properties of the JCM with the intensity-dependent coupling characterized by an additional parameter q. It is equivalent to the JCM in which the creation and annihilation operators of the radiation field are replaced by q-deformed-oscillator operators [19]. The case $q = 1$ corresponds to the standard JCM.

2. *The q-deformed JCM*

The Hamiltonian of the deformed JCM (DJCM) is of the form

$$H = \omega_0\sigma_3 + \omega N_A + g(A^\dagger\sigma_- + A\sigma_+), \tag{1}$$

where σ_3, $\sigma_\pm$ are the standard pseudospin atomic two-level transitions operators ($[\sigma_+, \sigma_-] = 2\sigma_3\,,\ [\sigma_3, \sigma_\pm] = \pm\sigma_\pm$). The operators A and $A^\dagger$ are constructed from the single-mode field operators a, $a^\dagger$ and N ($[a, a^\dagger] = 1$, $[a, N] = a$, $N = a^\dagger a$). In the case of intensity-dependent coupling, the operators are

$$\begin{aligned} &A = af(N) = f(N+1)a, \\ &A^\dagger = f(N)a^\dagger = a^\dagger f(N+1), \\ &N_A = N, \end{aligned} \tag{2}$$

where, generally, $f(N)$ is an arbitrary real function of N. These operators satisfy the deformed-oscillator commutation relations:

$$\begin{aligned} &[A, A^\dagger] = [N+1] - [N], \\ &[A, N] = A, \\ &[A^\dagger, N] = -A^\dagger, \end{aligned} \tag{3}$$

where

$$[N] = Nf^2(N).$$

The q-deformed JCM, $(\mathrm{DJCM})_q$, is obtained when we choose

$$[N] = \frac{1-q^N}{1-q}\,. \tag{4}$$

There are also other possibilities [9]. The operators A and $A^\dagger$ then satisfy the following q-deformed commutation relations:

$$AA^\dagger - qA^\dagger A = 1, \qquad q \in \mathbf{R},$$

or

$$AA^\dagger - A^\dagger A = q^N. \tag{5}$$

The time-evolution operator $U(t) = e^{-iHt}$ for this case was found in Ref. 19, where the time behaviour of the mean photon number $\overline{n}(t) \equiv \langle \psi(t)|a^\dagger a|\psi(t)\rangle$ and the population inversion $\langle \sigma_3(t)\rangle \equiv \langle \psi(t)|\sigma_3|\psi(t)\rangle$ were investigated for different values of the deformation parameter q.

3. Squeezing

The squeezing properties of the radiation field are usually studied by introducing two Hermitian time-dependent quadrature operators:

$$\begin{aligned} a_1 &= \left(ae^{i\omega t} + a^\dagger e^{-i\omega t}\right)/2, \\ a_2 &= \left(ae^{i\omega t} - a^\dagger e^{-i\omega t}\right)/2i, \end{aligned} \tag{6}$$

satisfying $[a_1, a_2] = i/2$. The corresponding uncertainty relation is $(\Delta a_1)(\Delta a_2) \geq$ $\geq 1/4$, where variances $(\Delta a_{1,2})$ are defined by $(\Delta a_{1,2})^2 = \langle a_{1,2}^2\rangle - \langle a_{1,2}\rangle^2$. A state of the field is considered squeezing if either (Δa_1) or (Δa_2) are smaller than 1/2.

Let us define the relative variances with respect to $(\Delta a_{1,2})_{\mathrm{coh}} = 1/4$:

$$S_{1,2}(t) = 4(\Delta a_{1,2})^2 - 1. \tag{7}$$

The squeezing condition becomes

$$S_i(t) < 0, \qquad i = 1 \quad \text{or} \quad 2. \tag{8}$$

In terms of the photon operators we have

$$S_1(t) = 2\left\langle a^\dagger a\right\rangle + \left\langle a^2\right\rangle e^{2i\omega t} + \left\langle a^{\dagger 2}\right\rangle e^{-2i\omega t} - \left(\langle a\rangle e^{i\omega t} + \left\langle a^\dagger\right\rangle e^{-i\omega t}\right)^2, \tag{9}$$

and similarly for $S_2(t)$.

