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COMPOUND MULTIPLICITY DISTRIBUTION IN NUCLEUS-NUCLEUS INTERACTIONS – PHASE TRANSITION STUDY

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Experimental data on compound multiplicity distribution produced in ^{24}Mg-AgBr and ^{12}C-AgBr interactions, both at $4.5A$ GeV, have been analysed in terms of scaled factorial moments. We have calculated the values of generalized dimension D_q for $q = 2, 3, 4, 5, 6$ and 7 with the help of intermittency exponent α_q. From the knowledge of D_q, the multifractal specific heat is calculated for different multiplicity bin ranges. The parameter λ_q ($\lambda_q = (\alpha_q + 1)/q$) is calculated from α_q to look for possible non-thermal phase transition. The analysis reveals no evidence of non-thermal phase transition. Instead, different specific heat in different multiplicity bins has been observed.

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1. Introduction

In the field of multiparticle production process in high-energy hadronic or nuclear interactions, the concept of intermittency, a new methodology, was first introduced by Bialas and Peschanski [1 − 2]. Intermittency is signaled by the power-law behaviour of the scaled factorial moments (SFM) with increasing spatial resolution of the particle detection. The pioneers suggested a method which involves the statistical counting variable called the scaled factorial moment F_q that is the main tool for studying intermittency. The definition of SFM is obtained from turbulence theory [3] where it gives a measure of the amount of intermittency in a turbulent system. It has been shown that the average scaled factorial moment [1] is equal to the moment of a true probability distribution of particle density, without any statistical bias.

According to Bialas and Peschanski, power-law growth of the factorial moments with decreasing phase space interval size signals the onset of intermittency in the context of high-energy interactions. Intermittency differs from Gaussian fluctuation with the self-similarity at various scales. It has been conjectured that intermittency is a general mechanism for producing large local fluctuations, may be a manifestation of phase transition [4 − 5]. But no unambiguous experimental evidence has yet been obtained, which has shown that phase transition and similar mechanisms occur in particle production process. The nature of such phase transition is another crucial problem to be addressed for a meaningful understanding of the dynamics of particle production.

In most of previous works on intermittency, linear best fits were drawn in the total bin range from some pre-conceived ideas. The properties of intermittency have been studied [6] to investigate the structure of different phases of a self-similar multiparticle system using the random cascade model. It is a matter of fact that a self-similar cascade is not consistent with particle production during one phase but instead requires a non-thermal phase transition [7]. The signals of non-thermal phase transitions can be studied with the help of the parameter $\lambda_q = (\alpha_q + 1)/q$ where α_q is the intermittency exponent of q-th order. The condition for non-thermal phase transition may occur when the function λ_q has a minimum value at $q = q_c$ [5 − 7]. The regions $q < q_c$ and $q > q_c$ are dominated by numerous small fluctuation and rare large fluctuations, respectively. There is a co-existence of the liquid phase of the many small fluctuations and the dust phase of a few grains of very high density depending on whether we probe the system by a moment of order $q < q_c$ or $q > q_c$, respectively.

Some simple thermodynamic approximations could be applicable to the multiparticle production process. Recently Bershadskii [8] showed that the constant specific heat approximation (CSH), which is widely applicable in the ordinary thermodynamics, is also applicable to the multifractal data. According to Bershadskii, we have also calculated the multifractal specific heats for both the interactions in emission angle space individually from the knowledge of generalized dimensions D_q obtained from the scaled factorial moment (F_q). We have divided the full bin range into several sub-intervals to observe the existence of non-thermal phase transition.

Investigations, generally in high-energy nuclear collisions, are carried out on the produced pions with a common belief that these particles are the most informative as they are produced immediately after the collisions. Very few attempts have been made on the medium energy (30 − 400 MeV) knocked-out target protons, which are supposed to carry some information about the interaction dynamics, because the time scale of emission of these particles is of the same order ($\approx 10^{-22}$ s) as that of the produced particles. These target protons, which manifest themselves as grey tracks in nuclear emulsion, are the low energy part of the internuclear cascade formed in high-energy interactions. If the number of fast target fragments of an event generally known as the grey particles in emulsion media, is combined with produced pions of the same event, known as the shower tracks in the same media, a new parameter named as "compound multiplicity" ($n_c = n_g + n_s$ where $n_c =$ compound multiplicity, $n_g =$number of grey tracks and $n_s =$number of shower tracks) is formed which can play an important role in understanding the reaction dynamics in high-energy nuclear interactions. So far, very few attempts have been made to work with this parameter [9]. The behaviour of the compound multiplicity spectra thoroughly is to be probed with the available sophisticated tools.

