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EVIDENCE OF MONOFRACTAL PROTON EMISSION IN ^{24}Mg – AgBr INTERACTIONS AT 4.5 A GeV

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This paper presents a fractal analysis of emission of medium-energy target protons from ^{24}Mg – AgBr interactions at 4.5 A GeV, in terms of their emission angle (θ) and azimuthal angle (ϕ), following the method proposed by F. Takagi. The generalised dimensions D_q have been calculated for $q = 2$, 3 and 4. The values are almost independent of the order for both phase spaces, suggesting mono-fractality of proton emission in heavy ion interactions.

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1. *Introduction*

The study of fractal nature of the multi-particle production process has recently attracted much attention. The word fractal was first coined by Mandelbrot [1] who discovered that there is a fractal face to the geometry of nature. He opened a new window, namely fractal geometry, for looking into the world of apparent irregularities. Scale invariance plays a fundamental role in many natural phenomena, and is often related to the appearance of irregular forms which are non describable in terms of usual differential geometry. Fractal geometry allows one to mathematically describe systems that are intrinsically irregular at all scales. A fractal structure has the property that if one magnifies a small portion of it, this shows the same complexity as the entire system. Usually the term fractal is used to characterize systems with properties of self-similarity in general. If these properties can be described by a single exponent, one has a simple or homogeneous fractal. In a more complex case, the term multi-fractality is used when discussing generalised scaling. Approaches based on the concept of fractal studies may be related to phase transitions, self-similar cascades, chaos, entropy etc. The importance of the subject has inspired different parallel approaches [2] for extracting fractal behaviour of multi-particle production process, most of which present an inherent drawback in the sense that experimental data sets do not show the linearity in a log-log plot of moment against the bin-size, as expected from the mathematical formulations. This might be due to the fact that the assumed mathematical limit (number of points tend to infinity) is not valid for the real experimental data where the number of particles in each event is always finite. To eliminate such deficiencies in experimental data, Takagi has recently introduced a new method [3] in which he proposed a simple measure to probe the fractal structure of the multi-particle production process and has applied this method successfully to electron – positron anihilation [4,5] and UA5 data on proton – antiproton collisions [6] to extract fractal characteristics.

So far most of attempts made to study the fractal nature in the multi-particle production process involve relativistically produced pions [7], but an in-depth study of the medium-energy (30 – 400 MeV) knocked-out target protons, termed as grey tracks according to emulsion terminology, may also lead to significant conclusions. The general belief about these protons is that they are the low energy part of the internuclear cascade formed in high-energy interactions. They have their own importance as they are supposed to carry relevant information about the hadronisation mechanism, since the time scale of emission of these particles is of the same order as that of the produced particles. In spite of such potential, only some elementary work has been made on these particles, and even the emission process of these protons has not yet been fully explained theoretically. With the help of the sophisticated tools in hand, those protons should be thoroughly investigated for proper understanding of the multi-particle production process.

The objective of this work is to study the fractal behaviour of the medium-energy protons, emitted in ^{24}Mg – AgBr interactions at 4.5 $A\,$GeV, in terms of emission angle (θ) and azimuthal angle (ϕ) (with respect to the beam direction), following the method introduced by Takagi [3].

The data set used in our analysis was obtained by irradiating stacks of NIKFI BR2 nuclear emulsion plates by a ^{24}Mg beam of incident energy 4.5 $A\,$GeV at JINR Dubna. More information about our data can be found from our earlier work [8]. The scanning of the plates was carried out with the help of a high resolution Leitz Metalloplan microscope provided with an online computer system. The scanning was done using objective 10× in conjunction with 25× ocular lenses.

The events were chosen using the following criteria:

a) The beam track should not be at an angle greater than 3° to the mean beam direction in the pellicle.

b) The interaction should not be within 20 μm from the top or bottom surface of the pellicle.

c) All the primary beam tracks are followed back to ensure that the events chosen do not include interactions from the secondary tracks of other interactions.

When they are observed to do so, the corresponding events are removed from the sample.