The matrix elements of the operators a, a^2, $a^\dagger a$, ... are calculated using the wave function $|\psi(t)\rangle = U(t)|\psi(0)\rangle$ with a given initial state

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} Q_n \,|n\rangle\, |\psi_{\mathrm{atom}}(0)\rangle\,, \tag{10}$$

where $|\psi_{\mathrm{atom}}(0)\rangle$ denotes the atom in the ground state $|-\rangle$ $(\sigma_-|-\rangle = 0)$ and $|n\rangle$ denotes the normalized Fock states of the operators A, $A^\dagger$ [19]:

$$|n\rangle = \frac{(A^\dagger)^n}{\sqrt{[n]!}}\,|0\rangle\,, \qquad A\,|0\rangle = 0, \tag{11}$$

with $[n]! \equiv [n][n-1]\dots[1]$ and $[0]! = 1$. The initial distribution of A quanta is assumed to be a deformed Poisson distribution [19]:

$$|Q_n|^2 = |\langle n|\alpha\rangle|^2 = e_A\left(|\alpha|^2\right)^{-1} \frac{|\alpha|^{2n}}{[n]!}, \tag{12}$$

where $\alpha = |\alpha|e^{i\phi}$ and $e_A(x)$ is the deformed exponential function:

$$e_A(x) = \sum_{n=0}^{\infty} \frac{x^n}{[n]!}. \tag{13}$$

Note that the mean number of A-quanta at $t = 0$ is

$$\overline{n} = |\alpha|^2 \frac{e'_A\left(|\alpha|^2\right)}{e_A\left(|\alpha|^2\right)}. \tag{14}$$

Introducing the notation $\langle a\rangle = e^{-i(\omega t-\phi)}A_1(t)$, $\langle a^2\rangle = e^{-2i(\omega t-\phi)}A_2(t)$ and $\langle a^\dagger a\rangle =$ $= A_0(t)$, we obtain

$$\begin{aligned} S_1(t) &= 2[A_0(t) - A_2(t)] + 4\cos^2\phi[A_2(t) - A_1^2(t)], \\ S_2(t) &= 2[A_0(t) - A_2(t)] + 4\sin^2\phi[A_2(t) - A_1^2(t)], \end{aligned} \tag{15}$$

where in the resonant case $\omega_0 = \omega$:

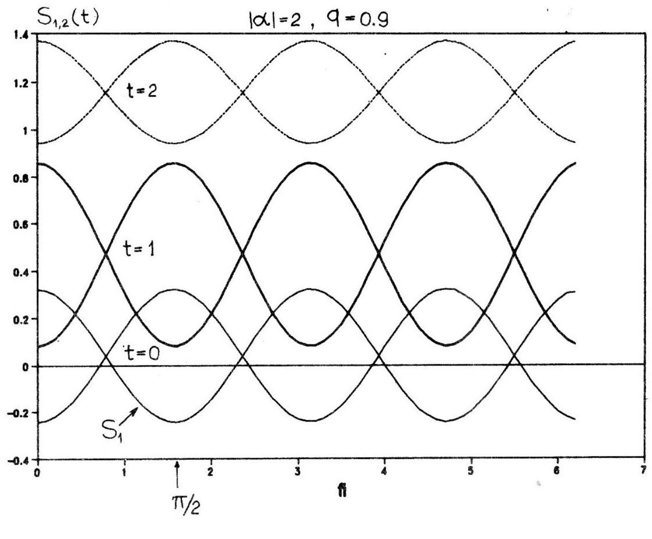

Fig. 1. Behaviour of $S_1(t)$ and $S_2(t)$ as functions of the phase ϕ of α for $|\alpha| = 2$, $q = 0.9$ at $t = 0, 1$ and 2.

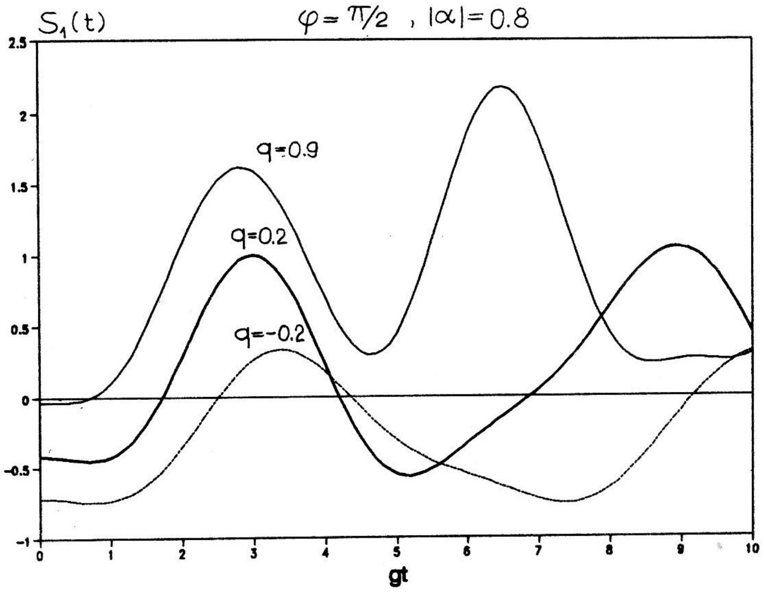

Fig. 2. Time evolution of $S_1(t)$ for $|\alpha| = 0.8$ and $\phi = \pi/2$. The various curves correspond to $q = -0.2$, 0.2 and 0.9.

