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RECENT PROGRESS IN THE RELATIVISTIC DESCRIPTION OF FEW-BODY SYSTEMS

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Some features of relativistic few body calculations are very briefly reviewed, with emphasis on the covariant spectator theory and its unique ability to explain the high Q^2 measurements of the deuteron form factors.

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1. *Part I: Review of relativistic approaches*

First, a brief review is presented of the various choices of relativistic dynamics, and then how one of these choices (the covariant spectator theory) describes the deuteron form factor measurements recently completed at the Jefferson Laboratory (JLab). Then, in Part II, a very brief overview is given of the foundations and some other applications of the relativistic spectator theory.

1.1. Choices of dynamics

Several methods that have been recently used to treat few body systems relativistically are outlined and diagrammed in Fig. 1. There are two major approaches, with different options within each approach:

- Hamiltonian dynamics is based on the conventional quantum mechanics of systems with a fixed number of particles. Quantum mechanical states are vectors in a Hilbert space, initially defined on a surface in space-time. Dirac originally identified three distinct cases:
  - *Instant form* quantum mechanics, with states defined on the surface $t = 0$;
  - *Front form* quantum mechanics with states defined on the light front $t - z = 0$; and
  - *Point form* quantum mechanics with states defined on the hyperboloid $t^2 - r^2 =$ constant.

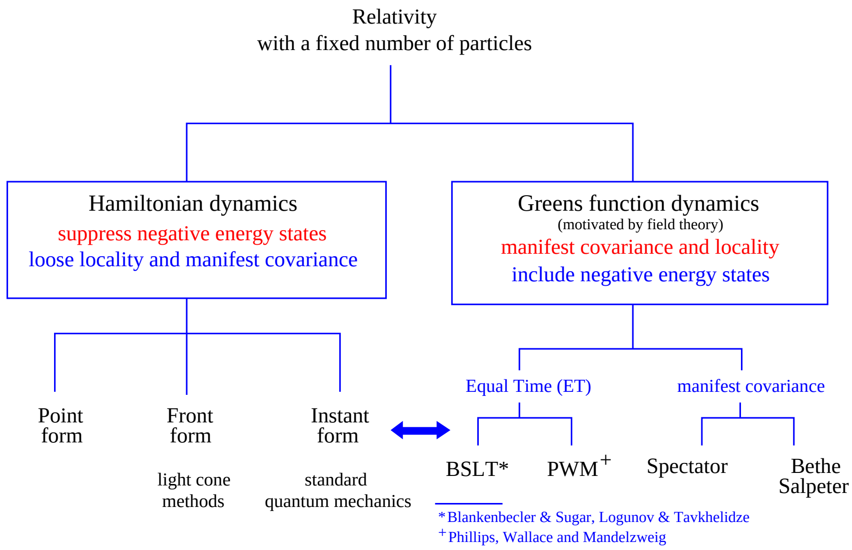

Fig. 1. Diagrammatic representation of the choices of relativistic dynamics discussed in the text.

In each of these cases some of the 10 generators of the Poincaré group carry the states away from the initial surface, and their action on the state cannot be evaluated without solving a generalized Schrödinger equation. The remaining are kinematic (i.e. do not require the solution of a dynamical equation). In the familiar instant form, for example, the effect of four of the Poincaré generators (the Hamiltonian and the three boost operators) all require solution of a dynamical equation, and for this reason the boost of a state cannot be calculated without first solving the dynamical problem. This is a significant disadvantage of the Hamiltonian forms of dynamics.