Following the standard emulsion technique, we identified the shower tracks as the tracks for which ionization is less than $1.4\,I_0$ (I_0 is the minimum ionization). They are mainly due to the produced pions. The target fragments with ionization greater than $1.4\,I_0$ produce either black or grey tracks. The black tracks with a range less than 3 mm represent target evaporation particles of energy less than 30 MeV, singly or multiply charged. The grey tracks with a range ≥ 3 mm are mainly images of fast target protons of the energy range up to 400 MeV. To ensure that all the grey tracks are most likely due to protons, we have followed the tracks up to their end-points, and none of them show signals of decay or interaction. In order to avoid contamination with projectile fragments, we exclude the particles emitted within a forward cone of about 5°. The grey tracks are identified for each event, and their emission angle θ and azimuthal angle ϕ (with respect to the beam direction) are determined by measuring space coordinates of the interaction centre and points on the incident and secondary tracks using $100\times$ oil immersion objective.

2. *Takagi's methodology*

In the multi-particle production process at a certain incident energy, the particle distribution is considered in a phase space x. Consider a single event containing n hadrons distributed in the interval $x_{\min} < x < x_{\max}$. The multiplicity n changes from event to event according to the distribution $P_n(\Delta x)$ where $\Delta x = x_{\max} - x_{\min}$. The chosen phase space interval of length Δx is divided into m bins of equal size $\delta x = \Delta x/m$. The multiciplity distribution for a particular bin is denoted as $P_n(\delta x)$ for $n = 0, 1, 2, 3, \ldots$ where it can be assumed that the inclusive particle distribution $\mathrm{d}n/\mathrm{d}x$ is constant and $P_n(\delta x)$ is independent of the location of the bin. The hadrons produced in Ω independent events are distributed in Ωm bins of size δx. Let N be the total number of hadrons produced in these Ω events and n_{aj} be the multiplicity of the hadrons in the j-th bin of the a-th event. The theory has been motivated [9] to consider the normalized density P_{aj} defined by

$$P_{aj} \quad = \quad \frac{n_{aj}}{N}, \tag{1}$$

and to consider the quantity

$$T_q(\delta x) \quad = \quad \ln \sum_{a=1}^{\Omega} \sum_{j=1}^{m} P_{aj}^q, \quad \text{for} \quad q > 0, \tag{2}$$

which behaves like a linear function of the logarithm of the 'resolution' $R(\delta x)$ (q is the order number)

$$T_q(\delta x) \quad = \quad A_q + B_q \ln\big(R(\delta x)\big). \tag{3}$$

Here A_q and B_q are constants independent of δx. If such a behaviour is observed for a considerable range of $R(\delta x)$, a generalized dimension may be determined as

$$D_q = \frac{B_q}{q-1}. \tag{4}$$

Here the case with $q = 1$ is obtained by taking an appropriate limit [9].

Evaluating the double sum of P_{aj}^q, Takagi showed that for sufficiently large Ω

$$\sum_{a=1}^{\Omega}\sum_{j=1}^{m} P_{aj}^q = \sum_{n=0}^{\alpha} \Omega m P_n(\delta x)(n/N)^q = \frac{\langle n^q \rangle}{N^{q-1}\langle n \rangle}\,, \tag{5}$$

where $\langle n^q \rangle = \sum_{n=0}^{\alpha} n^q P_n(\delta x)$ and $\langle n \rangle = \sum_{n=0}^{\alpha} n P_n(\delta x)$. The relation $\Omega m = N/\langle n \rangle$ has been used. Since

$$\langle n \rangle = \frac{N}{\Omega \Delta x}\delta x, \tag{6}$$

T_q may be expressed as

$$\begin{aligned} T_q(\delta x) &= \ln\left(\frac{\langle n^q \rangle}{N^{q-1}\dfrac{N\delta x}{\Omega \Delta x}}\right) \\ &= \ln\langle n^q \rangle - \ln(\delta x) + const. \end{aligned} \tag{7}$$

