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ON HADRON PRODUCTION FROM THE QUARK–GLUON PLASMA PHASE IN ULTRA-RELATIVISTIC HEAVY-ION COLLISIONS

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We describe the quark gluon plasma (QGP) as a thermalized quark–gluon system, the thermalized QGP phase of QCD. The hadronization of the thermalized QGP phase is given in a way resembling a coalescence model with correlated quarks and antiquarks. The input parameters of the approach are the spatial volumes of the hadronization. We introduce three dimensionless parameters, C_{M}, C_{B} and $C_{\bar{\mathrm{B}}}$, related to the spatial volumes of the production of low-lying mesons (M), baryons (B) and antibaryons ($\bar{\mathrm{B}}$). We show that at the temperature $T = 175\,\mathrm{MeV}$ our predictions for the ratios of multiplicities agree well with the presently available set of hadron ratios measured for various experiments given by the NA44, NA49, NA50 and WA97 Collaborations on Pb+Pb collisions at 158 GeV/nucleon, the NA35 Collaboration on S+S collisions and the NA38 Collaboration on O+U and S+U collisions at 200 GeV/nucleon.

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1. Introduction

Recently [1], we have suggested a kind of coalescence model [2] for the description of the hadronization from the quark–gluon plasma (QGP) phase of QCD. We have considered the QGP as a thermalized quark–gluon system [3] at high densities and temperature in which quarks, antiquarks and gluons, being at the deconfined phase, collide frequently each other. There is the belief [4] that the QGP phase can be realized in ultra-relativistic heavy-ion collision ($E_{\rm cms}$/nucleon $\gg 1\,$GeV) experiments.

At these energies, colliding heavy-ions form a system of highly relativistic and very dense quarks and gluons. Due to the asymptotic freedom, the particles are almost free. The high density leads to frequent collisions and an equilibrium state. Considering such a state as a thermalized QGP, the probabilities of light massless quarks $n_{\rm q}(\vec{p}\,)$ and light massless antiquarks $n_{\bar{\rm q}}(\vec{p}\,)$, with q = u or d and a momentum p, can be described by the Fermi–Dirac distribution functions [3,5]

$$n_{\rm q}(\vec{p}\,) = \frac{1}{{\rm e}^{-\nu(T) + p/T} + 1}, \qquad n_{\bar{\rm q}}(\vec{p}\,) = \frac{1}{{\rm e}^{\nu(T) + p/T} + 1}, \tag{1}$$

where the temperature T is measured in MeV, $\nu(T) = \mu(T)/T$ and $\mu(T)$ is the chemical potential of the light massless quarks q = u, d, that depends on the temperature T [5]. Since the chemical potential of light antiquarks is $-\mu(T)$, a positive value of $\mu(T)$ describes an abundance of light quarks with respect to light antiquarks in the thermalized state [1,4]. The chemical potential $\mu(T)$ is a phenomenological parameter and will be determined below [1].

The gluon momenta are assumed to follow the Bose–Einstein distribution function

$$n_{\rm g}(\vec{p}\,) \quad = \quad \frac{1}{{\rm e}^{p/T} - 1}. \tag{2}$$

Since strangeness of the colliding heavy-ions is zero, the densities of strange quarks and antiquarks should be equal. This implies zero-values of the chemical potentials, $\mu_{\rm s} = \mu_{\bar{\rm s}} = 0$. In this case, the probabilities of strange quarks and antiquarks can be described by

$$n_{\rm s}(\vec{p}\,) = n_{\bar{\rm s}}(\vec{p}\,) = \frac{1}{{\rm e}^{\sqrt{\vec{p}^{\,2} + m_{\rm s}^2}/T} + 1}, \tag{3}$$

where $m_{\rm s} = 135\,$MeV [6] is the mass of strange quark. The value of the current s-quark mass, $m_{\rm s} = 135\,$MeV, has been successfully used in the calculations of chiral corrections to the amplitudes of low-energy interactions, form factors and mass spectra of low-lying hadrons [7] and charmed heavy–light mesons [8]. Unlike the massless antiquarks $\bar{\rm u}$ and $\bar{\rm d}$, for which the suppression is caused by the chemical potential $\mu(T)$, the strange quarks and antiquarks are suppressed by virtue of the non-zero mass $m_{\rm s}$.

In Ref. [1], we have supposed that the chemical potential $\mu(T)$, a phenomenological parameter for the description of the QGP as a thermalized quark–gluon system at temperature T, is an intrinsic characteristic of a thermalized quark–gluon system. Thereby, if the QGP is an excited state of the QCD vacuum, a chemical potential should exist not only in ultra-relativistic heavy-ion collisions. Quark distribution functions of a thermalized quark–gluon system at temperature T should be characterized by a chemical potential $\mu(T)$ for any external state and any external conditions. Since any state of a thermalized system is closely related to the external conditions, in order to obtain $\mu(T)$ we only need to specify the external conditions of the thermalized quark–gluon system convenient for the determination of $\mu(T)$.

Indeed, it is well-known [5] that the Helmholtz free energy $F(T,V,N)$, defining the partition function $Z(T,V,N)$, $F(T,V,N) = -T\ln Z(T,V,N)$, plays a central role in studying thermalized systems. It has a meaning of the work for an isothermic process. Keeping $T = const.$ and measuring the work, one can get a full information about the Helmholtz free energy $F_{\rm exp}(T,V,N)$ and of the partition function $Z_{\rm exp}(T,V,N)$ describing the thermalized system at any T.

Following this idea, we have fixed the chemical potential $\mu(T)$ in the form [1]

$$\frac{\mu(T)}{\mu_0} = \left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\pi^6}{27}\left(\frac{T}{\mu_0}\right)^6}\,\right]^{1/3} - \left[-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\pi^6}{27}\left(\frac{T}{\mu_0}\right)^6}\,\right]^{1/3}. \quad (4)$$

In the low-temperature limit $T \to 0$, we get

$$\mu(T) \;=\; \mu_0\left[1 - \frac{\pi^2}{3}\frac{T^2}{\mu_0^2} + O\Big(T^6\Big)\right], \quad (5)$$

where $\mu_0 = \mu(0) = 250\,{\rm MeV}$ is the chemical potential at zero temperature [1,9].

In the high-temperature limit $T \to \infty$, the chemical potential $\mu(T)$ defined by Eq. (4) decreases like T^{-2}

$$\mu(T) \;=\; \frac{\mu_0^3}{\pi^2}\,\frac{1}{T^2} + O\Big(T^{-7}\Big). \quad (6)$$

The chemical potential decreases very fast when the temperature increases. Indeed, at $T = 160\,{\rm MeV}$, we obtain $\mu(T) \simeq \mu_0/4$, while at $T = \mu_0$, the value of the chemical potential makes up about a tenth of μ_0, i. e. $\mu(T) \simeq \mu_0/10$. This implies that at very high temperatures the function $\nu(T) = \mu(T)/T$ becomes small and the contribution of the chemical potential of light quarks and antiquarks can be taken into account perturbatively. This assumes in particular that at temperatures $T \geq \mu_0 = 250\,{\rm MeV}$, the number of light antiquarks will not be suppressed by a chemical potential relative to the number of light quarks.

In our approach, we define the multiplicities of hadron production in terms of quark and antiquark distribution functions in a way similar to a simple coalescence model [2] but for correlated quarks and antiquarks. Indeed, in a coalescence model, quarks and antiquarks are uncorrelated [2]. This allows to introduce separately the numbers of light quarks q, light antiquarks $\bar{\rm q}$, strange quarks s and of strange antiquarks $\bar{\mathrm{s}}$ [2]. The subsequent calculation of multiplicities of hadrons in a simple coalescence model resembles the quark counting. In fact, the multiplicities of hadrons are proportional to the products of the numbers of quarks (q,s) and antiquarks $(\bar{\mathrm{q}}, \bar{\mathrm{s}})$ in accord with the naive quark structure of hadrons. Since quarks and antiquarks do not correlate, the multiplicities of hadrons turn out to be independent on the momenta of hadrons. We would like to emphasize that the numbers of quarks (q,s) and antiquarks $(\bar{\mathrm{q}}, \bar{\mathrm{s}})$ and the coefficients of proportionality are free parameters of the simple coalescence model. Therefore, a simple coalescence model contains seven free parameters, but only five of them can be fixed from experimental data [2].

In our *coalescence model of correlated quarks*, the multiplicities of hadrons produced from the QGP phase are described by momentum integrals of quark and antiquark distribution functions. Unlike in the simple coalescence model, these integrals depend explicitly on the momenta of hadrons, temperature T and chemical potential $\mu(T)$. The relative motion of quarks is taken into account in terms of wave functions of quarks and antiquarks. In the first approximation, we describe the wave functions for the relative motion of quarks and antiquarks inside hadrons by constants. This corresponds to the limit of *asymptotic freedom* of QCD. We show below that such an approximation for quark correlations describes effectively the dynamics of quark–antiquark pairing into mesons and baryon production. It agrees well with the experimental data on relative multiplicities of hadron production.