- Green's function dynamics is formulated in terms of the matrix elements of field operators. Two methods are particularly easy to describe in this language. When applied to a two-body bound state (for example, denoted by D for the deuteron)

  – the *Bethe-Salpeter (BS)* theory works with the matrix element

$$\Psi(x_1,x_2) \equiv \langle 0|\, T\left(\psi(x_1)\psi(x_2)\right) |D\rangle \ , \tag{1}$$

  – while the *covariant spectator theory* works with the matrix element

$$\Psi(x) \equiv \langle n|\, \psi(x_1)\, |D\rangle \ . \tag{2}$$

The so-called *equal time (ET)* formalisms are less easy to classify in this language, and I include them here because they are more similar to Green's function methods than to Hamiltonian methods. These include approaches developed by

- *Blankenbecler-Sugar and Logunov-Tavkhelidze (BSLT)*; and
- *Phillips, Wallace, Mandelsweig (PWM).*

Note that the BS matrix element connects the initial bound state to two final *off-shell* particles, while one of the final particles is *on-shell* in the spectator matrix element. Note also that the matrix elements (1) and (2) are manifestly covariant under *all 10 generators* of the Poincaré group. This considerable advantage can be maintained only by writing equations for these amplitudes that use covariant propagators, and this necessarily introduces negative energy states into the formalism. Those who prefer not to deal with negative energy states, and the indefinite norms that sometimes accompany them, must use one of the Hamiltonian formalisms.

The two *equal time* formalisms constrain the relative off-shell energy of the two equal mass intermediate particles to zero: $k_0 = (k_{10} - k_{20})/2 = 0$, but in all other respects are guided by field theory. The equal energy constraint introduces many simplifications, but violates covariance and cluster separability when applied to systems of more that two particles.

For references and a discussion and comparison of these methods see the recent review papers [1], [2] and [3].

1.2. Applications to the deuteron form factors

Recent measurements of the deuteron elastic scattering observables A, B, and $\tilde{t}_{20}$ have stimulated theory, and all of the relativistic methods mentioned in the previous section have been recently used to calculate these observables. Comparison with the new high precision data is very interesting; for a review see Refs. [1], [2] and [3]. Here I want to focus only on one aspect of this recent work, which illustrates the state of the art.

The exact current operator in the spectator theory is shown in Fig. 2 ; other

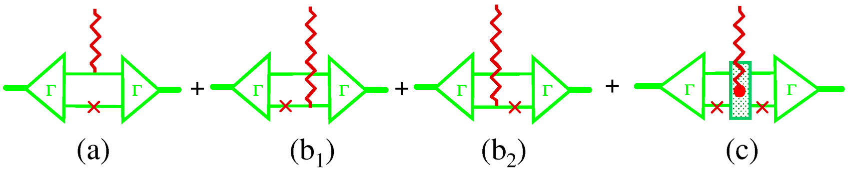

Fig. 2. Diagrammatic representation of two-body current for the deuteron form factors in the spectator theory. In diagram (a) the spectator is on-shell (denoted by the ×) and it is this configuration that gives the spectator theory its name. Diagrams (b_i) are nearly equal to (a), but the small differences are important for the proof of gauge invariance. Diagram (c) is the interaction current.

aspects of this theory will be discussed briefly in Part II of this talk. As the figure shows, the impulse diagrams (a) and (b) require knowledge of the off-shell single nucleon current. This feature is common to all relativistic approaches based on *composite hadronic* degrees of freedom (as opposed to *point-like quark* degrees of freedom), and it is important to realize that the needed off-shell nucleon current operators cannot be *uniquely* fixed by the theory. In the spectator theory [4] these currents are well constrained by the the Ward-Takashi identity

$$q_\mu \, j_N^\mu(p', p) = S^{-1}(p) - S^{-1}(p') \,, \tag{3}$$

where $S(p)$ is the propagator of a nucleon with four-momentum p. In applications of the spectator theory to the NN system, Ref. [5] uses a dressed nucleon propagator of the form

$$S(p) = \frac{h^2(p)}{m - \not{p}} = \frac{h^2(p)}{\Lambda_-(p)} \,, \tag{4}$$

where $h(p)$ is a phenomenological scalar function of p^2. The *simplest* solution of Eq. (3) gives the following one nucleon current

$$j_N^\mu(p', p) = F_0 \left\{ F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{\mathrm{i}\sigma^{\mu\nu} q_\nu}{2m} \right\} + G_0 F_3(Q^2)\Lambda_-(p')\gamma^\mu \Lambda_-(p) \,, \tag{5}$$

where terms proportional to q^μ have been dropped (they are required by the identity (3) but vanish when contracted into the conserved electron current) and the functions F_0 and G_0 are

$$\begin{aligned} F_0 &= \frac{h(p)(m^2 - p'^2)}{h(p')(p^2 - p'^2)} - \frac{h(p')(m^2 - p^2)}{h(p)(p^2 - p'^2)} \,, \\ G_0 &= \left[\frac{h(p')}{h(p)} - \frac{h(p)}{h(p')} \right] \frac{4m^2}{p^2 - p'^2} \,. \end{aligned} \tag{6}$$