In this paper, we have analyzed the fluctuation of compound multiplicity produced in ^{24}Mg-AgBr and ^{12}C-AgBr interactions, both at 4.5 A GeV, in emission angle space in the framework of scaled factorial moment. Also, the multifractal specific heat has also been studied in both data sets.

2. Experimental details

The data set used in this present analysis was obtained by exposing NIKFI-BR2 emulsion plates by ^{24}Mg and ^{12}C beam, both at incident energy of 4.5 A GeV at JINR Dubna, Russia. The scanning of the plates is carried out with the help of a high-resolution Leitz metalloplan microscope provided with semi-automatic scanning and measuring system. The scanning is done using objective 10× in conjunction with a 25× ocular lens. To increase the scanning efficiency, two independent observers scanned the plates independently. For measurement, 100× oil immersion objective is used in conjunction with 25× ocular lens. The measuring system fitted with it had 1 μm resolution along the X and Y axes and 0.5 μm resolution along the Z-axis. From the scanned events only those events are chosen which satisfy the following criteria:

a) The beam track should lie within the angle 3° to the mean beam direction of the pellicle.

b) The events, which are within 20 μm thickness from the top or bottom surface of the plate, should be rejected.

c) The events for which the primary beam track is observed to be a secondary track of other interaction should not be analysed and are rejected.

According to the emulsion technique, the particles emitted after interactions are classified as:

a. Black particles: They are target fragments with ionization greater or equal to $10I_0$, I_0 being the minimum ionization of a singly-charged particle. Their range is less than 3 mm, velocity less than $0.3c$ and energy less than 30 MeV, where c is the velocity of light in vacuum.

b. Grey particles: They are mainly fast target recoil protons with energy up to 400 MeV. They have ionization $1.4I_0 \leq I < 10I_0$. Their range is greater than 3mm and have velocities $0.3c \leq V \leq 0.7c$.

c. Shower particles: The relativistic shower tracks with ionization less than or equal to 1.4 I_0 are namely produced by pions and are not generally confined within the emulsion pellicle.

d. The projectile fragments are a different class of tracks with constant ionisation, long range and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy-ionizing tracks of (black+grey). For our present analysis, we have considered the combination of grey and shower tracks for studying the characteristics of compound multiplicity distribution. According to the above selection procedure, we have chosen 800 events of ^{24}Mg-AgBr interactions and 792 events of ^{12}C-AgBr interactions both at 4.5A GeV. The emission angle (θ) is measured for each track by taking the coordinates of the interaction point (X_0, Y_0, Z_0), coordinates (X_1, Y_1, Z_1) at any point of the linear portion of each secondary track and coordinate (X_i, Y_i, Z_i) of a point on the incident beam. It is worthwhile to mention that emulsion technique possesses very high spatial resolution which makes it a very effective detector for studying the intermittency phenomenon.

3. Method of study

We here adopt the scaled factorial moment method to investigate the fluctuation pattern of compound multiplicity distribution. The number of particles considered is finite at the available projectile energy. Hence, the statistical fluctuation is very much obvious and the standard moments fail to reveal the dynamical fluctuation of particle density distribution. The scaled factorial moment method is free from the hazards of statistical noise pollution.

Consider a certain interval in some 'x' space, defined as $\Delta x = x_{\rm max} - x_{\rm min}$. The considered region is divided into M bins of equal size having width $\delta x = \Delta x/M$. The normalized scaled factorial moment is defined as [1]

$$\langle F_q \rangle = \langle M^{q-1} \sum_{m=1}^{M} \frac{k_m(k_m-1)\cdots(k_m-q+1)}{\langle k_m \rangle^q} \rangle \,, \tag{1}$$

where k_m is the number of particles in the $m^{\rm th}$ bin $(m = 1, 2, \cdots M)$ and angular bracket denotes an average over the whole sample of events, q is the order of moment which can assume any positive integral value starting from 2.

