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HADRONIC CORRELATIONS ABOVE THE CHIRAL/DECONFINEMENT TRANSITION

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The statistical bootstrap model is critically revised in order to include a medium-dependent resonance width in it. We show that a thermodynamic model with a vanishing width below the Hagedorn temperature $T_{\rm H}$ and a Hagedorn spectrum-like width above $T_{\rm H}$ may not only eliminate the divergence of the thermodynamic functions above $T_{\rm H}$, but also gives a satisfactory description the lattice quantum chromodynamics data on the energy density above the chiral/deconfinement transition as the main result of this contribution. This model allows us to explain the absence of heavy resonance contributions in the fit of the experimentally measured particle ratios at SPS and RHIC energies.

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1. *Introduction*

Ultrarelativistic heavy-ion collision experiments at SPS and RHIC are performed with the aim to create conditions of temperature and density under which hadronic matter undergoes a phase transition to the hypothetical quark–gluon plasma (QGP) and to investigate the properties of this state of matter once it was created. The strongest theoretical support for the existence of the QGP comes from lattice quantum chromodynamics (QCD) simulations at finite temperature T which show a step-like enhancement of the effective number of degrees of freedom (energy density in units of T^4) at a critical temperature $T_{\rm c}$, see Fig. 1. This be-

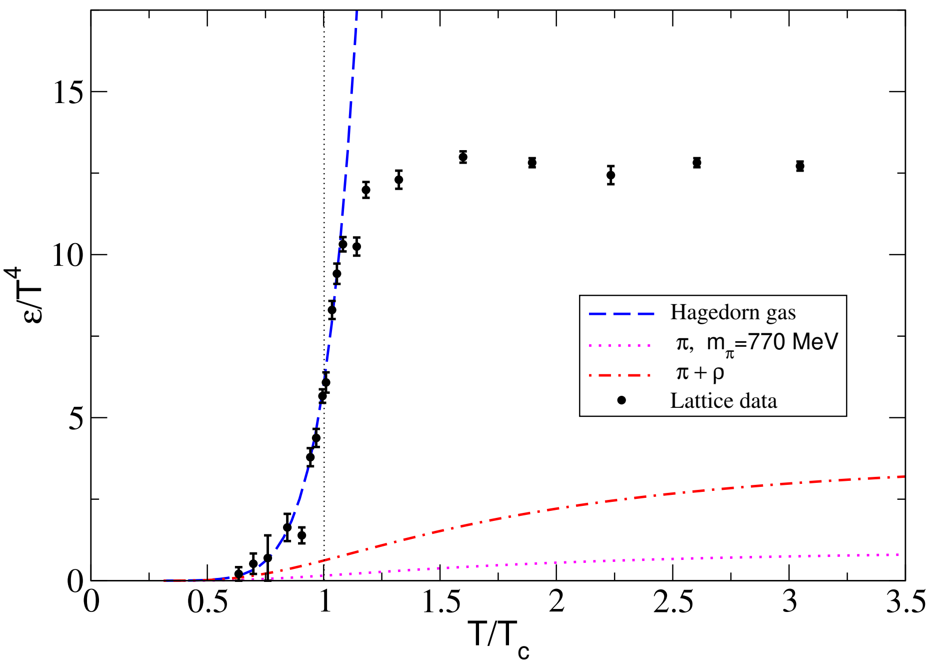

Fig. 1. Energy density of lattice QCD (circles) and Hagedorn gas (long dashed line) as functions of the reduced temperature $T/T_{\rm c}$. The contributions of pion gas (dotted line) and gas of pions and ρ mesons (dash-dotted line) are shown for a comparison. Note that the pion mass on the lattice is not the physical one.

havior is conventionally interpreted as the transition from a few mesonic degrees of freedom (mainly π and ρ mesons) to those of quasifree quarks and gluons. A microscopic description of this transition is, however, still missing. On the one hand, it has been shown [1] that a resonance gas model can perfectly explain the steep rise in the number of degrees of freedom at $T \approx T_{\rm c}$. On the other hand, lattice QCD has also revealed that hadronic correlations persist for $T > T_{\rm c}$ [2]. The question arises whether it is more appropriate to describe hot QCD matter in terms of hadronic correlations rather than in terms of quarks and gluons. In the present contribution, we introduce a generalization of the Hagedorn resonance gas (statistical bootstrap) model as such a description.