Now let us turn to the definition of multiplicities of hadrons produced from the QGP in our approach. As an example, we consider the multiplicity of π^+-meson production, $N_{\pi^+}(\vec{q}, T)$, where $\vec{q}$ is the 3-momentum of the π^+ meson. We define the multiplicity $N_{\pi^+}(\vec{q}, T)$ as follows

$$N_{\pi^\pm}(\vec{q}, T) = \int \frac{\mathrm{d}^3 p_{\mathrm{u}}}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 p_{\bar{\mathrm{d}}}}{(2\pi)^{3/2}} \, n_{\mathrm{u}}(\vec{p}_{\mathrm{u}}) \, n_{\bar{\mathrm{d}}}(\vec{p}_{\bar{\mathrm{d}}}) \, \Phi(\vec{p}_{\mathrm{u}} - \vec{p}_{\bar{\mathrm{d}}}) \, \delta^{(3)}(\vec{q} - \vec{p}_{\mathrm{u}} - \vec{p}_{\bar{\mathrm{d}}}), \quad (7)$$

where $\vec{p}_{\mathrm{u}}$ and $\vec{p}_{\bar{\mathrm{d}}}$ are the momenta of quark u and antiquark $\bar{\mathrm{d}}$, $\Phi(\vec{p}_{\mathrm{u}} - \vec{p}_{\bar{\mathrm{d}}})$ is the wave function of a $\mathrm{u}\bar{\mathrm{d}}$ pair and the δ-function $\delta^{(3)}(\vec{q} - \vec{p}_{\mathrm{u}} - \vec{p}_{\bar{\mathrm{d}}})$ describes momentum conservation reflecting the obvious fact that the momentum of a centre of mass of a quark–antiquark pair $\mathrm{u}\bar{\mathrm{d}}$ should move only with the momentum of the π^+ meson. The wave function $\Phi(\vec{p}_{\mathrm{u}} - \vec{p}_{\bar{\mathrm{d}}})$ can be calculated, e.g., in the way suggested by Hwa and Lam [10].

The multiplicity of π^+-meson production given by Eq. (7) can be reduced to the form postulated by Das and Hwa within the recombination mechanism approach [11]. To this aim, we suggest to make a change of variables

$$\begin{aligned} \vec{p}_{\mathrm{u}} &= x_1 \vec{q} + \vec{p}_{\mathrm{u}\perp}, \\ \vec{p}_{\bar{\mathrm{d}}} &= x_2 \vec{q} + \vec{p}_{\bar{\mathrm{d}}\perp}, \end{aligned} \quad (8)$$

where $\vec{p}_{\mathrm{u}\perp} \cdot \vec{q} = \vec{p}_{\bar{\mathrm{d}}\perp} \cdot \vec{q} = 0$.

In terms of the variables x_1, x_2, $\vec{p}_{\mathrm{u}\perp}$ and $\vec{p}_{\bar{\mathrm{d}}\perp}$, the multiplicity Eq. (7) reads

$$N_{\pi^\pm}(\vec{q},T) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int \frac{\mathrm{d}^2 p_{\mathrm{u}\perp}}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^2 p_{\bar{\mathrm{d}}\perp}}{(2\pi)^{3/2}}\, q\, \delta(1-x_1-x_2)\, \delta^{(2)}(\vec{p}_{\mathrm{u}\perp}+\vec{p}_{\bar{\mathrm{d}}\perp}) \times n_{\mathrm{u}}(x_1\vec{q}+\vec{p}_{\mathrm{u}\perp})\, n_{\bar{\mathrm{d}}}(x_2\vec{q}+\vec{p}_{\bar{\mathrm{d}}\perp})\, \Phi\big((x_1-x_2)\vec{q}+\vec{p}_{\mathrm{u}\perp}-\vec{p}_{\bar{\mathrm{d}}\perp}\big). \quad (9)$$

Integrating over $\vec{p}_{\bar{\mathrm{d}}\perp}$ and setting $\vec{p}_{\mathrm{u}\perp} = \vec{p}_\perp$, we obtain

$$N_{\pi^\pm}(\vec{q},T) = \int_0^1 \frac{\mathrm{d}x_1}{x_1} \int_0^1 \frac{\mathrm{d}x_2}{x_2}\, F_{\mathrm{u}}(x_1) F_{\bar{\mathrm{d}}}(x_2)\, \rho(x_1,x_2)\, R(x_1,x_2)\, \delta(1-x_1-x_2), \quad (10)$$

where we have denoted $F_{\mathrm{u}}(x_1) = n_{\mathrm{u}}(x_1\vec{q})$, $F_{\bar{\mathrm{d}}}(x_2) = n_{\bar{\mathrm{d}}}(x_2\vec{q})$; $R(x_1,x_2) = 6\,x_1x_2$ is *the probability of recombination of just the* $\mathrm{u}\bar{\mathrm{d}}$ *pair* (Das and Hwa) [11]. Then, $\rho(x_1,x_2)$ is determined by

$$\rho(x_1,x_2) = \frac{1}{6}\int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^3}\, \frac{n_{\mathrm{u}}(x_1\vec{q}+\vec{p}_\perp)}{n_{\mathrm{u}}(x_1\vec{q})} \cdot \frac{n_{\bar{\mathrm{d}}}(x_2\vec{q}-\vec{p}_\perp)}{n_{\bar{\mathrm{d}}}(x_1\vec{q})}\, q\, \Phi\big((x_1-x_2)\vec{q}+2\vec{p}_\perp\big). \quad (11)$$

The function $\rho(x_1,x_2)$ is a phenomenological function of the valon-recombination approach [11]. *It is difficult to determine* $\rho(x_1,x_2)$ *in general* (Das and Hwa) [11]. Hence, up to the definition of the function $\rho(x_1,x_2)$, our model agrees with the valon-recombination approach by Hwa et al. [11].

In the simplest approximation, we suggest to neglect the momentum dependence of the wave function $\Phi\big((x_1-x_2)\vec{q}+2\vec{p}_\perp\big)$ and set

$$\Phi\big((x_1-x_2)\vec{q}+2\vec{p}_\perp\big) = N_{\mathrm{C}} V_\pi, \quad (12)$$

where $N_{\mathrm{C}} = 3$ is the number of quark colours, and V_π is the volume inside of which a thermalized quark–antiquark–gluon ensemble produces π^+ mesons. The approximation in Eq. (12) is not in contradiction with the limit of *asymptotic freedom* of QCD valid in the regime of very high energies. The application of this limit can be justified by the fact that in the thermalized QGP, quarks, antiquarks and gluons are decoupled and only collide with each other. We would like to emphasize that the high-energy limit of *asymptotic freedom* does not contradict to confinement which should accompany the hadronization of quarks and antiquarks from the QGP. Indeed, the application of Fermi–Dirac distribution functions, Eq. (1), for the description of thermalized quarks and antiquarks, provides a concentration of integrands around momenta $p \sim T$. This pushes quarks and antiquarks to spatial regions of the of order $\Delta r \le 1/T \sim 1\,\mathrm{fm}$.

We determine the volume V_π as [1]

$$V_\pi = \frac{C_{\mathrm{M}}}{(M_\pi F_\pi)^{3/2}}, \quad (13)$$

where $C_{\rm M}$ is the parameter of the approach, and $M_\pi = 140\,{\rm MeV}$ and $F_\pi = 131\,{\rm MeV}$ are the mass and leptonic coupling constant of the π^+ meson [12].

Another approximation for multiplicities of hadron production from the QGP phase, which we employ in our *coalescence model of correlated quarks*, concerns the following. According to the experimental data, the phase volume of hadrons produced in ultra-relativistic A + A collisions is not spherically symmetric with respect to the collision axis. This can imply that the thermalization of the quark–gluon system, leading to the formation of the thermalized QGP, should be different in the transversal and longitudinal direction relative to the collision axis of ultra-relativistic A + A collisions. However, due to the very high complexity of the theoretical description of hadronization from the thermalized QGP phase, we suggest to approximate multiplicities of hadron production by spherically symmetric distribution functions, assuming a spherically symmetric thermalization of the quark–gluon system.