If the non-statistical fluctuations are self-similar in nature, in the limit of small bin size, factorial moment obeys the relation

$$\langle F_q \rangle \propto M^{\alpha_q} \,. \tag{2}$$

From (2) we get

$$\ln \langle F_q \rangle = \alpha_q \ln M + C \,. \tag{3}$$

In analogy with turbulent fluid dynamics, this property is called "intermittency". In connection with high-energy interactions, α_q measures the strength of intermittency and is called the intermittency exponent. From the intermittency exponent α_q, one can find out the signals of non-thermal phase transition with the help of the relation

$$\lambda_q = \frac{1 + \alpha_q}{q} \tag{4}$$

The generalized dimension (D_q) is calculated from α_q by using the relation

$$D_q = 1 - \frac{\alpha_q}{q-1} \,. \tag{5}$$

It is well known that in usual thermodynamics, constant specific heat (CSH) approximation is widely applicable in many important cases, e.g., the specific heat of gases and solids is constant, independent of temperature, over a smaller or greater temperature interval [10]. This approximation is applicable to multifractal data of multiparticle production process, as has already been shown by Bershadskii [8]. If the q-dependence of D_q satisfies the condition $D_q > D_{q'}$ for $q < q'$, then the spectra are said to exhibit multifractality, and the multifractal specific heat can be obtained from the generalized dimension, D_q by the relation [8].

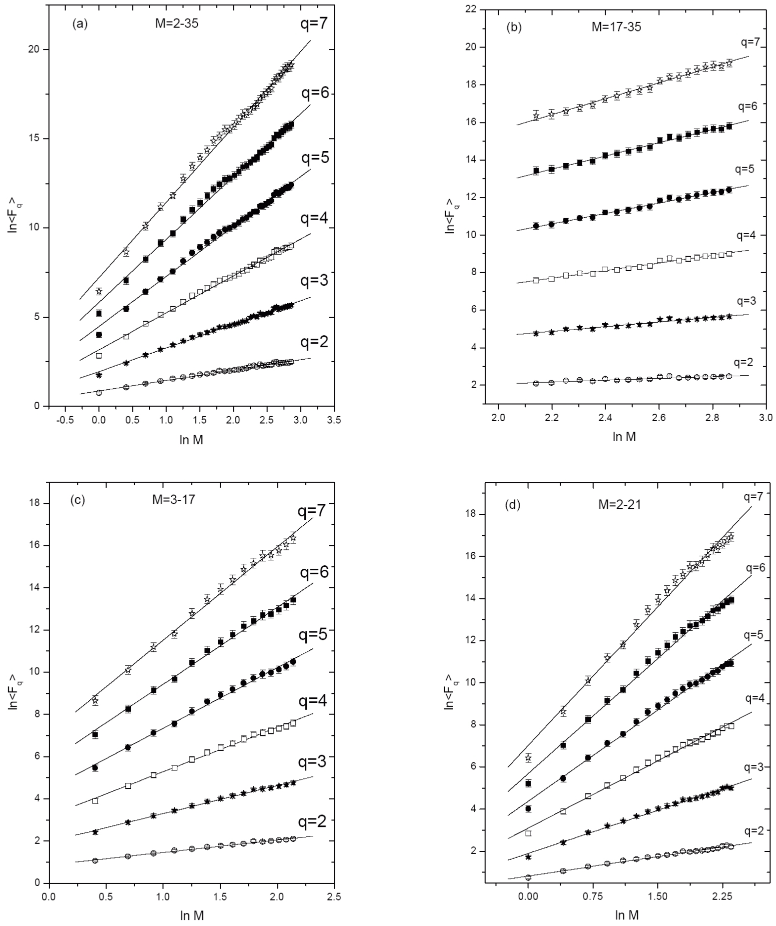
$$D_q = (a - c) + c \ln q/(q - 1) \quad \text{for} \quad q > 1\,, \tag{6}$$

where c is the specific heat, a is some other constant and q can be interpreted as the inverse of temperature [10].