The statistical bootstrap model (SBM) [3 − 6] is based on the hypothesis that hadrons are made of hadrons, with constituent and compound hadrons being treated on the same footing. This implies an exponentially growing form of the hadronic mass spectrum $\rho_{\rm H}(m) \approx C_{\rm H} m^{-a} \exp\left\{m/T_{\rm H}\right\}$ for $m \to \infty$. The parameter $T_{\rm H}$, Hagedorn temperature, was interpreted as a limiting temperature reached at infinite energy density.

The extensive investigation of the SBM has led to a formulation of both the important physical ideas and the mathematical methods for modern statistical mechanics of strongly interacting matter. Thus it has been clarified that the original SBM requires two crucial modifications:

- hadrons should be considered as composite objects (the simplest way are hadrons as MIT bags [7] of quarks and gluons), and their proper volume v has to be taken into account [8,9] in the partition function;
- only $SU_c(3)$ color singlet clusters of quarks and gluons contribute [10 − 13] to the partition function of the system with their masses m_i and volumina v_i.

The SBM with proper volume was solved analytically by the Laplace transform to the isobaric ensemble [14] and, indeed, its solution showed various possibilities for the phase transition between the QGP and the hadron gas. Since then, this technique has been successfully applied not only to solve far more sophisticated versions [10 − 13,15] of the SBM, but also to find an analytical solution of a simplified version of the statistical multifragmentation model [16,17] for the nuclear liquid-gas phase transition. However, up to now the formulation of the SBM has encountered severe problems. The first one is the absence of a width for the heavy resonances. From the Particle Data Group [18], we know that heavy resonances with masses $m \geq 3.5$ GeV may have width comparable to their masses. Taking the widths into account will effectively reduce the statistical weight of the resonance. As we shall show below, this change may eventually remove the divergence of the SBM thermodynamics. The second problem arises while discussing the results of the hadron gas (HG) model [19,20]. The HG model describes remarkably well the light-hadron multiplicities measured in nucleus-nucleus collisions at CERN SPS [19] and BNL RHIC [20] energies. This model is nothing else than the SBM of light hadrons which accounts for the proper volume of hadrons with masses below 2.5 GeV, but neglects the contribution of the exponentially growing mass spectrum.

In other words, in order to calculate the particle ratios within the HG model it is, on the one hand, necessary to consider all strong decays of resonances according to their partial width collected in Ref. [18], and, on the other hand, it is necessary to truncate the hadron spectrum for masses above 2.5 GeV. Thus, one immediately faces the following problem: Why the heavy resonances with masses above 2.5 GeV predicted by the SBM do not appear in the particle spectra measured in heavy-ion collisions at SPS and RHIC energies? Note that the absence of heavy-resonance contributions in the particle ratios cannot be due to the statistical suppression of the Hagedorn mass spectrum because the latter should not be strong in the quark–hadron phase transition region, where those ratios are believed to be formed [19,20].

In the present contribution, we suggest that the introduction of a finite width of the resonances can solve the above problems of the SBM. In the next section we formulate a simple statistical model that incorporates besides the Hagedorn mass spectrum also medium dependent resonance widths due to the hadronic Mott effect, and analyze its mathematical structure. In Sect. 3 we discuss a model fit to recent lattice data of QCD thermodynamics [1] and some possible consequences for the heavy-ion physics.

2. *Resonance width model: Mott transition*

According to QCD, hadrons are not elementary, pointlike objects but rather color singlet bound states of quarks and gluons with a finite spatial extension of their wave function. While at low densities a hadron gas description can be sufficient, at high densities and temperatures, when hadronic wave functions overlap, nonvanishing quark exchange matrix elements between hadrons occur in order to fulfill the Pauli principle. This leads to a Mott–Anderson type delocalization transition with frequent rearrangement processes of color strings (string-flip [21]) so that hadronic resonances become off-shell with a finite, medium-dependent width. Such a Mott transition has been thoroughly discussed for light hadron systems in Ref. [22] and has been named *soft deconfinement*. The Mott transition for heavy mesons may serve as the physical mechanism behind the anomalous J/ψ suppression phenomenon [23].