Thus, we describe the multiplicities for the production of $\pi^\pm$ and $K^\pm$ mesons by the momentum integrals

$$
\begin{aligned}
N_{\pi^\pm}(\vec{q},T) &= \frac{3C_{\rm M}}{(M_\pi F_\pi)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\;\frac{1}{{\rm e}^{\nu(T)+p/T}+1},\\
N_{K^+}(\vec{q},T) &= \frac{3C_{\rm M}}{(M_K F_K)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\;\frac{1}{{\rm e}^{\sqrt{\vec{p}^{\,2}+m_{\rm s}^2}/T}+1},\\
N_{K^-}(\vec{q},T) &= \frac{3C_{\rm M}}{(M_K F_K)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\;\frac{1}{{\rm e}^{\sqrt{\vec{p}^{\,2}+m_{\rm s}^2}/T}+1},
\end{aligned}
\tag{14}
$$

where $\vec{p}$ is the relative momentum of quarks and antiquarks coalesced into a meson with 3-momentum $\vec{q}$ at temperature T. Since the main contribution to the integrals comes from the relative momenta of order $p \sim T$, so that quarks and antiquarks coalesce at relative momenta of order $p \sim T$. This agrees with the order of momenta of hadrons, $q_\perp \sim 2/3\,T$ transversal to the collision axis of colliding heavy-ions, produced in the centre-of-mass frame of heavy-ion collisions. The factor 3 corresponds to the number of quark colour degrees of freedom, $M_K = 500\,{\rm MeV}$ and $F_K = 160\,{\rm MeV}$ are the mass and the leptonic coupling constant of the K mesons [12]. The dimensionless parameter $C_{\rm M}$, a free parameter of the approach, is the same for all low-lying mesons.

We define the multiplicities of vector meson production, e.g. for $K^{*\pm}$ and $\rho^\pm$, as

$$
\begin{aligned}
N_{K^{*+}}(\vec{q},T) &= \frac{3C_{\rm M}}{(M_{K^*} F_K)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\;\frac{1}{{\rm e}^{\sqrt{\vec{p}^{\,2}+m_{\rm s}^2}/T}+1},\\
N_{K^{*-}}(\vec{q},T) &= \frac{3C_{\rm M}}{(M_{K^*} F_K)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\;\frac{1}{{\rm e}^{\sqrt{\vec{p}^{\,2}+m_{\rm s}^2}/T}+1},
\end{aligned}
$$

$$N_{\rho^{\pm}}(\vec{q},T)=\frac{3C_{\rm M}}{(M_\rho F_\pi)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{{\rm e}^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\,\frac{1}{{\rm e}^{\nu(T)+p/T}+1},\quad(15)$$

where $M_{K^*}=892\,{\rm MeV}$ and $M_\rho=770\,{\rm MeV}$ are the masses of the K^* and ρ mesons, respectively [10].

In the case of baryons and antibaryons, we suggest to define the multiplicities using the diquark–quark picture of baryons and antibaryons. As an example, we write the multiplicities for protons (p) and antiprotons ($\bar{\rm p}$)

$$N_{\rm p}(\vec{q},T)=\frac{3!}{3!}\,\frac{C_{\rm B}}{(M_{\rm p}F_\pi)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{\Big({\rm e}^{-\nu(T)+p/T}+1\Big)^2}\,\frac{1}{{\rm e}^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1},$$

$$N_{\bar{\rm p}}(\vec{q},T)=\frac{3!}{3!}\,\frac{C_{\bar{\rm B}}}{(M_{\rm p}F_\pi)^{3/2}}\int\frac{{\rm d}^3p}{(2\pi)^3}\,\frac{1}{\Big({\rm e}^{\nu(T)+p/T}+1\Big)^2}\,\frac{1}{{\rm e}^{\nu(T)+|\vec{p}-\vec{q}\,|/T}+1},\quad(16)$$

where the momentum $\vec{p}$ has the meaning of the relative momentum of the diquark–quark system, $M_{\rm p}=940\,{\rm MeV}$ is the mass of protons and antiprotons. As in the meson case, the main contribution to the momentum integrals comes from the momenta of order $p\sim T$, providing the coalescence of three quarks (three antiquarks) into baryons (anti–baryons) at momenta of the order $p\sim T$. That is again of the order of the momenta of the produced hadrons, $q_\perp\sim 2/3\,T$ transversal to the collision axis of the colliding heavy-ions, coupled in the centre-of-mass frame of heavy-ion collisions. The factor 3! in the numerator is related to the quark colour degrees of freedom and defined by $\varepsilon_{ijk}\,\varepsilon^{ijk}=3!$, where i,j and k are colour indices and run over $i=1,2,3$ each. In turn, the factor 3! in the denominator takes into account the identity of three light quarks (qqq) and three antiquarks ($\bar{\rm q}\bar{\rm q}\bar{\rm q}$). In the isotopical limit, we do not distinguish u and d quarks as well as $\bar{\rm u}$ and $\bar{\rm d}$ antiquarks. The dimensionless parameters $C_{\rm B}$ and $C_{\bar{\rm B}}$ are free parameters of the approach. Each of them is the same for all components of the octets of baryons and antibaryons, respectively, but $C_{\rm B}\neq C_{\bar{\rm B}}$.

Now let us specify the type of equilibrium which we assume in our approach. As usual, the thermal models are used in two different ways for the description of hadronization in ultra-relativistic heavy-ion collisions based on global and local equilibrium of the quark–gluon plasma [13]. In our approach, we assume *global equilibrium* of the quark–gluon plasma and *chemical equilibrium* of the multicomponent hadron gas produced from the quark–gluon plasma [14,15]. We understand the possible dependence of temperature T and multiplicities of hadron production on the space–time point $(\vec{r},t)$ understand in the sense of the Chapman–Enskog approach to non-equilibrium systems [14]. Due to *the asymptotic freedom*, quarks, antiquarks and gluons are decoupled and only collide with each other. Therefore, the Chapman–Enskog method can be qualitatively applied to the understanding of a space–time dependence of temperature and multiplicities. According to the Chapman–Enskog method, the distribution function of the non-equilibrium system can be expanded in the Enskog series in powers of ε, where $1/\varepsilon$ measures the frequency of collisions. Let $f(\vec{q},\vec{r},t)$ be a distribution function. If ε were small, collisions would be very frequent and the system would behave like a continuum in which local equilibrium is everywhere maintained [14]. The distribution function $f(\vec{q},\vec{r},t)$ can be expanded in the Enskog series [14]

$$f(\vec{q},\vec{r},t) \;=\; f^{(0)}(\vec{q},\vec{r},t)+\varepsilon\, f^{(1)}(\vec{q},\vec{r},t)+\varepsilon^2\, f^{(2)}(\vec{q},\vec{r},t)+\ldots. \tag{17}$$

In the case of $\varepsilon \ll 1$, when the frequency of collisions is much greater than unity ($1/\varepsilon \gg 1$), the first term $f^{(0)}(\vec{q},\vec{r},t)$ dominates and does not really depend on $(\vec{r},t)$. Indeed, relative changes in the density, $n(\vec{q},\vec{r},t)$, and temperature, $T(\vec{q},\vec{r},t)$, within the scattering length $\ell_{\rm coll} \ll 1\,{\rm fm}$ and the collision time $\tau_{\rm coll} \ll 1\,{\rm fm/c}$, are small compared with unity [14]

$$\begin{aligned} &\ell_{\rm coll}\left|\frac{1}{n(\vec{q},\vec{r},t)}\cdot\frac{\partial n(\vec{q},\vec{r},t)}{\partial\vec{r}}\right| \ll 1, \qquad \tau_{\rm coll}\left|\frac{1}{n(\vec{q},\vec{r},t)}\cdot\frac{\partial n(\vec{q},\vec{r},t)}{\partial t}\right| \ll 1,\\ &\ell_{\rm coll}\left|\frac{1}{T(\vec{q},\vec{r},t)}\cdot\frac{\partial T(\vec{q},\vec{r},t)}{\partial\vec{r}}\right| \ll 1, \qquad \tau_{\rm coll}\left|\frac{1}{T(\vec{q},\vec{r},t)}\cdot\frac{\partial T(\vec{q},\vec{r},t)}{\partial t}\right| \ll 1. \end{aligned} \tag{18}$$

In the QGP, the frequency of collisions $1/\varepsilon$ is much greater that unity [1] and quarks, antiquarks and gluons are decoupled due to *asymptotic freedom*. As result, the temperature and multiplicities of hadron production, having the meaning of densities of multicomponent quasi-equilibrium system, are slightly varying functions on $\vec{r}$ and t.

Thereby, for the description of multiplicities of hadron production by Eqs. (14)–(16) valid in the limit of *asymptotic freedom*, one can neglect the space–time dependence and set the temperature T constant in space and time.