For the simplest choice of 'resolution', Eq. (2) becomes

$$T_q(\delta x) = A_q + B_q \ln(\delta x). \tag{8}$$

Then comparing Eqs. (4) and (7), Takagi obtained the relation

$$\begin{aligned} \ln\langle n^q \rangle &= A_q + \big(B_q + 1\big)\ln(\delta x) \\ &= A_q + \big((q-1)D_q + 1\big)\ln(\delta x). \end{aligned} \tag{9}$$

The plot of $\ln\langle n \rangle$ against δx was observed to saturate for large x region [3,10] when real data were analysed [4–6]. The deviation may be due to the non-flat behaviour of $\mathrm{d}n/\mathrm{d}x$ in the large x region. Takagi then introduced $\langle n \rangle$ as a better choice of the 'resolution' R, because $\mathrm{d}n/\mathrm{d}\langle n \rangle$ is flat by definition [3,10–12]. Choosing $R(\delta x) = \langle n \rangle$, one has

$$\ln\langle n^q \rangle = A_q + \big((q-1)D_q + 1\big)\ln\langle n \rangle, \tag{10}$$

a simple linear relation between $\ln\langle n^q \rangle$ and $\ln\langle n \rangle$. The generalized dimension D_q can be obtained from the slope in the $\ln\langle n^q \rangle$ – and $\ln\langle n \rangle$ diagram. If D_q decreases with increasing order, we can conclude that the fractal study of the concerned particle shows multi-fractality. But if D_q remains the same with increasing order, mono-fractality is the conclusion.

3. Results and discussion

In the present case, we have considered that particles are distributed in the total emission angle (θ) space. The phase-space interval is divided into overlapping bins whose size is increased symmetrically in steps of 0.125, taking the central value at zero. $\ln\langle n^q\rangle$ are calculated and plotted against $\ln\langle n\rangle$ for $q = 2$, 3 and 4. As expected, a nice linear behaviour is obtained in the above log-log plot, as shown in Fig. 1. We found no problem to calculate the slopes from the best linear fits. The generalized dimensions D_2, D_3 and D_4 were calculated using Eqs. (4) and (10) and are presented in Table 1.

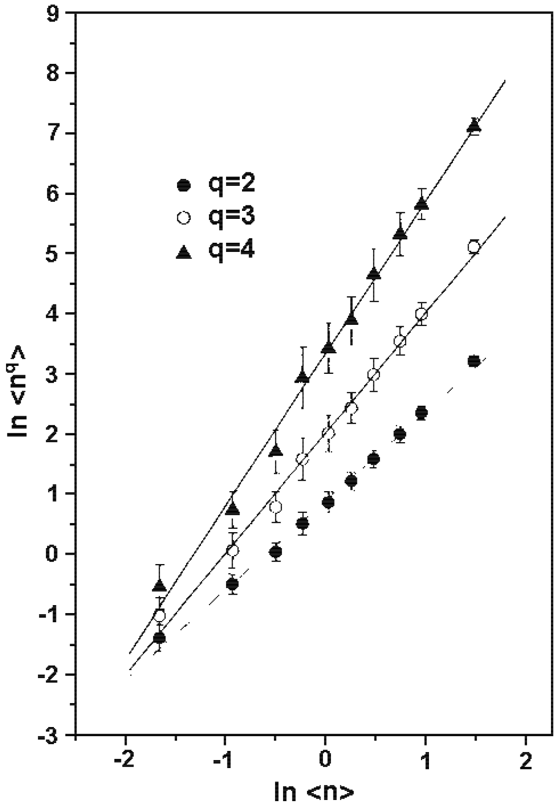

Fig. 1. Plot of $\ln\langle n^q\rangle$ *against* $\ln\langle n\rangle$ *for* $q = 2, 3, 4$ *for* $^{24}\mathrm{Mg} - \mathrm{AgBr}$ *interactions at 4.5* $A\,\mathrm{GeV}$ *in* $\cos\theta$ *space*

TABLE 1. The values of generalized dimensions D_q *(*$q = 2, 3, 4$*) for phase space variable* $\cos\theta$*.*