We introduce the width Γ of a resonance in the statistical model with the Hagedorn mass spectrum through the spectral function

$$A(s,m) = N_s \frac{\Gamma\ m}{(s-m^2)^2 + \Gamma^2\ m^2}\ , \tag{1}$$

a Breit–Wigner distribution of virtual masses with a maximum at $\sqrt{s} = m$ and the normalization factor

$$N_s = \left[\int\limits_{m_b^2}^{\infty} \mathrm{d}s \frac{\Gamma m}{(s-m^2)^2 + \Gamma^2 m^2} \right]^{-1} = \frac{1}{\frac{\pi}{2} + \arctan\left(\dfrac{m^2 - m_b^2}{\Gamma m} \right)}\ . \tag{2}$$

The energy density of this model with zero resonance proper volume for given temperature T and baryonic chemical potential μ can be cast in the form

$$\begin{aligned} \varepsilon(T,\mu) \quad &= \sum_{i=\pi,\rho,\ldots} g_i\ \varepsilon_M(T,\mu_i;m_i) \\ &\quad + \sum_{A=M,B} \int\limits_{m_a}^{\infty} \mathrm{d}m \int\limits_{m_b^2}^{\infty} \mathrm{d}s\ \rho_H(m)\ A(s,m)\ \varepsilon_A(T,\mu_A;\sqrt{s}), \end{aligned} \tag{3}$$

where the energy density per degree of freedom with a mass m is

$$\varepsilon_A(T,\mu_A;m) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\sqrt{k^2+m^2}}{\exp\left\{ \dfrac{\sqrt{k^2+m^2} - \mu_A}{T} \right\} + \delta_A}\ , \tag{4}$$

with the degeneracy g_A and the baryonic chemical potential μ_A of hadron A. For mesons, $\delta_M = -1$, $\mu_M = 0$ and for baryons $\delta_B = 1$ and $\mu_B = \mu$. According to

Eq. (3), the energy density of hadrons consists of the contribution of light hadrons for $m_i < m_a$ and the contribution of the Hagedorn mass spectrum $\rho_{\rm H}(m)$ for $m \geq m_a$.

A new element of Eq. (3) in comparison to the SBM is the presence of the $\sqrt{s}$-dependent spectral function. The analysis shows that, depending on the behavior of the resonance width Γ in the limit $m \to \infty$, there are the following possibilities:

- For vanishing resonance width, $\Gamma = 0$, Eq. (3) evidently reproduces the usual SBM.
- For finite values of the resonance width, $\Gamma =$ const, Eq. (3) diverges for all temperatures T because, in contrast to the SBM, the statistical factor in Eq. (3) behaves as $\left[\exp\left\{(m_b - \mu_A)/T\right\} + \delta_A\right]^{-1}$ so that it cannot suppress the exponential divergence of the Hagedorn mass spectrum $\rho_{\rm H}(m)$.
- For a resonance width growing with mass like the Hagedorn spectrum $\Gamma \sim C_\Gamma \exp\left\{m/T_{\rm H}\right\}$ or faster, Eq. (3) converges again.

Indeed, in the latter case the Breit–Wigner spectral function behaves as

$$N_s \frac{\Gamma m}{(s-m^2)^2 + \Gamma^2 m^2}\Bigg|_{m\to\infty} \quad \to \quad \frac{2}{\pi\,\Gamma} \sim \exp\left(-\frac{m}{T_{\rm H}}\right) \tag{5}$$

and cancels the exponential divergence of the Hagedorn mass spectrum. Hence, the energy density remains finite. Note that both the analytical properties of model (3) and the right hand side of Eq. (5) remain the same if a Gaussian shape of the spectral function is chosen instead of the Breit–Wigner one.

It can be shown that the behavior of the width at finite resonance masses is not essential for the convergence of the energy density (3). In other words, for a convergent energy density (3) above $T_{\rm H}$ it is sufficient to have a very small probability density (5) (or smaller) for a resonance of mass m to be found in the state with the virtual mass $\sqrt{s}$. Since there is no principal difference between the high and low mass resonances, we can use the same functional dependence of the width Γ for all masses. Thus, for the following model ansatz

$$\Gamma(T) = \begin{cases} 0\,, & \text{for} \quad T \leq T_{\rm H}\,, \\ C_\Gamma \left(\dfrac{m}{T_{\rm H}}\right)^{N_m} \left(\dfrac{T}{T_{\rm H}}\right)^{N_T} \exp\left(\dfrac{m}{T}\right)\,, & \text{for} \quad T > T_{\rm H}\,, \end{cases} \tag{6}$$

the energy density (3) is finite for all temperatures and the divergence of the SBM is removed. At $T = T_{\rm H}$, depending on choice of parameters, it may have either a discontinuity or its partial T derivative may be discontinuous. As discussed above, for $T \leq T_{\rm H}$ such a model corresponds to the usual SBM, but for high temperatures $T > T_{\rm H}$ it remains finite for a wide choice of powers N_m.