Our description of hadron production within the *ansatz* of a thermalized QGP runs parallel to the approach of inter-species *chemical equilibrium* by Becattini and Heinz [15], and Cleymans [16]. In our case, this corresponds to a hadron gas of fireballs with sizes $V_{\rm h}$ ($\rm h = \pi, K, p, \bar{p},\ldots$), where $V_{\rm h}$ is a fireball volume in its rest frame [15]. Unlike Becattini and Heinz [15], we classify fireballs with respect to the quantum numbers of a produced hadron h. The total sum over the volumes of fireballs can be, in principle, less than the total volume V occupied by the QGP, i. e. $\sum_{\rm h} V_{\rm h} < V$. All h-hadron components of a multicomponent hadron gas are in *chemical equilibrium* state with a common temperature T [15,16].

In the ideal case, one should expect that the multiplicity in Eq. (7) of π^+ meson production should be fitted by the Boltzmann distribution function [16]

$$\frac{{\rm d}^3 n_{\pi^+}(\vec{q},T)}{{\rm d}^3 q} = N_{\pi^+}(\vec{q},T) = \frac{1}{(2\pi)^3}\,{\rm e}^{-\sqrt{\vec{q}^{\,2}+M_\pi^2}/T+\mu_{\pi^+}(T)/T}, \tag{19}$$

where $\mu_{\pi^+}(T)$ is the chemical potential of the π^+-meson gas. We show below that for $q \gg T$, the momentum integral describing the multiplicity $N_{\pi^+}(\vec{q},T)$ by Eq. (14)

can be represented in the form

$$N_{\pi^+}(\vec{q},T) \;=\; \frac{1}{(2\pi)^3}\,\mathrm{e}^{-q/T + \mu_{\pi^+}(T)/T}, \tag{20}$$

where the chemical potential $\mu_{\pi^+}(T)$ is defined by

$$\mu_{\pi^+}(T) \;=\; T\ln\big[(2\pi)^3 V_\pi I_\pi(T)\big] \tag{21}$$

with $I_\pi(T)$ given by Eq. (23).

The paper is organized as follows. In Sect. 2, we calculate the theoretical values of multiplicities of hadron production from the thermalized QGP phase. The theoretical predictions and experimental data are adduced in Table 1. In the Conclusion, we discuss the calculated results. A possible estimate of the absolute values of our input parameters is discussed through the application of our approach to the calculation of the number of baryons and antibaryons relative to the number of photons at the early stage of the evolution of the Universe, assuming that this evolution went through the intermediate thermalized QGP phase.

2. Multiplicities of hadron production from the thermalized QGP phase

Now let us proceed to evaluate the multiplicities of hadron production from the thermalized QGP phase of QCD. We compare the theoretical predictions for different ratios of hadron multiplicities with experimental data in Table 1 (see Ref. [17]). These are the data on Pb+Pb collisions at 158 GeV/nucleon from various experiments obtained by NA44, NA49, NA50 and WA97 collaborations. We also compare our results with the experimental data obtained by the NA35 Collaboration on S+S collisions and the NA38 Collaboration on O+U and S+U collisions at 200 GeV/nucleon. From Table 1 of this paper, one can see that the typical momenta q of the produced hadrons are much greater than T, $q \gg T$. This means that the typical momenta of the produced hadrons are much greater than $T = 175\,\mathrm{MeV}$. Since the integrands of the momentum integrals describing the multiplicities of hadron production are concentrated around $p \sim T$, the momenta of the produced hadrons are also much greater than the relative momenta of quarks, $q \gg p \sim T$. Therefore, for the simplification of the analytical calculations, we calculate the multiplicities for $q \gg T$ [1]. That allows to expand the interands of the momentum integrals defining the multiplicities of hadron production in powers of exponents $\mathrm{e}^{-q/T}$ and keep only the leading contributions of the order $O(\mathrm{e}^{-q/T})$.

In this approximation, the multiplicities of hadron production defined by Eqs. (14) to (16) can be recast into the form

$$N_{\pi^+}(\vec{q},T) \;=\; N_{\pi^-}(\vec{q},T) \;=\; N_{\pi^0}(\vec{q},T) = \frac{3\,C_{\mathrm{M}}}{(M_\pi F_\pi)^{3/2}}\,\mathrm{e}^{-q/T}\,I_\pi(T),$$

$$
\begin{aligned}
N_{K^+}(\vec{q},T) &= N_{K^0}(\vec{q},T) \;=\; \frac{3\,C_{\rm M}}{(M_K F_K)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{+\nu(T)}\,I_K(T),\\
N_{K^-}(\vec{q},T) &= N_{\bar{K}^0}(\vec{q},T) \;=\; \frac{3\,C_{\rm M}}{(M_K F_K)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{-\nu(T)}\,I_{\bar{K}}(T),\\
N_{K^0_S}(\vec{q},T) &= \frac{1}{2}\,N_{K^0}(\vec{q},T)+\frac{1}{2}\,N_{\bar{K}^0}(\vec{q},T)=\frac{1}{2}\,\frac{3\,C_{\rm M}}{(M_K F_K)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{+\nu(T)}\,I_{K^0_S}(T),\\
N_\eta(\vec{q},T) &= \sin^2\bar{\theta}\,\frac{3\,C_{\rm M}}{(M_\eta F_\pi)^{3/2}}\,{\rm e}^{-\,q/T}\,I_\pi(T)+\cos^2\bar{\theta}\,\frac{3\,C_{\rm M}}{(M_\eta F_{\rm S})^{3/2}}{\rm e}^{-\,q/T}\,I_\eta(T),\\
N_\phi(\vec{q},T) &= \frac{3\,C_{\rm M}}{(M_\phi F_{\rm S})^{3/2}}\,{\rm e}^{-\,q/T}\,I_\phi(T),\\
N_{\rm p}(\vec{q},T) &= \frac{C_{\rm B}}{(M_{\rm p} F_\pi)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{+\,\nu(T)}\,I_{\rm p}(T),\\
N_\Lambda(\vec{q},T) &= \frac{3\,C_{\rm B}}{(M_\Lambda F_K)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{+2\,\nu(T)}\,I_\Lambda(T),\\
N_\Xi(\vec{q},T) &= \frac{3\,C_{\rm B}}{(M_\Xi F_{\rm S})^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{+\nu(T)}\,I_\Xi(T),\\
N_\Omega(\vec{q},T) &= \frac{C_{\rm B}}{(M_\Omega F_{\rm S})^{3/2}}\,{\rm e}^{-q/T}\,I_\Omega(T),\\
N_{\bar{\rm p}}(\vec{q},T) &= \frac{C_{\bar{\rm B}}}{(M_{\rm p} F_\pi)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{-\nu(T)}\,I_{\bar{\rm p}}(T),\\
N_{\bar\Lambda}(\vec{q},T) &= \frac{3\,C_{\bar{\rm B}}}{(M_\Lambda F_K)^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{-\,2\,\nu(T)}\,I_{\bar\Lambda}(T),\\
N_{\bar\Xi}(\vec{q},T) &= \frac{3\,C_{\bar{\rm B}}}{(M_\Xi F_{\rm S})^{3/2}}\,{\rm e}^{-q/T}\,{\rm e}^{-\nu(T)}\,I_{\bar\Xi}(T),\\
N_{\bar\Omega}(\vec{q},T) &= \frac{C_{\bar{\rm B}}}{(M_\Omega F_{\rm S})^{3/2}}\,{\rm e}^{-q/T}\,I_{\bar\Omega}(T),
\end{aligned}
\tag{22}
$$

where the structure functions $I_i(T)\,(i=\pi,K,\bar{K},\ldots)$ are defined by

$$
\begin{aligned}
I_\pi(T) &= {\rm e}^{+\nu(T)}\int\limits_0^\infty\frac{{\rm d}p}{4\pi^2}\,\frac{p^2}{{\rm e}^{+\nu(T)\,+\,p/T}+1}+{\rm e}^{-\nu(T)}\int\limits_0^\infty\frac{{\rm d}p}{4\pi^2}\,\frac{p^2}{{\rm e}^{-\nu(T)+p/T}+1}\\
&= \frac{T^3}{4\pi^2}\left[{\rm e}^{+\nu(T)}\int\limits_0^\infty\frac{{\rm d}x\,x^2}{{\rm e}^{+\nu(T)+x}+1}+{\rm e}^{-\nu(T)}\int\limits_0^\infty\frac{{\rm d}x\,x^2}{{\rm e}^{-\nu(T)+x}+1}\right]\\
&= 3.153\,\frac{T^3}{4\pi^2},
\end{aligned}
$$