4. *Results and discussion*

To analyze and measure the factorial moments in the emission angle ($\cos\theta$)-space in compound multiplicity, the two type of data sets (^{24}Mg-AgBr and ^{12}C-AgBr interactions, both at 4.5A GeV), have been divided into 2 to 35 subintervals ($M = 2$ to 35). The normalized scaled factorial moments of order 2 to 7 are calculated for each interval with Eq. (1). Figure 1a shows the nature of the variation of normalized scaled factorial moment of different orders ($q = 2$ to 7) with logarithm of the number of multiplicity bins in the emission-angle space for ^{24}Mg-AgBr interactions at 4.5A GeV. Figure 2a shows the same plot for ^{12}C-AgBr interactions at 4.5A GeV. Both plots do not show perfect linear behaviour in the full bin range. Therefore, we divide the full bin range into parts to probe whether the linear behaviour is prominent in different subintervals of M. Variation of the factorial moment with $\ln M$ in selected intervals ($17 \le M \le 35$, $3 \le M \le 17$, $2 \le M \le 21$ and $9 \le M \le 18$) gives a clear picture of the intermittent pattern of particle production. Those are represented in Figs. 1b – e and 2b – e, respectively, for the ^{24}Mg-AgBr and ^{12}C-AgBr interactions, both at 4.5A GeV.

The intermittency exponents α_q's are evaluated by performing best linear fits according to Eq. (5). Table 1 shows the value of the intermittency exponents α_q with different orders in $\cos\theta$-space for different bin ranges in ^{24}Mg-AgBr interactions at 4.5A GeV. Table 2 shows the same thing for ^{12}C-AgBr interactions, also at 4.5A GeV. It is evident from the tables that the value of the intermittency exponents gradually increases with the increase of orders for all multiplicity bin ranges in both data sets. Here we have analysed the scaled factorial moments in $\cos\theta$-space to search for the signals of non-thermal phase transition. λ_q-values are calculated from the value of the intermittency exponents according to the relation (4). The results of our study on the variation of λ_q with q for ^{24}Mg-AgBr



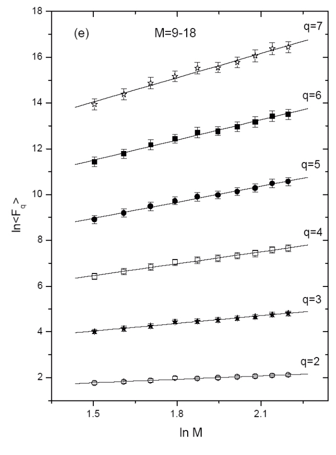


Fig. 1. Dependence of $\ln\langle F_q\rangle$ *on* $\ln M$ *for different bin ranges: (a)* $2 \le M \le 35$, *(b)* $17 \le M \le 35$, *(c)* $3 \le M \le 17$, *(d)* $2 \le M \le 21$ *and (e)* $9 \le M \le 18$, *for different order of moments (*$q = 2$, *3, 4, 5, 6 and 7) in* ^{24}Mg-AgBr *interactions at 4.5A* GeV *in* $\cos\theta$*-space.*