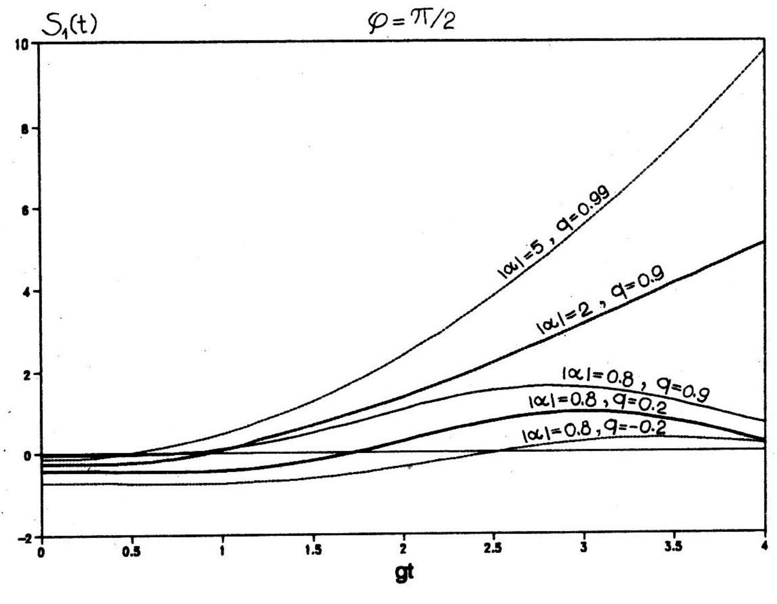

Fig. 3. Time evolution of $S_1(t)$ for $\phi = \pi/2$ and different $|\alpha|$ and q values.

$$
\begin{aligned}
A_0(t) &= \overline{n}(t) \\
&= \overline{n} - \sum_{n=0}^{\infty} |Q_n|^2 \sin^2\left(gt\sqrt{[n]}\right), \\
A_1(t) &= |\alpha| \sum_{n=0}^{\infty} \frac{|Q_n|^2}{\sqrt{[n+1]}} \left[\sqrt{n}\sin\left(gt\sqrt{[n]}\right)\sin\left(gt\sqrt{[n+1]}\right)\right. \\
&\quad \left. + \sqrt{n+1}\cos\left(gt\sqrt{[n]}\right)\cos\left(gt\sqrt{[n+1]}\right)\right], \\
A_2(t) &= |\alpha|^2 \sum_{n=0}^{\infty} \frac{|Q_n|^2}{\sqrt{[n+1][n+2]}} \left[\sqrt{n(n+1)}\sin\left(gt\sqrt{[n]}\right)\sin\left(gt\sqrt{[n+2]}\right)\right. \\
&\quad \left. + \sqrt{(n+1)(n+2)}\cos\left(gt\sqrt{[n]}\right)\cos\left(gt\sqrt{[n+2]}\right)\right].
\end{aligned}
\tag{16}
$$

The dependence of the functions $S_{1,2}$ $(t = 0, 1, 2)$ on the phase ϕ of α is shown in Fig. 1. We observe that the maximum squeezing for $S_1(t)$ can be obtained for $\phi = \pi/2$. The time evolution of the function $S_1(t)$ for $\phi = \pi/2$, $|\alpha| = 0.8$ and different values of the deformation parameter q is shown in Fig. 2. We see that the initial squeezing disappears quickly for $q \gtrsim 1$, whereas it oscillates for q small or negative. In Fig. 3 we show the time behaviour of $S_1(t)$ for $|\alpha| = 2$ and 5, $\phi = \pi/2$ and $q \approx 1$. We conclude that squeezing is rather weak if $q \gtrsim 1$. For small positive and negative q values, large magnitudes of squeezing are obtained. We expect that the long-time behaviour of $S_1(t)$ will be characterized by recoveries of squeezing at least for small q values.