$$
\begin{aligned}
I_K(T) &= \int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1} + \mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{-\nu(T)+p/T}+1} \\
&= \frac{T^3}{4\pi^2}\left[\frac{m_\mathrm{s}^3}{T^3}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1} + \mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{-\nu(T)+x}+1}\right] \\
&= 2.924\,\frac{T^3}{4\pi^2}, \\
I_{\bar K}(T) &= \int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1} + \mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{+\nu(T)+p/T}+1} \\
&= \frac{T^3}{4\pi^2}\left[\frac{m_\mathrm{s}^3}{T^3}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1} + \mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{+\nu(T)+x}+1}\right] \\
&= 3.463\,\frac{T^3}{4\pi^2}, \\
I_\eta(T) &= I_\phi(T) = \int\limits_0^\infty \frac{\mathrm{d}p}{2\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{\vec{p}^{\,2}+m_\mathrm{s}^2}/T}+1} = \frac{m_\mathrm{s}^3}{2\pi^2}\int\limits_0^\infty \frac{\mathrm{d}\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1} \\
&= 3.522\,\frac{m_\mathrm{s}^3}{2\pi^2}, \\
I_\mathrm{p}(T) &= \int\limits_0^\infty \frac{\mathrm{d}p}{2\pi^2}\,\frac{1}{\left(\mathrm{e}^{-\nu(T)+p/T}+1\right)^2} = \frac{T^3}{2\pi^2}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{-\nu(T)+x}+1\right)^2} = 0.253\,\frac{T^3}{2\pi^2}, \\
I_\Lambda(T) &= \mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1}\;\frac{1}{\mathrm{e}^{-\nu(T)+p/T}+1} \\
&\quad + \mathrm{e}^{-2\,\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{-\nu(T)+p/T}+1\right)^2} \\
&= \frac{m_\mathrm{s}^3}{4\pi^2}\left[\mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1}\;\frac{1}{\mathrm{e}^{-\nu(T)+(m_\mathrm{s}/T)\,x}+1}\right. \\
&\quad \left. + \mathrm{e}^{-2\,\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{-\nu(T)+(m_\mathrm{s}/T)\,x}+1\right)^2}\right] = 0.582\,\frac{m_\mathrm{s}^3}{4\pi^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{\Xi}(T) \;=\; & \int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1\right)^2} + \mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1} \\
& \times\frac{1}{\mathrm{e}^{-\nu(T)+p/T}+1} = \frac{m_\mathrm{s}^3}{4\pi^2}\Bigg[\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1\right)^2} \\
& +\mathrm{e}^{-\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1}\,\frac{1}{\mathrm{e}^{-\nu(T)+(m_s/T)\,x}+1}\Bigg] = 0.528\,\frac{m_\mathrm{s}^3}{4\pi^2},
\end{aligned}
$$

$$
I_{\Omega}(T) = \int\limits_0^\infty \frac{\mathrm{d}p}{2\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1\right)^2} = \frac{m_\mathrm{s}^3}{2\pi^2}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1\right)^2} = 0.251\,\frac{m_\mathrm{s}^3}{2\pi^2},
$$

$$
I_{\bar{\mathrm{p}}}(T) = \int\limits_0^\infty \frac{\mathrm{d}p}{2\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{\nu(T)+p/T}+1\right)^2} = \frac{T^3}{2\pi^2}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{\nu(T)+x}+1\right)^2} = 0.097\,\frac{T^3}{2\pi^2},
$$

$$
\begin{aligned}
I_{\bar{\Lambda}}(T) \;=\; & \mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1}\cdot\frac{1}{\mathrm{e}^{+\nu(T)+p/T}+1} \\
& +\mathrm{e}^{+2\,\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{+\nu(T)+p/T}+1\right)^2} \\
=\; & \frac{m_\mathrm{s}^3}{4\pi^2}\Bigg[\mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1}\cdot\frac{1}{\mathrm{e}^{+\nu(T)+(m_\mathrm{s}/T)\,x}+1} \\
& +\mathrm{e}^{+\,2\,\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{+\nu(T)+(m_\mathrm{s}/T)\,x}+1\right)^2}\Bigg] = 0.686\,\frac{m_\mathrm{s}^3}{4\pi^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{\bar{\Xi}}(T) \;=\; & \int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\left(\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1\right)^2} + \mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}p}{4\pi^2}\,\frac{p^2}{\mathrm{e}^{\sqrt{p^2+m_\mathrm{s}^2}/T}+1} \\
& \times\frac{1}{\mathrm{e}^{+\nu(T)+p/T}+1} = \frac{m_\mathrm{s}^3}{4\pi^2}\Bigg[\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\left(\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1\right)^2} \\
& +\mathrm{e}^{+\nu(T)}\int\limits_0^\infty \frac{\mathrm{d}x\,x^2}{\mathrm{e}^{(m_\mathrm{s}/T)\sqrt{1+x^2}}+1}\,\frac{1}{\mathrm{e}^{+\nu(T)+(m_\mathrm{s}/T)\,x}+1}\Bigg] = 0.558\,\frac{m_\mathrm{s}^3}{4\pi^2},
\end{aligned}
$$

$$I_{\bar{\Omega}}(T) \quad = \quad I_{\Omega}(T). \tag{23}$$

The numerical values of the integrals are obtained for $m_{\rm s} = 135\,{\rm MeV}$ and $T = 175\,{\rm MeV}$.

We define the theoretical ratios of multiplicities of hadron production, which we compare with the experimentally measured values, as follows

$$\begin{aligned}
R_{K^+K^-}(q,T) &= \frac{N_{K^+}(\vec{q},T)}{N_{K^-}(\vec{q},T)} = {\rm e}^{+\,2\,\nu(T)}\,\frac{I_K(T)}{I_{\bar{K}}(T)} = 1.520,\\
R_{K^+\pi^+}(q,T) &= \frac{N_{K^+}(\vec{q},T)}{N_{\pi^+}(\vec{q},T)} = \left(\frac{M_\pi F_\pi}{M_K F_K}\right)^{3/2} {\rm e}^{+\,\nu(T)}\,\frac{I_K(T)}{I_\pi(T)} = 0.139,\\
R_{K^-\pi^-}(q,T) &= \frac{N_{K^-}(\vec{q},T)}{N_{\pi^-}(\vec{q},T)} = \left(\frac{M_\pi F_\pi}{M_K F_K}\right)^{3/2} {\rm e}^{-\,\nu(T)}\,\frac{I_{\bar{K}}(T)}{I_\pi(T)} = 0.090,\\
R_{K^0_S\pi^-}(q,T) &= \frac{N_{K^0_S}(\vec{q},T)}{N_{\pi^-}(\vec{q},T)} = \frac{1}{2}\left(\frac{M_\pi F_\pi}{M_K F_K}\right)^{3/2} {\rm e}^{+\,\nu(T)}\,\frac{I_{K^0_S}(T)}{I_{\pi^-}(T)} = 0.113,\\
R_{\Xi\Lambda}(q,T) &= \frac{N_{\Xi}(\vec{q},T)}{N_{\Lambda}(\vec{q},T)} = \left(\frac{M_\Lambda F_K}{M_\Xi F_{\rm S}}\right)^{3/2} {\rm e}^{-\,\nu(T)}\,\frac{I_\Xi(T)}{I_\Lambda(T)} = 0.108,\\
R_{\Omega\Xi}(q,T) &= \frac{N_{\Omega}(\vec{q},T)}{N_{\Xi}(\vec{q},T)} = \frac{1}{3}\left(\frac{M_\Xi}{M_\Omega}\right)^{3/2} {\rm e}^{-\,\nu(T)}\,\frac{I_\Omega(T)}{I_\Xi(T)} = 0.166,\\
R_{\bar{\Lambda}\bar{\rm p}}(q,T) &= \frac{N_{\bar{\Lambda}}(\vec{q},T)}{N_{\bar{p}}(\vec{q},T)} = 3\left(\frac{M_p F_\pi}{M_\Lambda F_K}\right)^{3/2} {\rm e}^{-\,\nu(T)}\,\frac{I_{\bar{\Lambda}}(T)}{I_{\bar{p}}(T)} = 2.081,\\
R_{\bar{\Xi}\bar{\Lambda}}(q,T) &= \frac{N_{\bar{\Xi}}(\vec{q},T)}{N_{\bar{\Lambda}}(\vec{q},T)} = \left(\frac{M_\Lambda F_K}{M_\Xi F_{\rm S}}\right)^{3/2} {\rm e}^{+\,\nu(T)}\,\frac{I_{\bar{\Xi}}(T)}{I_{\bar{\Lambda}}(T)} = 0.173,\\
R_{\bar{\Omega}\bar{\Xi}}(q,T) &= \frac{N_{\bar{\Omega}}(\vec{q},T)}{N_{\bar{\Xi}}(\vec{q},T)} = \frac{1}{3}\left(\frac{M_\Xi}{M_\Omega}\right)^{3/2} {\rm e}^{+\,\nu(T)}\,\frac{I_{\bar{\Omega}}(T)}{I_{\bar{\Xi}}(T)} = 0.282,\\
R_{\bar{\Omega}\Omega}(q,T) &= \frac{N_{\bar{\Omega}}(\vec{q},T)}{N_{\Omega}(\vec{q},T)} = \frac{C_{\bar{\rm B}}}{C_{\rm B}} = R^{\rm exp}_{\bar{\Omega}\Omega} = 0.46 \pm 0.15,\\
R_{\bar{\Lambda}\Lambda}(q,T) &= \frac{N_{\bar{\Lambda}}(\vec{q},T)}{N_{\Lambda}(\vec{q},T)} = \frac{C_{\bar{\rm B}}}{C_{\rm B}} \times {\rm e}^{-\,4\,\nu(T)}\,\frac{I_{\bar{\Lambda}}(T)}{I_\Lambda(T)} = 0.168 \pm 0.055,\\
R_{\bar{\Xi}\Xi}(q,T) &= \frac{N_{\bar{\Xi}}(\vec{q},T)}{N_{\Xi}(\vec{q},T)} = \frac{C_{\bar{\rm B}}}{C_{\rm B}} \times {\rm e}^{-\,2\,\nu(T)}\,\frac{I_{\bar{\Xi}}(T)}{I_\Xi(T)} = 0.270 \pm 0.088,
\end{aligned}$$