3. *Applications for lattice QCD and heavy-ion collisions*

As one can see from Fig. 1, the Hagedorn gas model correctly reproduces the lattice QCD results below the critical temperature T_c and just in a vicinity above T_c, but not for large temperatures. Figure 2 shows a comparison of the same lattice QCD data [1] with the Mott–Hagedorn gas (6) where the parameters of the spectral function are $N_T = 2.325$, $N_m = 2.5$ and $T_H = 165$ MeV and $m_a = m_b = 1$ GeV. The successful description of the lattice energy density [1] indicates that above T_c the strongly interacting matter may be well described in terms of strongly correlated hadronic degrees of freedom. This result is based on the concept of soft deconfinement and provides an alternative to the conventional explanation of the deconfinement transition as the emergence of quasifree quarks and gluons.

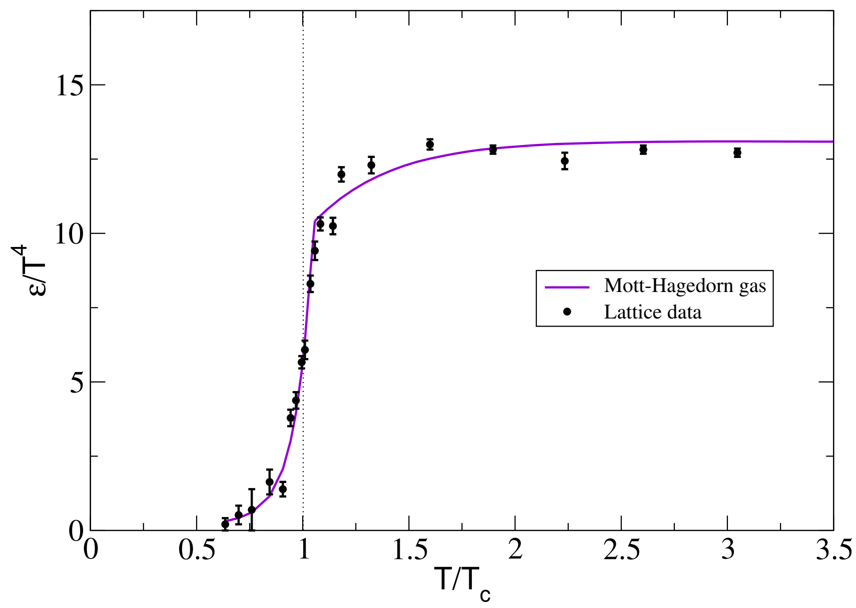

Fig. 2. Fit of the lattice QCD data [1] with the Mott–Hagedorn resonance gas model (6). For details see text.

Another interesting feature of the model (6) is that it allows a natural explanation for the absence of heavy resonance contributions to the particle yields measured at highest SPS and at all RHIC energies, where QGP conditions are expected [19,20]. In order to find out whether a given resonance has a chance to survive untill the freeze-out, it is necessary to compare its lifetime with the typical timescale in the system. There are two typical timescales usually discussed in nucleus-nucleus collisions, the equilibration time τ_{eq} and the formation time τ_f. The equilibration time tells when the matter created in collision process reaches a thermal equilibrium which allows the use of the hydrodynamic and thermodynamic descriptions.

For Au + Au collisions at RHIC energies, it was estimated to be about $\tau_{\rm eq} \approx 0.5$ fm [24]. On the other hand, in transport calculations, the formation time is used: the time for constituent quarks to form a hadron. The formation time depends on the momentum and energy of the created hadron, but is of the same order $\tau_{\rm f} \approx 1 - 2$ fm [25] as the equilibration time.

