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LETTER TO THE EDITOR

HYPERFINE STRUCTURE IN Ω-NUCLEUS SYSTEM

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Dedicated to the memory of Professor Dubravko Tadić

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We discuss the hyperfine structure of a bound state of the Ω hyperon and a nucleus with an electric quadrupole moment. We note a new type of contact term, analogous to the Fermi contact interaction for magnetic dipoles, this time in P-wave states.

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The Ω hyperon decays weakly, so that it can form exotic atoms by Coulomb binding to nuclei. Due to the angular momentum of 3/2, it can have (in addition to magnetic dipole moment) an electric quadrupole and also a magnetic octupole moment. It is a challenge to determine these higher multipole moments.

Sternheimer and Goldhaber [1] proposed long time ago the study of fine structure of Ω^--Pb atoms as a method to measure the magnetic moment and electric quadrupole moment of Ω^-. At that time, the electromagnetic properties of the hyperons were not yet known and the authors assumed that the magnetic moment is a few magnetons and the quadrupole moment a few fm^2. With these values, the fine structure would be dominated by the quadrupolar coupling, while the magnetic interaction would be a small perturbation. The magnetic moment of the Ω hyperon has now been measured [2] and is somewhat larger than previously assumed, but the quadrupole moment, while not measured, has been estimated [3] in "realistic" quark models to be a few hundreths of a fm^2, much smaller than assumed in Ref. [1]. Thus, the fine structure of the Ω^--Pb atom will be dominated by the magnetic moment, contrary to Ref. [1] (see also Ref. [4]). There are no Ω^--atom data as yet, but the data will come. We propose the *hyperfine* structure of Ω exotic atoms as a method to measure the Omega quadrupole moment. One needs an exotic atom whose nucleus has a quadrupole moment which couples to the quadrupole moment of the Ω^-. Our discussion here while not complete, describes the appearance of a new type of contact coupling for quadrupoles, analogous to the Fermi contact interaction for dipoles [5].

The interaction of two electric quadrupoles is familiar for two homopolar molecules. The lowest non-zero multipole moment for a homopolar molecule, like the hydrogen molecule, is the electric quadrupole, and these interactions have been studied extensively by molecular physicists [6].

The interaction of two quadrupoles Q^a and Q^b may be written in terms of cartesian components

$$\begin{aligned} H_{\text{tensor}} &= \frac{1}{48\pi} \sum Q^a_{ij} \partial_i \partial_j \left(r_k r_l \frac{Q^b_{kl}}{r^5} \right) \qquad (1) \\ &= \frac{1}{48\pi r^5} \left[2\, Q^a \cdot Q^b - 20\, Q^a_{ij} Q^b_{ik} n_i n_k + 35\, Q^a_{ij} Q^b_{kl} n_i n_j n_k n_l \right]. \end{aligned}$$

In Eq. (1), repeated indices are summed; in the first term, the two quadrupoles are fully contracted and in the next terms $n_i = r_i/r$, etc. The interaction of two quadrupoles decreases very rapidly with the distance, like r^{-5}, as follows from dimensions, much faster than the fine structure coupling, which decreases only like r^{-3}. As a result, the quadrupolar coupling is more important for smaller orbits, at low orbital quantum numbers. Therefore, we will focus on low lying P-states of the atom. It can be checked that the tensor quadrupolar interaction (Eq. (1)) is ambiguous in P-states of the atom; the angular averaging vanishes because the angular dependence of the interaction (when expressed in spherical harmonics) involves $Y_{4m}(\theta, \phi)$, where the angles θ and ϕ are those of the vector connecting the two quadrupoles [6]. The expectation value of a spherical harmonic Y_4 vanishes in a P-wave state. The radial integration on the other hand diverges. This ambiguity is similar to the case of the dipole-dipole interaction in S-waves, where the Fermi contact interaction solves the ambiguity. Similarly here, instead of the tensor interaction of two quadrupoles we have a contact term, involving the derivative of a delta function, which generalizes the Fermi contact interaction where there is a delta function. Although the quadrupolar contact interaction follows from the Coulombs law and usual quantum mechanics, it has not been described before, as far as we know. The quadrupolar contact coupling is a scalar interaction coupling the two quadrupole tensors and second derivatives of a delta function (as required by dimensional considerations) to a scalar. The precise formula for the quadrupolar contact term is given below

$$H_{\text{contact}} = -\frac{1}{36} \sum Q^a_{ij} Q^b_{ik} \left[\frac{4}{7} \left(\partial_j \partial_k - \frac{1}{3}\, \delta_{jk} \Delta \right) \delta^3(\boldsymbol{r}) + \frac{2}{15}\, \delta_{jk} \Delta\, \delta^3(\boldsymbol{r}) \right]. \qquad (2)$$

The derivation of Eq. (2) will be given elsewhere. In this formula Q^a and Q^b are the quadrupole moments of the hyperon and the nucleus, respectively, and Δ is the Laplacian operator.