$$\begin{aligned}
R_{\eta\pi^0}(q,T) &= \frac{N_\eta(\vec{q},T)}{N_{\pi^0}(\vec{q},T)} = \sin^2\bar{\theta}\left(\frac{M_\pi}{M_\eta}\right)^{3/2} + \cos^2\bar{\theta}\left(\frac{M_\pi F_\pi}{M_\eta F_S}\right)^{3/2}\frac{I_\eta(T)}{I_{\pi^+}(T)} \\
&= 0.088, \\
R_{\phi\pi}(q,T) &= \frac{N_\phi(\vec{q},T)}{N_\pi(\vec{q},T)} = \left(\frac{M_\pi F_\pi}{M_\phi F_{\rm S}}\right)^{3/2}\frac{I_\phi(T)}{I_\pi(T)} = 7.84\times 10^{-3}, \\
R_{\phi(\rho^0+\omega^0)}(q,T) &= \frac{N_\phi(\vec{q},T)}{N_{\rho^0}(\vec{q},T)+N_{\omega^0}(\vec{q},T)} = \left(\frac{M_\rho F_\pi}{M_\phi F_{\rm S}}\right)^{3/2}\frac{I_\phi(T)}{I_\pi(T)} = 0.103, \\
R_{\phi K^0_S}(q,T) &= \frac{N_\phi(\vec{q},T)}{N_{K^0_S}(\vec{q},T)} = 2\left(\frac{M_K F_K}{M_\phi F_{\rm S}}\right)^{3/2} {\rm e}^{-\,\nu(T)}\,\frac{I_\phi(T)}{I_{K^0_S}(T)} = 0.071, \\
R_{\Lambda K^0_S}(q,T) &= \frac{N_\Lambda(\vec{q},T)}{N_{K^0_S}(\vec{q},T)} = \frac{C_{\rm B}}{C_{\rm M}}\times 2\,{\rm e}^{+\,\nu(T)}\,\frac{I_\Lambda(T)}{I_{K^0_S}(T)} = \frac{C_{\rm B}}{C_{\rm M}}\times 0.148 \\
&= R^{\rm exp}_{\Lambda K^0_S} = 0.65\pm 0.11 \to \frac{C_{\rm B}}{C_{\rm M}} = 4.39\pm 0.74, \qquad (24) \\
R_{{\rm p}K^+}(q,T) &= \frac{N_{\rm p}(\vec{q},T)}{N_{K^+}(\vec{q},T)} = \frac{C_{\rm B}}{C_{\rm M}}\times\frac{1}{3}\left(\frac{M_K F_K}{M_{\rm p} F_\pi}\right)^{3/2}\frac{I_{\rm p}(T)}{I_K(T)} = 0.136\pm 0.023, \\
R_{\bar{\rm p}K^-}(q,T) &= \frac{N_{\bar{\rm p}}(\vec{q},T)}{N_{K^-}(\vec{q},T)} = \frac{C_{\bar{\rm B}}}{C_{\rm M}}\times\frac{1}{3}\left(\frac{M_K F_K}{M_{\rm p} F_\pi}\right)^{3/2}\frac{I_{\bar{\rm p}}(T)}{I_{\bar{K}}(T)} = 0.020\pm 0.007.
\end{aligned}$$

The constant $F_{\rm S} = 3.5\,F_\pi$ is related to the leptonic constant of the pseudoscalar meson containing only s-quarks, $s\bar{s}$ [19]. We have estimated $F_{\rm S}$ in agreement with the experimental data on the $\eta(550)/\pi^0$ and $\phi(1020)/\pi$ production. For the description of the multiplicity of the $\eta(550)$ meson production, we have taken into account that the low-energy meson phenomenology [12,19,20] gives the following quark structure of the $\eta(550)$ meson

$$\eta(550) = ({\rm q}\bar{\rm q})\,\sin\bar{\theta} + ({\rm s}\bar{\rm s})\,\cos\bar{\theta}, \qquad (25)$$

where $\bar{\theta} = \vartheta_0 - \vartheta_P$ with $\vartheta_0 = 35.264^0$, the ideal mixing angle, and ϑ_P, the octet–singlet mixing angle. A recent analysis of the value of the octet–singlet mixing angle carried out by Bramon, Escribano and Scadron [20] gives $\vartheta_P = -\,16.9\pm 1.7^{\,0}$. For the $\phi(1020)$ meson, we have supposed the $s\bar{s}$ quark structure [12,19].

3. Conclusion

The theoretical and experimental values of the ratios of hadron production are shown in Table 1. From the table, one can see a good agreement between

TABLE 1. The theoretical ratios of multiplicities of the hadron production are compared with the experimental data obtained by NA44, NA49, NA50 and WA97 Collaborations on Pb+Pb *collisions at 158 GeV/nucleon, NA35 Collaboration on* S+S *collisions and NA38 Collaboration on* O+U *and* S+U *collisions at 200* GeV/nucleon. *The theoretical multiplicities are calculated at the temperature $T = 175$ MeV.*

N	Ratio	Model	Data	Coll.	Rapidity	Ref.
1	$\bar{\mathrm{p}}/\mathrm{p}$	0.097(32)	0.055(10)	NA44	2.3–2.9	[21]
		0.097(32)	0.085(8)	NA49	2.5–3.3	[22]
2	$\bar{\Lambda}/\Lambda$	0.168(55)	0.128(12)	WA97	2.4–3.4	[23]
3	$\bar{\Xi}/\Xi$	0.270(88)	0.227(33)	NA49	3.1–3.85	[24]
		0.270(88)	0.266(28)	WA97	2.4–3.4	[23]
4	$\bar{\Omega}/\Omega$	fit	0.46(15)	WA97	2.4–3.4	[23]
5	Ξ/Λ	0.108	0.127(11)	NA49	3.1–3.85	[24]
		0.108	0.093(7)	WA97	2.4–3.4	[23]
6	Ω/Ξ	0.166	0.195(28)	WA97	2.4–3.4	[23]
7	$\bar{\Xi}/\bar{\Lambda}$	0.173	0.180(39)	NA49	3.1–3.85	[24]
		0.173	0.195(23)	WA97	2.4–3.4	[23]
8	$\bar{\Lambda}/\bar{\mathrm{p}}$	2.081	3(1)	NA49	3.1–3.85	[25]
9	$\bar{\Omega}/\bar{\Xi}$	0.282	0.27(6)	WA97	2.4–3.4	[26]
10	K^+/K^-	1.520	1.85(9)	NA44	2.4–3.5	[21]
		1.520	1.8(1)	NA49	1.4–4.1	[27]
11	K^+/π^+	0.139	0.137(8)	NA35	1.4–4.1	[28]
12	K^-/π^-	0.090	0.076(5)	NA35	1.4–4.1	[28]
13	K^0_S/π^-	0.113	0.125(19)	NA49	–	[29]
14	η/π^0	0.088	0.081(13)	WA98	2.3–2.9	[30]
15	$2\phi/(\pi^+ + \pi^-)$	7.8×10^{-3}	$9.1(1.0) \times 10^{-3}$	NA50	2.9–3.9	[31]
16	$\phi/(\rho^0 + \omega^0)$	0.103	≈ 0.1	NA38	2.8–4.1	[32,33]
17	ϕ/K^0_S	0.071	0.084(11)	NA49	–	[34]
18	Λ/K^0_S	fit	0.65(11)	WA97	2.4–3.4	[35]
19	p/K^+	0.136(23)				
20	$\bar{\mathrm{p}}/K^-$	0.020(7)				
21	$\bar{\mathrm{p}}/\mathrm{p} \cdot K^+/K^-$	0.147(57)	0.102(19)	NA44	2.3–2.9	[21]
		0.147(57)	0.153(17)	NA49	2.5–3.3	[22,27]