This current has *one new form factor*, $F_3(Q^2)$, which is *completely undetermined* except for the requirement that $F_3(0) = 1$. This form factor must be determined phenomenologically by comparison with data, just as F_1 and F_2 are determined from the on-shell scattering measurements.

Another source of uncertainty is the famous $\rho\pi\gamma$ exchange current, which is included in the diagram in Fig. 2c. This current is separately gauge invariant and strongly dependent on the $\rho\pi\gamma$ form factor, $f_{\rho\pi\gamma}(Q^2)$ [6]. The value of $f_{\rho\pi\gamma}(0)$ is constrained by meson radiative decays, and its contribution to the deuteron form factors at small Q^2 is negligible. Its importance at high Q^2 depends strongly on the assumed dependence of $f_{\rho\pi\gamma}(Q^2)$, which is unknown but can be estimated from quark models. One of the best calculations of this form factor is that of the Rome group [7].

In Ref. [8] (denoted VODG), the current operator shown in Fig. 2 was used to calculate deuteron form factors and structure functions. The relativistic wave functions were taken from Ref. [5]. The VODG calculation used the "standard" dipole form for the unknown form factor F_3

$$F_3(Q^2) = \left(\frac{\Lambda^2}{\Lambda^2 - Q^2} \right)^2 \,, \tag{7}$$

with $\Lambda^2 = 0.71$ GeV^2.

For purposes of this discussion I define the VODG calculation in RIA (an approximation that replaces the sum of terms shown in Figs. 2b$_1$ and b$_2$ by that in Fig. 2a, and neglects all interaction currents shown in Figs. 2c, so that the total is twice that shown in Fig. 2a) as "standard," and consider the effect of (i) altering the Q^2 dependence of F_3, or (ii) adding the $\rho\pi\gamma$ exchange current. The "standard" calculation is illustated by the dotted lines shown in the top two panels and bottom right hand panel of Fig. 3. The effect of changing F_3 to a tripole form