Acknowledgements

This research was supported by the Ministry of Science of Croatia under the contract 1-03-212.

References

1) E. T. Jaynes and F. W. Cummings, Proc. IEEE **51** (1963) 89;

2) S. Stenholm, Phys. Rep. **6** (1973) 1;
H. I. Yoo and J. H. Eberly, Phys. Rep. **118** (1985) 239;
D. Meschede, Phys. Rep. **211** (1992) 201;

3) J. H. Eberly, N. B. Narozhny and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44** (1980) 1323;
N. B. Narozhny, J. J. Sanchez and J. H. Eberly, Phys. Rev. **A23** (1981) 236;

4) P. Meystre, E. Geneux, A. Quattropani and A. Faust, Nuovo Cim. **B25** (1975) 521;

5) P. Goy, P. M. Raimond, M. Gross and H. Haroche, Phys. Rev. Lett. **50** (1983) 1903;
G. Gabrielse and H. Dehmelt, Phys. Rev. Lett. **55** (1985) 67;
D. Meschede, H. Walther and G. Muller, Phys. Rev. Lett. **54** (1985) 551;
G. Rempe, H. Walther and N. Klein, Phys. Rev. Lett. **58** (1987) 353;
G. Rempe, F. Schmidt-Kaler and H. Walther, Phys. Rev. Lett. **64** (1990) 2783;

6) B. Buck and C. V. Sukumar, Phys. Lett. **81A** (1981) 132;
J. Phys. **A17** (1984) 885;

7) V. Bužek, Phys. Rev. **A39** (1989) 3196;

8) M. Chaichian, D. Ellinas and P. Kulish, Phys. Rev. Lett. **65** (1990) 980;

9) G. S. Agarwal, J. Opt. Soc. Am. **B2** (1985) 480;
C. C. Gerry, Phys. Rev. **A37** (1988) 2683;

10) C. V. Sukumar and B. Buck, Phys. Lett. **83A** (1981) 211
A. S. Shumovsky, Fam Le Kien and E. I. Aliskenderov, Phys. Lett. **A142** (1987) 351;

11) S. Singh, Phys. Rev. **A25** 3206;

12) P. Meystre and M. S. Zubairy, Phys. Lett. **89A** (1982) 390;
C. C. Gerry, Phys. Rev. **A37** (1988) 2683;
J. R. Kukeinski, and J. L. Madajczyk, Phys. Rev. **A37** (1988) 3175;

13) Ho Trung Dung, Fam Le Kien, E. I. Aliskenderov and A. S. Shumovsky, Phys. Lett. **136A** (1989) 489;

14) R. E. Shuster, L. W. Hollberg, B. Yurke, J. C. Mertx and J. F. Valley, Phys. Rev. Lett. **55** (1985) 2409;
R. M. Shelby, M. D. Levenson, S. H. Perlmuter, R. G. Devoe and D. F. Walls, Phys. Rev. Lett. **57** (1986) 691;
L. Wu, H. J. Kimble, J. L. Hall and H. Wu, Phys. Rev. Lett. **57** (1986) 2520;

15) J. Katriel, A. I. Solomon, G. D'Ariano and M. Rasetti, Phys. Rev. **D34** (1986) 2332;

16) E. Celeghini, M. Rasetti and G. Vitiello, Phys. Rev. Lett. **66** (1991) 2056;

17) M. Jimbo, Lett. Math. Phys. **10** (1985) 63; **11** (1986) 247;

18) L. C. Biedenharn, J. Phys. **A22** (1989) L873;
A. J. MacFarlane, J. Phys. **A22** (1989) 4581;

19) J. Črnugelj, M. Martinis and V. Mikuta-Martinis, *JCM and the deformed-oscillator algebra*, RBI-TH-7/93, submitted to Phys. Lett. A.

SVOJSTVA STJEŠNJAVANJA q-DEFORMIRANOG JAYNES-CUMMINGOVOG MODELA

JOSIP ČRNUGELJ, MLADEN MARTINIS i VESNA MIKUTA-MARTINIS

Institut "Ruđer Bošković", 41000 Zagreb

UDK 539.186

Originalni znanstveni rad

Proučavana su svojstva stješnjavanja Jaynes-Cummingovog modela s vezanjem ovisnim o intenzitetu i parametru q, u međudjelovanju s početnom q-deformiranom koherentom radijacijom. Opaženo je jako stješnjavanje za male pozitivne i male negativne vrijednosti q. Ako je $q \approx 1$ ili > 1, stješnjavanje je vrlo slabo.