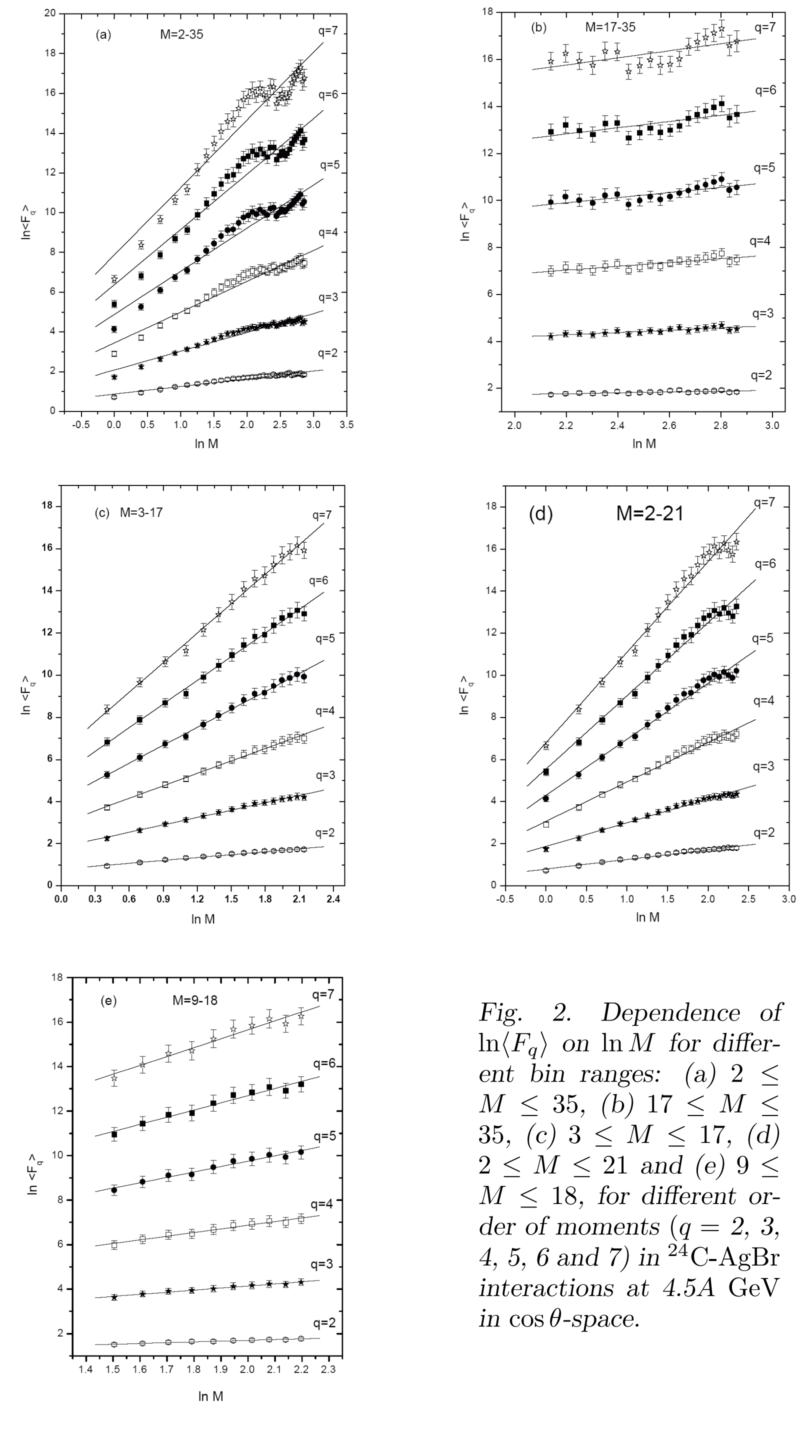


Fig. 2. Dependence of $\ln\langle F_q\rangle$ *on* $\ln M$ *for different bin ranges: (a)* $2 \leq M \leq 35$*, (b)* $17 \leq M \leq 35$*, (c)* $3 \leq M \leq 17$*, (d)* $2 \leq M \leq 21$ *and (e)* $9 \leq M \leq 18$*, for different order of moments (*q = 2, 3, 4, 5, 6 *and* 7*) in* ^{24}C-AgBr *interactions at 4.5A* GeV *in* $\cos\theta$*-space.*

TABLE 1. Intermittency exponents α_q for different order of moments (q = 2, 3, 4, 5, 6 and 7) for different bin ranges in compound multiplicity spectrum of ^{24}Mg-AgBr *interactions at 4.5A* GeV *in* $\cos\theta$*-space.*

Bin range	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$	$q = 7$
2≤M≤35	0.584±0.011	1.323±0.017	2.068±0.024	2.804±0.034	3.525±0.045	4.231±0.058
17≤M≤35	0.485±0.050	1.233±0.070	2.018±0.074	2.790±0.075	3.530±0.084	4.236±0.104
3≤M≤17	0.589±0.014	1.332±0.026	2.101±0.042	2.877±0.060	3.653±0.079	4.425±0.098
2≤M≤21	0.614±0.013	1.365±0.024	2.128±0.038	2.888±0.056	3.642±0.077	4.387±0.100
9≤M≤18	0.503±0.035	1.121±0.046	1.733±0.055	2.341±0.075	2.942±0.107	3.537±0.146

TABLE 2. Intermittency exponents α_q for different order of moments (q = 2, 3, 4, 5, 6 and 7) for different bin ranges in compound multiplicity spectrum of ^{24}C-AgBr *interactions at 4.5A* GeV *in* $\cos\theta$*-space.*

Bin range	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$	$q = 7$
2≤M≤35	0.383±0.015	0.947±0.036	1.553±0.061	2.170±0.094	2.788±0.132	3.401±0.173
17≤M≤35	0.180±0.039	0.488±0.074	0.811±0.124	1.090±0.208	1.322±0.317	1.511±0.435
3≤M≤17	0.457±0.014	1.162±0.026	1.978±0.039	2.841±0.055	3.748±0.076	4.671±0.100
2≤M≤21	0.455±0.013	1.126±0.030	1.871±0.052	2.661±0.078	3.476±0.107	4.302±0.138
9≤M≤18	0.346±0.019	0.933±0.049	1.639±0.096	2.420±0.153	3.220±0.215	3.996±0.079