$$F_3(Q^2) = \left(\frac{\Lambda^2}{\Lambda^2 - Q^2}\right)^3, \tag{8}$$

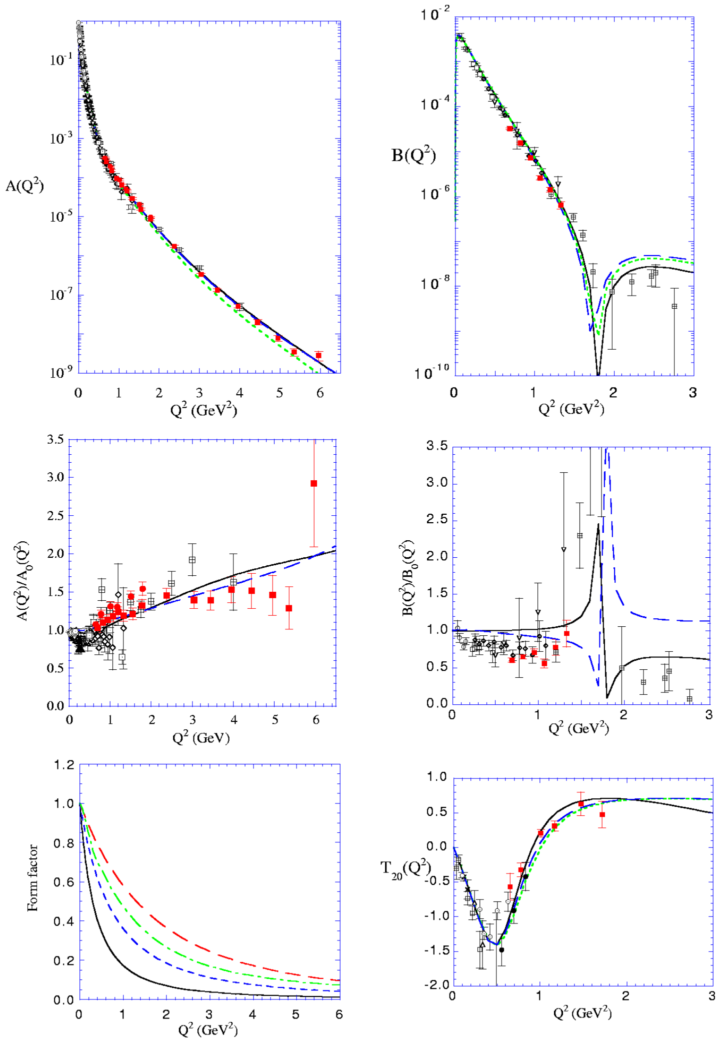

Fig. 3. Upper panels (A and B) and lower right panel ($\tilde{t}_{20}$) compare data to three theoretical models based on VODG: (i) "standard" case referred to in the text (dotted line), (ii) model with the tripole F_3 (solid line), and (iii) model with the dipole $f_{\rho\pi\gamma}$ (dashed line). The center two panels show the data and models (ii) and (iii) divided by model (i). The lower left panel shows form factors: standard dipole with $\Lambda^2 = 0.71$ (solid line), dipole with $\Lambda^2 = 1.5$ (short dashed line), Rome $f_{\rho\pi\gamma}$ (dot-dashed line) [7], and the tripole with $\Lambda^2 = 5$ (long dashed line).

with $\Lambda^2 = 5$ GeV^2 (and continuing to keep $f_{\rho\pi\gamma} = 0$) is shown by solid lines. Finally, the effect of adding a $\rho\pi\gamma$ exchange current using a dipole form factor with $\Lambda^2 = 1.5$ GeV^2 (and keeping the standard dipole form for F_3) is shown by the dashed curves. [The F_3 and $\rho\pi\gamma$ form factors themselves are compared to the Rome form factor and the standard dipole (7) in the lower left panel.] To see the effects on A and B more clearly, the middle two panels show the ratios of A/A_0 and B/B_0, where A_0 and B_0 are the standard calculation.

We see that a reasonable adjustment of F_3 at high Q^2 can give an excellent description of all of the elastic deuteron observables. This is a remarkable observation: there is no assurance that an F_3 that improves the fit to A will also improve the fit to B, and in fact we see that no choice of $f_{\rho\pi\gamma}$ improves the description of both A and B simultaneously. Somehow, an adjustment of the F_3 term is just what is needed to correct the theory. We emphasize that changing F_3 is perfectly permissible within the theory, since the form factor F_3, while it *must* be present, cannot be determined by on-shell data, and must be treated phenomenologically. From this point of view the deuteron data have now determined the unknown form factor F_3, and the isoscalar single nucleon current is now fixed. It remains to be seen whether the same F_3 will give excellent results for other electron scattering observables.

What are we to say about the $\rho\pi\gamma$ exchange current? My own belief is that this exchange current is being overestimated, even using the Rome form factor. While this current is certainly present, it is probably either negligible, or cancelled by the many other short range currents neglected in these calculations.

1.3. Conclusions to Part I

Some of the things we have learned from the struggle to understand the high Q^2 behavior of electron deuteron scattering (including some things not discussed here) are that

- relativistic calculations are essential at JLab energies – and JLab data have stimulated the development of the relativistic theory of composite few body systems;
- excitations to low mass final states (as in elastic electron–deuteron scattering, where the mass of the final state $W = M_d$) can be efficiently and correctly described by an effective theory based only on composite nucleon degrees of freedom;
- when W is large (in high energy photodisintegration, for example) additional physics, perhaps involving the explicit appearance of quark degrees of freedom, is needed;
- predictions will not be reliable unless the currents are constrained by the strong interaction dynamics (calculations must be consistent); and

- electromagnetic currents cannot be completely determined by an effective theory with composite degrees of freedom.