D_2	D_3	D_4
0.53 ± 0.03	0.55 ± 0.02	0.55 ± 0.03

We have followed an identical procedure for analysing fractal structure in the azimuthal angle space. The azimuthal angle interval was divided into overlapping bins whose size was increased symmetrically in steps of 18°. The central value was 180°. We have calculated $\ln\langle n^q\rangle$ and plotted against $\ln\langle n\rangle$, as shown in Fig. 2. The generalized dimensions D_2, D_3 and D_4 were calculated from these slopes using Eqs. (4) and (10). The values of D_2, D_3 and D_4 for each step size are given in Table 2. For both phase-space variables $\cos\theta$ and azimuthal angle ϕ, it is observed that for each step size, D_q remains almost the same for different orders. Overall discussion

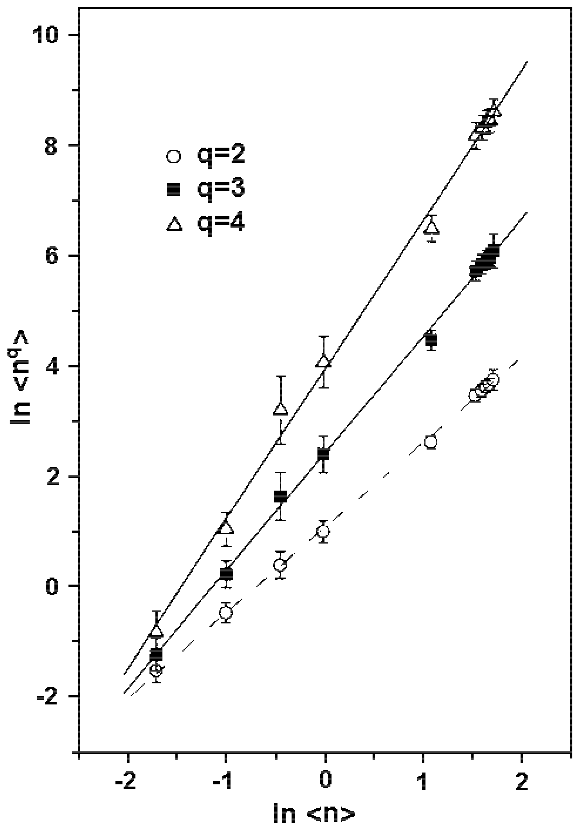

Fig. 2. Plot of $\ln\langle n^q\rangle$ *against* $\ln\langle n\rangle$ *for* $q = 2, 3, 4$ *for* ^{24}Mg – AgBr *interactions at 4.5 A* GeV *for* ϕ.

TABLE 2. The values of generalized dimensions D_q *(*$q = 2, 3, 4$*) in azimuthal angle space.*

D_2	D_3	D_4
0.56 ± 0.01	0.57 ± 0.01	0.57 ± 0.02

of our result can be summed up as follows:

1) For both $\cos\theta$ and ϕ, we get excellent linear behaviour when $\ln\langle n^q\rangle$ values are plotted against $\ln\langle n\rangle$ for $q = 2$, 3 and 4.

2) The generalized dimension does not depend on the order of the moment for both phase spaces, revealing first ever evidence of mono-fractality of proton emission in ^{24}Mg – AgBr interactionat 4.5 A GeV.

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POKAZATELJI MONO-FRAKTALNOSTI EMISIJE PROTONA U SUDARIMA ^{24}Mg – AgBr NA 4.5 A GeV

Predstavljamo fraktalnu analizu emisije protona srednjih energija iz mete u sudarima ^{24}Mg – AgBr na 4.5 A GeV, prema kutu njihove emisije (θ) i azimutalnom kutu (ϕ), slijedeći metodu F. Takagija. Izračunali smo poopćene dimenzije D_q za $q = 2$, 3 i 4. Te su vrijednosti gotovo neovisne o redu za oba fazna prostora, što ukazuje na mogućnost mono-fraktalne emisije protona u sudarima teških iona.