Since within our model the QGP is equivalent to a resonance gas with medium dependent widths, all hadronic resonances with life time $\Gamma^{-1}(m)$ shorter than $\max\{\tau_{\rm f}, \tau_{\rm eq}\}$ will have no chance to be formed in the system. Therefore, the upper limit of the the integrals over the resonance mass m and over the virtual mass $\sqrt{s}$ in Eq. (3) should be reduced to a resonance mass defined by

$$\Gamma(m)^{-1} = \max\{\tau_{\rm f}, \tau_{\rm eq}\}\,. \tag{7}$$

This reduction may essentially weaken the energy density gap at the transition temperature or even make it vanish. Thus, the explicit time dependence should be introduced into the resonance width model (3) while applying it to nuclear collisions, and this finite time (size) effect, as we discussed, may change essentially the thermodynamics of the hadron resonances formed in the nucleus-nucleus collisions.

4. *Conclusions and remarks*

The statistical bootstrap model allows the interpretation of the QGP as the hadron resonance gas dominated by the state of infinite mass (and infinite volume). We argue that it is necessary to include the resonance width into the SBM in order to avoid the contradiction with the experimental data on hadron spectroscopy. We found that the simple model (3)–(6), with a vanishing width below the Hagedorn temperature $T_{\rm H}$ and a Hagedorn spectrum-like width above $T_{\rm H}$, can eliminate the divergence of the thermodynamic functions above $T_{\rm H}$, but also it is able to successfully describe the lattice QCD data [1] for energy density with three fitting parameters only. The model also allows a natural explanation of the absence of heavy resonance contributions in the fit of the experimentally measured particle ratios at SPS and RHIC energies.

However, such a modification of the SBM requires an essential change in our view of QGP: it is conceivable that hadrons of very large masses, which should be associated with QGP, cannot be formed in nucleus–nucleus collisions because of their very short lifetime.

In the analysis of experimental data, the presented model should be applied with care: it can be successfully applied to describe either the quantities associated with the chemical freeze-out, i.e. particle ratios, or the spectra of Ω hyperons, ϕ, J/ψ and ψ' mesons that are freezing out at hadronization [26−29]. But, as discussed in Refs. [30−32], the model presented here should not be used for the post freeze-out momentum spectra of other hadrons produced in the nucleus-nucleus collisions. Perhaps only such weakly interacting hadrons like Ω, ϕ, J/ψ and ψ' will allow us to test the presented model.

The question how to derive the non-zero width below T_{H} has to be investigated. To solve this problem, it will be necessary to include into the present model a non-zero proper volume of hadrons. This, however, will require to consider a mixture of hadrons of different sizes, as studied recently in Refs. [33] and [34] or even a relativistic modification of the excluded volume of hadrons [34,35].

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References

[1] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C **29** (2003) 549.

[2] I. Wetzorke, F. Karsch, E. Laermann, P. Petreczky and S. Stickan, Nucl. Phys. Proc. Suppl. **106** (2002) 510; M. Asakawa, T. Hatsuda and Y. Nakahara, Nucl. Phys. A **715** (2003) 863.

[3] R. Hagedorn, Suppl. Nuovo Cimento **3** (1965) 147; R. Hagedorn and J. Ranft, Suppl. Nuovo Cimento **6** (1968) 169.

[4] R. Hagedorn, *The long Way to the Statistical Model*, in *Proc. NATO Adv. Research Workshop on Hot Hadronic Matter: Theory and Experiment*, eds. J. Letessier, H. H. Gutbrod and J. Rafelski, Divonne, France (1994) p. 13.

[5] S. Frautschi, Phys. Rev. D **3** (1971) 2821.

[6] A. Tounsi, *Statistical Bootstrap and Thermodynamical Model of High Energy Strong Interactions,* in *Phenomenology of Particles at High energy*, Academic Press (1974).

[7] A. Chodos et al., Phys. Rev. D **19** (1974) 3471.

[8] J. Baacke, Acta. Phys. Pol. B **8** (1977) 625.

[9] R. Hagedorn and J. Rafelski, Phys. Lett. B **97** (1980) 136.

[10] M. I. Gorenstein et al., Teor. Mat. Fiz. **52** (1982) 346.

[11] M. I. Gorenstein, S. I. Lipskikh and G. M. Zinovjev, Z. Phys. C **22** (1984) 189.

[12] J. Letessier and A. Tounsi, Phys. Rev. D **40** (1989) 2914.