the presently available set of hadron ratios measured by NA44, NA49, NA50 and WA97 collaborations on Pb+Pb collisions at 158 GeV/nucleon, NA35 Collaboration on S+S collisions and NA38 Collaboration on O+U and S+U collisions at 200 GeV/nucleon and theoretical predictions for the ratios of multiplicities of hadron production from the thermalized QGP phase at a temperature $T = 175\,\mathrm{MeV}$ [21–35]. Except for the ratios $\bar{\Lambda}/\bar{\mathrm{p}}$, $(\bar{\Lambda}/\bar{\mathrm{p}})_{\mathrm{th}} = 2.081$ and $(\bar{\Lambda}/\bar{\mathrm{p}})_{\mathrm{exp}} = 3 \pm 1$, the deviations of theoretical results from the experimental ones are less than 18%.

In our approach, multiplicities of hadron production are defined by momentum integrals of Fermi–Dirac distribution functions for quarks and antiquarks in accordance with the phenomenological quark structure of hadrons. For the analysis of multiplicities of baryon and antibaryon production in terms of quark and antiquark distribution functions, we have followed the diquark–quark picture for baryons and antibaryons. This has allowed to describe multiplicities of baryon, antibaryon and meson production on the same footing.

For the analytical analysis of these multiplicities, we have used an approximation replacing the wave functions of the relative motion of quarks and antiquarks by constants. This approximation corresponds to the limit of *asymptotic freedom* of QCD with almost free quarks, antiquarks and gluons. This is realized in the thermalized QGP when quarks, antiquarks and gluons are decoupled and only collide with each other. Due to the high frequency of collisions, a non-equilibrium quark–gluon system produced in ultra-relativistic heavy-ion collisions becomes thermalized for $\Delta\tau \le 1\,\mathrm{fm/c}$ (Heinz) [15]. The limit of *asymptotic freedom* applied to the approximation of wave functions of quarks and antiquarks does not contradict to the confinement which should accompany hadronization from the QGP. In fact, by virtue of the Fermi–Dirac distribution functions used for the description of thermalized quarks and antiquarks at temperature T, the relative momenta p of quarks and antiquarks are concentrated around $p \sim T$. This provides a natural confinement of quarks and antiquarks in the spatial region $\Delta r \le 1/T \sim 1\,\mathrm{fm}$.

For a simplification of analytical calculations, we have suggested the following approximation of the multiplicities of hadron production from the QGP phase. According to experimental data, the phase volume of the hadrons produced in ultra-relativistic A + A collisions is not spherically symmetric with respect to the collision axis. This can imply that the thermalization of the quark–gluon system, leading to the formation of the thermalized QGP, should be different in the transversal and longitudinal direction relative to the collision axis of ultra-relativistic A + A collisions. However, due to the very high complexity of the theoretical description of hadronization from the thermalized QGP phase, we suggest to approximate the multiplicities of hadron production by spherically symmetric distributions, assuming spherically-symmetric thermalization of the quark–gluon system.

For the explanation of the experimental data on hadron production in ultra-relativistic heavy-ion collisions, we have used three input parameters $C_{\bar{\mathrm{B}}}/C_{\mathrm{B}}$, $C_{\mathrm{M}}/C_{\mathrm{B}}$ and F_{S}. These parameters are related to the spatial volumes of hadronization of the quarks and antiquarks from the thermalized QGP phase. The first two parameters have been fixed from the experimental data on the ratios $(\bar{\Omega}/\Omega)_{\mathrm{exp}} = 0.46 \pm 0.15$, $(\Lambda/K^0_S)_{\mathrm{exp}} = 0.65 \pm 0.11$. These give $C_{\bar{\mathrm{B}}}/C_{\mathrm{B}} = 0.46 \pm 0.15$ and $C_{\mathrm{M}}/C_{\mathrm{B}} = 0.23 \pm 0.04$. In turn, the value of the parameter $F_S = 3.5\,F_\pi = 458.5\,\mathrm{MeV}$ is a result of a smooth fit of the ratios of hadrons containing the ss and $\mathrm{s}\bar{\mathrm{s}}$ components in the quark structure. In the bulk our approach to the hadronization from the QGP has succeeded in describing 21 experimental data on ultrarelativistic heavy-ion collisions.

Unlike other approaches [2–4], the ratio of the $\bar{\Omega}$ and Ω baryon production is an input parameter in our model ($C_{\bar{\mathrm{B}}}/C_{\mathrm{B}}$). In our approach, the ratio $\bar{\Omega}/\Omega$ does not depend both on the momenta of baryons and on the temperature. The former is due to the zero-value of the s-quark chemical potential, $\mu_{\mathrm{s}} = \mu_{\bar{\mathrm{s}}} = 0$. As a result, the ratio $\bar{\Omega}/\Omega$ can be fitted only in our approach. By fitting the ratio $\bar{\Omega}/\Omega$ to the experimental data and applying this value to the description of other ratios of baryon and antibaryon production from the thermalized QGP phase, we have found good agreement with experimental data. This confirms the self-consistency of the approach.

The distinction between the parameters C_{B} and $C_{\bar{\mathrm{B}}}$ can be related to the well-known fact of the baryon–antibaryon asymmetry in the Universe, which one could put phenomenologically at the early stage of the evolution of the Universe, if the baryon synthesis in it went through the intermediate QGP phase. Indeed, as has been stated by Börner [36]: *Within the standard big-bang model, however, there seems to be little chance of achieving a physical separation of baryon and antibaryon phases in an initially baryon-symmetric cosmological model. If the baryon number was exactly conserved – as it is assumed to be in the standard model – the small asymmetry necessary for our existence must be postulated initially. Grand unified theories offer the possibility of creating this small asymmetry from physical processes.*

In our approach, the baryon–antibaryon asymmetry at the hadronic level can be realized phenomenologically in terms of a different rate of hadronization of baryons and antibaryons caused by the input parameter $C_{\bar{\mathrm{B}}}/C_{\mathrm{B}} = 0.46 \pm 0.15$, fixed by the experimental data on the ratio $\bar{\Omega}/\Omega$ in ultrarelativistic heavy-ion collisions. For the total number of antibaryons $N_{\bar{\mathrm{B}}}$ relative to the total number of baryons N_{B}, produced in the baryon synthesis in the early stage of the evolution of the Universe at temperature $T = 175\,\mathrm{MeV}$ [37], and gone through an intermediate thermalized QGP phase, we predict

$$\frac{N_{\bar{\mathrm{B}}}}{N_{\mathrm{B}}} = 0.41 \times \frac{C_{\bar{\mathrm{B}}}}{C_{\mathrm{B}}} = 0.19 \pm 0.06. \tag{26}$$

This result can be supported by an estimate for equilibrium baryon and antibaryon gases. In such an approximation, the ratio $N_{\bar{\mathrm{B}}}/N_{\mathrm{B}}$ is defined by

$$\frac{N_{\bar{\mathrm{B}}}}{N_{\mathrm{B}}} = \mathrm{e}^{-2\mu_{\mathrm{B}}(T)/T} = \mathrm{e}^{-6\mu(T)/T} = 0.17, \tag{27}$$

where the chemical potential $\mu(T)$ is given by Eq. (4), and $T = 175\,\mathrm{MeV}$.

Thus, at the early stage of the evolution of the Universe the number of antibaryons should have been an order of magnitude smaller than the number of baryons, $N_{\bar{\mathrm{B}}} \sim 0.2\,N_{\mathrm{B}}$. According to Börner [38], this is more than enough for the existence of life in the Universe. Recall that the standard approach [38] predicts for 10^9 antibaryons only $(10^9 + 1)$ baryons. As has been stated by Börner: *It is to that one part in* 10^9 *excess of ordinary matter that we owe our existence*! [38].