We illustrate the effect of this contact interaction for a simple system: a nucleus of spin one and an omega hyperon in a P-wave orbit around this nucleus. The total degeneracy of the system is $4 \times 3 \times 3 = 36$ states. These states split into 8 multiplets of total angular momentum F, which takes the values $F = 1/2, 1/2, 3/2, 3/2, 3/2, 5/2, 5/2$ and $7/2$. Check:$36 = 2+2+4+4+4+6+6+8$. Even in this simple case, the detailed spectrum depends on the relative size of the magnetic and electric interactions. To compute the spectrum, we assume that the magnetic fine structure dominates over the quadrupole-quadrupole hyperfine interaction.

This is the case for light nuclei; a simple example is the ^{14}N nucleus which has nuclear spin one. To start with, we couple the spin of the Ω, S^{Ω} to the orbital angular momentum L to obtain an intermediate angular momentum J, which can take the values $1+3/2 = 5/2, 3/2$ or $1/2$. Finally, J couples to the nuclear spin I to obtain a total angular momentum F. If the magnetic fine structure dominates over electric quadrupole interactions, one can neglect the mixing of states with different J. Then the states at $J = 1/2$ remain degenerate ($E(J = 1/2, F = 1/2) = E(J = 1/2, F = 3/2)$), because they cannot have a quadrupolar interaction and we have to find the splitting of the multiplets at $J = 3/2$ (with $F = 1/2, 3/2$ and $5/2$) and of the multiplets at $J = 5/2$ (with $F = 3/2, 5/2$ and $7/2$). The computation of these splittings is a little tedious and here we quote only the results for the case nuclear spin $I = 1$ and P-wave ($L = 1$). The contact energy ΔE_{QQ} has the form

$$\Delta E_{QQ} = -\frac{1}{21}\,|f(0)|^2\, Q^{\rm N} Q^{\Omega} Z(J)\, T(J,F)\,, \tag{3}$$

where

$$|f(0)|^2 = \frac{1}{3\pi}\,\frac{n^2-1}{n^5}(m_{\Omega} Z\alpha)^5\,,$$

$$T(J,F) = \frac{1}{2}\left[4\,(J\cdot I)^2 + 2(J\cdot I)\right] - \frac{2}{3}\,J(J+1)I(I+1)\,,$$

$$Z(J) = \frac{1}{J(J+1)}$$

$$\times\left\{\frac{13}{45}\left(J(J+1)-\frac{7}{4}\right)-\frac{1}{16}\left(J(J+1)-\frac{25}{4}\right)\left(J(J+1)+\frac{49}{4}\right)-\frac{15}{32}\left(J(J+1)-\frac{77}{12}\right)\right\}.$$

From these formulae, we find $Z(3/2) = 1.15$ and $Z(5/2) = -0.02$. This implies that only when $J = 3/2$ is the splitting significant.

For magnetic fine structure, we follow the equations given in Ref. [1]; denoting the magnetic fine structure energy $E_{\rm mag}$, it is expressed by the equation

$$E_{\rm mag} = RZ^4\alpha^2\,\frac{1}{n^3 L(L+1/2)(L+1)}\,\boldsymbol{L}\cdot\boldsymbol{S}\,(g_0+2g_1)\,. \tag{4}$$

In Eq. [4], R is 44.6 keV, the Rydberg energy for an Ω-hydrogen, the two hydrogenic quantum numbers are $L = 1$ for P-waves and $n = 2$ for the lowest P-wave state. The quantities g are defined by Bethe and Salpeter [7] and are used to describe the Dirac and anomalous magnetic moment of the Ω^- (in Ω magnetons). Using the measured magnetic moment of Ω^- of (minus) 2.02 nuclear magnetons we, find for the last bracket in Eq. [4] the value 6.2 about double the value assumed in Ref. [1]. We have not corrected for reduced masses as our estimates are approximate. As a rough estimate, the ratio of the quadrupolar contact energy to the magnetic energy of Eq. [4] can be estimated to be about 10^{-2} for a nucleus with $Z = 7$ and quadrupole moments (for the hyperon and the nucleus) of order 10^{-2} fm^2. This makes quadrupolar energies of the order of a keV. These preliminary estimates will be expanded elsewhere, where the choice of the most favourable target nucleus will also be discussed. We believe that the hyperon quadrupole moment can be measured in this way.

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HIPERFINA STRUKTURA SUSTAVA Ω-JEZGRA

Raspravljamo o hiperfinoj strukturi vezanog stanja Ω hiperona i jezgre s električnim kvadrupolnim momentom. Ističemo novu vrstu kontaktnog međudjelovanja, koja je slična Fermijevom kontaktnom međudjelovanju za magnetske dipole, ali ovdje u P-stanjima.