2. *Part II: Review of the covariant spectator theory*

The second part of this paper is a very superficial sketch of the spectator theory and its applications to few body systems.

2.1. Two and three-body covariant equations

Assuming that the dynamics of the two and three body systems is governed by an irreducible two-body kernel (with no irreducible three body kernels), the equations that describe two and three body systems of identical particles are illustrated in Figs. 4 and 5. In these diagrams the symbol $\times$ on each line means that that particle is on its mass shell, and the shaded box is the two body irreducible kernel.

The physical motivation for the spectator theory is best illustrated by examination of the Faddeev picture showing how the three body interaction emerges from the infinite sequence of successive two body scatterings. One of the infinite

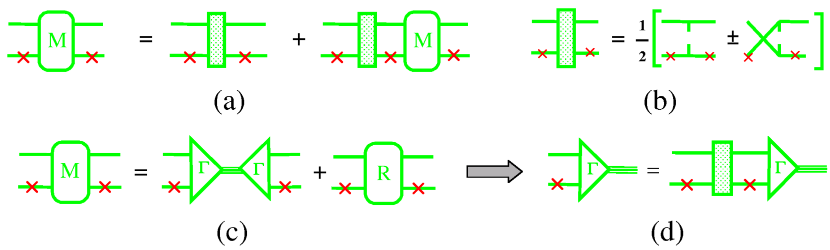

Fig. 4. Diagrammatic representation of two-body spectator equations: (a) The scattering equation, with (b) symmetrization of the one boson exchange potential for identical particles. If the scattering matrix has a bound state pole, represented by (c), then consistency of the scattering equation near the pole requires that the bound state vertex function Γ *(represented by the triangle) must satisfy equation (d).*

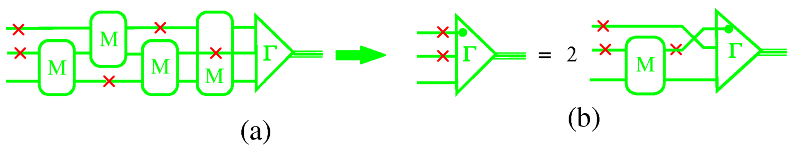

Fig. 5. Diagrammatic representation of three-body spectator equations: Figure (a) displays the definition of Γ^1_2*, the vertex function with particle 1 the spectator (always on shell) and particle 2 the on-shell interacting particle. This definition leads to equation (b) for the bound state vertex function* Γ^1_2*.*

sequence of these Faddeev diagrams is illustrated in Fig. 5a, which shows that the bound state vertex function Γ^1_2 is defined to be the infinite class of interactions that "end" with particle 1 as a spectator, and particle 2 also on-shell. If the three particles were not identical, this would lead to a coupled set of six equations for the six amplitudes Γ^i_j (with $i{\neq}j$ and $i, j \in \{1, 2, 3\}$), but if the particles are identical all of these amplitudes are related by particle interchange to a single amplitude that is the solution of the equations shown in Fig. 5.

This equation has been solved exactly for a family of one boson exchange models of nuclear forces [5, 9]. It was found that the three body interaction (as measured by the binding energy of the triton, E_T) is very sensitive to a parameter ν that measures off-shell coupling of isoscalar and isovector mesons to nucleons. Specifically, defining the scalar-NN coupling constant

$$g_s\Lambda(p', p) = g_s \left[1 + \nu\frac{(2m - \not{p}' - \not{p})}{2m}\right] , \tag{9}$$

so that the interaction depends on ν only when either the incoming or outgoing nucleon is off-shell, gives the ν dependence of the binding energy shown in the top panel of Fig. 6. Note that the experimental value of the binding energy is obtained