In the early Universe, the total number of baryons and antibaryons was roughly equal to the number of photons $N_{\rm ph}$ [38]:

$$\frac{N_{\bar{\rm B}} + N_{\rm B}}{N_{\rm ph}} \simeq 1. \tag{28}$$

Since the density of photons is [39]

$$\frac{N_{\rm ph}}{V} = \frac{2.404}{\pi^2}\,T^3, \tag{29}$$

where V is the volume of the early Universe, and the density of the total number of baryons and antibaryons $N_{\bar{\rm B}} + N_{\rm B}$ calculated in our approach at $T = 175\,{\rm MeV}$ amounts to

$$\frac{N_{\bar{\rm B}} + N_{\rm B}}{V} = C_{\rm B} \times 0.442 \times T^3, \tag{30}$$

we can estimate the numerical value of the parameter $C_{\rm B}$:

$$C_{\rm B} \simeq 0.55 \pm 0.08. \tag{31}$$

This gives an estimate of the input parameters $C_{\bar{\rm B}}$ and $C_{\rm M}$:

$$\begin{aligned} C_{\bar{\rm B}} &\simeq 0.25 \pm 0.08, \\ C_{\rm M} &\simeq 0.13 \pm 0.03. \end{aligned} \tag{32}$$

We are planning to carry out in our forthcoming publications the analysis of the influence of the input parameter $C_{\bar{\rm B}}/C_{\rm B} = 0.46 \pm 0.15$ on the evolution of the baryon–antibaryon asymmetry from the early Universe up to the present epoch and the formation of the dark and strange matter in the Universe [10].

Now we would like to discuss in more details our approach with respect to a simple coalescence model [2]. The main distinction of our approach from a simple coalescence model is in the correlation between quarks and antiquarks coalescing into hadrons. In fact, in a simple coalescence model, quarks and antiquarks are uncorrelated [2]. This has allowed to introduce separately the number of light quarks q and light antiquarks $\bar{\rm q}$ and the number of strange quarks s and strange antiquarks $\bar{\rm s}$ [2]. This has turned out to be of use in order to hide the dependence of quark and antiquark distribution functions on the temperature T and the chemical potential $\mu(T)$ in the numbers of light quarks q and light antiquarks $\bar{\rm q}$. A non-vanishing chemical potential of strange quarks $\mu_{\rm s}(T)$ is hidden in the number of strange quarks s and strange antiquarks $\bar{\rm s}$. Then, the calculation of multiplicities of hadrons produced from the QGP phase in a simple coalescence model resembles quark counting. In fact, the multiplicities of hadrons are proportional to the products of the number of quarks (q,s) and antiquarks $(\bar{\rm q},\bar{\rm s})$, in accordance with the naive quark structure of hadrons. Since quarks and antiquarks do not correlate, the multiplicities of hadrons turn out to be independent on the momenta of hadrons. Then, the numbers of quarks (q,s) and antiquarks ($\bar{\mathrm{q}}, \bar{\mathrm{s}}$) and the coefficients of proportionality, the coalescence coefficients C_{p}, C_{Λ}, C_{Ξ}, C_{Ω} and $C_{\bar{\mathrm{p}}}$, $C_{\bar{\Lambda}}$, $C_{\bar{\Xi}}$, $C_{\bar{\Omega}}$, are free parameters of a simple coalescence model. Therefore, a total number of free parameters appearing in a simple coalescence model for the description of baryon and antibaryon production is equal to twelve. By the assumption $C_{\mathrm{p}}/C_{\bar{\mathrm{p}}} = C_{\Lambda}/C_{\bar{\Lambda}} = C_{\Xi}/C_{\Xi} = C_{\Omega}/C_{\bar{\Omega}} = 1$, the number of free parameters has been reduced to five $(\mathrm{q}, \mathrm{s}, \bar{\mathrm{q}}, \bar{\mathrm{s}}, \mathrm{C})$, where C is a common coefficient for baryon and antibaryon coalescence. Two of these free parameters have been fixed by fitting to the experimental data on baryon and antibaryon production: $\bar{\mathrm{q}}/\mathrm{q} = 0.41 \pm 0.02$ and $\bar{\mathrm{s}}/\mathrm{s} = 0.75 \pm 0.06$ [2]. Thus, there are three free parameters left in a simple coalescence model applied to the description of baryon and antibaryon production from the QGP phase. It is also important to note that a simple coalescence model [2] explains only multiplicities of baryon and antibaryon production. In fact, except for the ratio of multiplicities of the K^+ and K^- mesons, no other multiplicities of pseudoscalar and vector mesons have been predicted within a simple coalescence model [2]. Therefore, it is not completely clear how many free parameters would be added in a simple coalescence model for the description of multiplicities of pseudoscalar and vector meson production.

In our approach, where quarks and antiquarks coalescing into hadrons are correlated, we have six parameters T, $\mu(T)$, C_{M}, C_{B}, $C_{\bar{\mathrm{B}}}$ and F_S. Five of these parameters $T = 175\,\mathrm{MeV}$, $\mu(T)$ given by Eq. (4), $F_S = 3.5\,F_\pi = 458.5\,\mathrm{MeV}$, $C_{\mathrm{M}}/C_{\mathrm{B}} = 0.23 \pm 0.04$ and $C_{\bar{\mathrm{B}}}/C_{\mathrm{B}} = 0.46 \pm 0.15$ are fixed. Therefore, there is only one free parameter left in the approach. Thus, if we take into account that within our approach, we have described not only multiplicities of baryon and antibaryon production from the QGP but also multiplicities of pseudoscalar and vector meson production, all together 21 experimental ratios, our approach to the thermalized QGP looks much more successful than the simple coalescence model. In our approach, due to correlations between quarks and antiquarks, we are able to follow the dependence of multiplicities of hadron production on the hadronic momenta. The advantage of our approach with respect to a simple coalescence model becomes obvious. The former is also supported by the fact that, as has been shown above, our model, up to the definition of a phenomenological function $\rho(x_1, x_2)$, agrees with the valon-recombination model by Hwa et al. [11].

Now, let us discuss the role of gluons in our approach. It is to the full extent the same as in the approaches developed by Becattini and Heinz [15] and Cleymans [16]. Quarks and antiquarks become thermalized due to rescattering by gluons (Heinz) [15]. The Bose–Einstein distribution function for thermalized gluons provides also a concentration of the integrands of momentum integrals around $p \le T$. This realizes confinement of gluons in a spatial region $\Delta r \sim 1/T \sim 1\,\mathrm{fm}$.

In our approach, as well as in the approaches by Becattini, Heinz and Cleymans [15,16], a non-trivial contribution of gluons to hadron production can be described by two- and three-gluon correlation functions which can be represented by the momentum integrals analogous to Eq. (14)–(16)

$$\begin{aligned} G_{\rm gg}(\vec{q},T) &\propto \int \frac{{\rm d}^3 p}{(2\pi)^3}\,\frac{1}{{\rm e}^{p/T}-1}\cdot\frac{1}{{\rm e}^{|\vec{p}-\vec{q}|/T}-1}, \\ G_{\rm ggg}(\vec{q},T) &\propto \int \frac{{\rm d}^3 p}{(2\pi)^3}\,\frac{1}{\left({\rm e}^{p/T}-1\right)^2}\cdot\frac{1}{{\rm e}^{|\vec{p}-\vec{q}|/T}-1}. \end{aligned} \tag{33}$$

These states can occupy, for example, the size $V_{\rm gg} + V_{\rm ggg} \le V - \sum_{\rm h} V_{\rm h}$. Unfortunately, the analysis of the role of the gluon correlation functions Eq. (33) for hadronization from the QGP is rather complicated and goes beyond the scope of this paper.

This problem together with the account for contributions of wave functions describing the relative motion of quarks and antiquarks in multiplicities of hadron production should be the matter of further development and improvement of our coalescence model of correlated quarks. We are planning to realize this program in our forthcoming publications.

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TVORBA HADRONA U FAZI KVARK-GLUON PLAZME ULTRARELATIVISTIČIH SUDARA TEŠKIH IONA

Opisujemo kvark-gluon plazmu (QGP) kao ravnotežni sustav kvarkova i gluona, tj. termaliziranu QGP fazu kvantne kromodinamike. Hadronizacija ravnotežne QGP faze opisuje se slično modelu skupljanja s koreliranim kvarkovima i antikvarkovima. Ulazni parametri ovog pristupa su prostorni volumeni hadronizacije. Uvodimo tri bezdimenzijska parametra, $C_{\rm M}$, $C_{\rm B}$ i $C_{\bar{\rm B}}$, vezanih s prostornim volumenima tvorbe lakih mezona (M), bariona (B) i antibariona ($\bar{\rm B}$). Pokazujemo da su na temperaturi $T = 175$ MeV naša predviđanja za omjere višestrukosti u skladu s poznatim skupom omjera hadrona određenim mjerenjima Pb + Pb sudara u eksperimentima NA44, NA49, NA50 i WA97 na 158 GeV/nukleon, NA35 suradnje za S + S sudare i NA38 suradnje za O + U i S + U sudare na 200 GeV/nukleon.