[13] M. Kataja et al., Z. Phys. C **55** (1992) 153.

[14] M. I. Gorenstein, V. K. Petrov and G. M. Zinovjev, Phys. Lett. B **106** (1981) 327.

[15] J. Letessier, J. Rafelski and A. Tounsi, Phys. Lett. B **328** (1994) 499, and references therein.

[16] K. A. Bugaev et al., Phys. Rev. C **62** (2000) 044320; arXiv:nucl-th/0007062; Phys. Lett. B **498** (2001) 144; arXiv:nucl-th/0103075.

[17] P. T. Reuter and K. A. Bugaev, Phys. Lett. B **517** (2001) 233.

[18] Particle Data Group, Phys. Rev. D **66** (2002) 1.

[19] P. Braun–Munzinger, I. Heppe and J. Stachel, Phys. Lett. B **465** (1999) 15; G. D. Yen and M. I. Gorenstein, Phys. Rev. C **59** (1999) 2788; F. Becattini et al., Phys. Rev. C **64** (2001) 024901.

[20] P. Braun–Munzinger et al., Phys. Lett. B **518** (2001) 41; N. Xu and M. Kaneta, Nucl. Phys. A **698**, (2002) 306; W. Florkowski, W. Broniowski and M. Michalec, Acta Phys. Pol. B **33** (2002) 761.

[21] D. Blaschke, F. Reinholz, G. Röpke and D. Kremp, Phys. Lett. B **151** (1985) 439; G. Röpke, D. Blaschke and H. Schulz, Phys. Rev. D **34** (1986) 3499.

[22] J. Hüfner, S. P. Klevansky and P. Rehberg, Nucl. Phys. A **606** (1996) 260.

[23] G. Burau, D. Blaschke and Yu. Kalinovsky, Phys. Lett. B **506** (2001) 297.

[24] X.-N. Wang, M. Gyulassy and M. Plümer, Phys. Rev. D **51** (1995) 3436; R. Baier et al., Phys. Lett. B **345** (1995) 277.

[25] See, for instance, S. A. Bass et al., Prog. Part. Nucl. Phys. **41** (1998) 225.

[26] More references can be found in K. A. Bugaev, M. Gazdzicki and M. I. Gorenstein, Phys. Lett. B **523** (2001) 255; Phys. Lett. B **544** (2002) 127; Phys. Rev. C **68** (2003) 017901.

[27] M. I. Gorenstein, K. A. Bugaev and M. Gazdzicki, Phys. Rev. Lett. **88** (2002) 132301.

[28] S. Bass and A. Dumitru, Phys. Rev. C **61** (2000) 064909;

[29] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. **86** (2001) 4783; nucl–th/0110037.

[30] K. A. Bugaev, Nucl. Phys. A **606** (1996) 559; J. Phys. G **28** (2002) 1981; Phys. Rev. Lett. **90** (2003) 252301.

[31] K. A. Bugaev and M. I. Gorenstein, nucl-th/9903072.

[32] K. A. Bugaev, M. I. Gorenstein and W. Greiner, J. Phys. G **25** (1999) 2147; Heavy Ion Phys. **10** (1999) 333.

[33] M. I. Gorenstein, A. P. Kostyuk and Y. D. Krivenko, J. Phys. G **25** (1999) L75.

[34] G. Zeeb et al., arXiv:nucl-th/0209011.

[35] K. A. Bugaev et al., Phys. Lett. B **485** (2000) 121.

HADRONSKE KORELACIJE IZNAD PRIJELAZA KIRALNOST – ODSUŽNJIVANJE

Kritički prerađujemo statistički model samopodizanja kako bi u njega uključili širine rezonancija ovisne o sredini. Pokazujemo da termodinamički model s iščezavajućom širinom ispod Hagedornove temperature T_{H}, i sa spektralnom širinom poput Hagedornovove iznad T_{H}, može ukloniti divergenciju termodinamičkih funkcija iznad T_{H}, ali on također daje dobar opis podataka kvantne kromodinamike na rešetci za gustoću energije iznad prijelaza kiralnost – odsužnjivanje, što je glavni doprinos ovog rada. Taj model objašnjava nepojavljivanje teških rezonancija u mjerenjima eksperimentalnih čestičnih omjera na energijama SPS i RHIC.