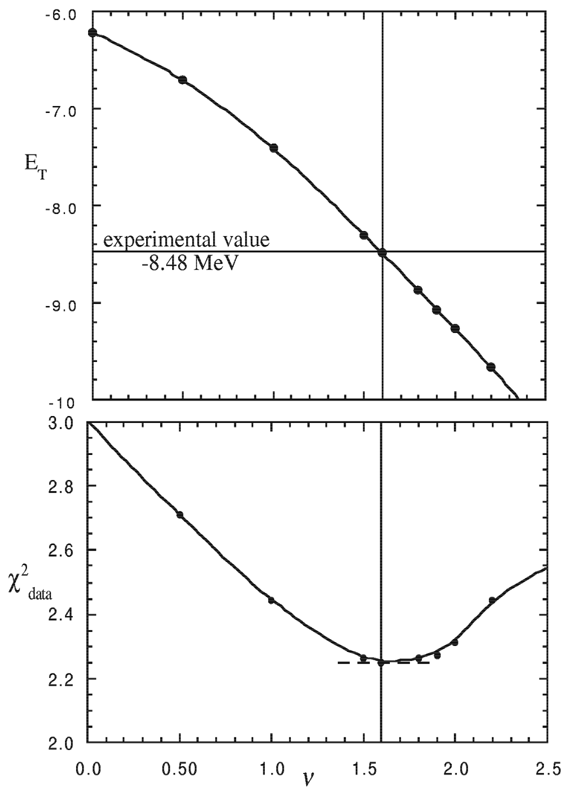

Fig. 6. Upper panel: Three body binding energy as a function of the off-shell coupling parameter ν. Lower panel: χ^2 for the fit to the two-body data as a function of the same parameter ν.

for the choice of $\nu \simeq 1.6$. The lower panel shows that the fit to the two body data is also improved as ν moves away form zero, and, remarkably, this same choice $\nu \simeq 1.6$ seems to also give the best fit to the two body data. The role of the parameter ν is discussed in some detail in Refs. [9] and [10].

2.2. Current operators for two and three body systems

The current operator for elastic two-body scattering was already displayed in Fig. 2 . The operator for two-body inelastic scattering is shown in Fig. 7. It has been shown that both of these currents are gauge invariant [4].

Construction of a three body current consistent with the Faddeev equation, Fig. 5b, is more difficult. This current operator was first derived by Kvinikhidze and Blankleider [11]. A more convenient form is currently being developed [12]. It has been shown that the three body normalization condition insures the conservation of charge.

An extensive review of the spectator theory is being prepared.

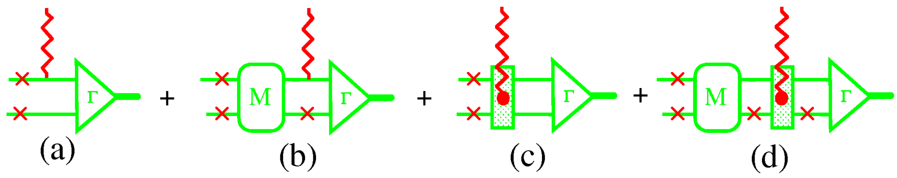

Fig. 7. Diagrammatic representation of two-body current for photo and electroproduction.

2.3. Conclusions to Part II

Space does not permit a more complete discussion here. The following features of the spectator theory have been discussed extensively in the literature:

- It is manifestly covariant, even under charge conjugation (if the total energy W is interpreted as $|W|$).
- It has a smooth nonrelativistic and one-body limit (if the mass of one particle becomes very large, the two-body equation reduces to a relativistic one body equation for the lighter particle moving in an instantenous potential).
- Bound states be normalized, and *gauge invariant* currents calculated consistently.
- Consistent solutions have been obtained for both two and three nucleon problems.

- Issues:
  - It has been shown (for neutral exchanges in the two-body sector) that restricting the heavy particle to its mass-shell gives the most accurate approximation to the generalized ladder (ladder and all crossed ladder) sum.
  - Identical particles are treated by explicitly symmetrizing the kernel. For one boson exchange models this introduces spurious singularities, which can be dealt with in a variety of ways.

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NOV NAPREDAK U RELATIVISTIČKOM OPISU MALOČESTIČNIH SUSTAVA

Kratko se razmatraju neke značajke relativističke teorije za maločestične sustave, s naglaskom na kovarijantnu teoriju promatrača i njenu jedinstvenu moć da objasni ishode za deuteronske faktore oblika na visokim Q^2.