TABLE 3. Specific heats for different bin ranges in compound multiplicity spectrum of ^{24}Mg-AgBr *and* ^{12}C-AgBr *interactions, both at 4.5A* GeV *in* $\cos\theta$*-space.*

Bin range	^{24}Mg-AgBr	^{12}C-AgBr
$2 \leq M \leq 35$	0.329	0.498
$17 \leq M \leq 35$	0.608	0.207
$3 \leq M \leq 17$	0.400	0.629
$2 \leq M \leq 21$	0.316	0.709
$9 \leq M \leq 18$	0.229	0.884

and ^{12}C-AgBr interactions for different intervals are shown in Figs. 3a and b. From the figure, one sees that λ_q gradually decreases with the increase of orders for all multiplicity bin ranges for both interactions and no such minimum value of λ_q is obtained at any value of q. The non-occurrence of minimum of λ_q speaks against the existence of non-thermal phase transition in the compound multiplicity spectrum. To obtain the multifractal specific heat, we have plotted the generalized dimensions D_q against $\ln q/(q-1)$. For both interactions, multifractal specific heats are calculated from the slope values of the best fitted straight lines which are tabulated in Table 3. The specific heats are different for different bin ranges for the two sets of interactions.

Thus, we may conclude that our present study reveals no evidence for non-thermal phase transition in compound multiplicity spectrum of ^{24}Mg-AgBr and ^{12}CAgBr interactions at $4.5A$ GeV in the emission angle space. Higher-dimensional analysis may help in getting better results.

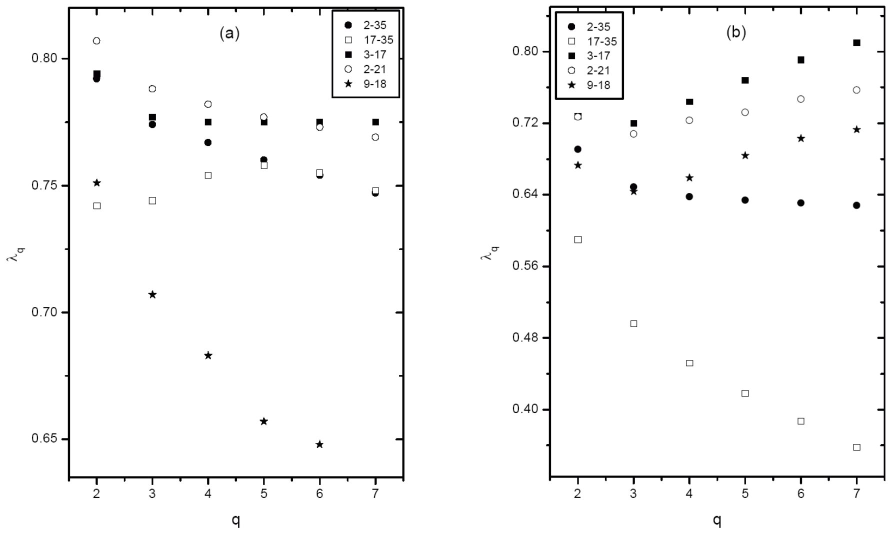


Fig. 3. Variation of λ_q against q for different orders and for different bin ranges in (a) ^{24}Mg-AgBr and (b) ^{12}C-AgBr interactions, both at 4.5A GeV in $\cos\theta$-space.

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RASPODJELE SLOŽENE VIŠESTRUKOSTI U SUDARIMA JEZGRA-JEZGRA – PROUČAVANJE FAZNOG PRIJELAZA

Analizirali smo eksperimentalne podatke o raspodjelama složene višestrukosti u sudarima ^{24}Mg-AgBr i ^{12}C-AgBr, oba na $4.5A$ GeV, primjenom prilagodnih faktorijalnih momenata. Izračunali smo vrijednosti poopćene dimenzije D_q za $q = 2, 3, 4, 5, 6$ i 7 pomoću eksponenta prekidanja α_q. Te vrijednosti D_q primijenili smo za izračunavanje višefraktalne specifične topline za niz pretinaca. Parametar λ_q ($\lambda_q = (\alpha_q + 1)/q$) smo izračunali na osnovi α_q tražeći netermički fazni prijelaz. Analiza nije pokazala prisutnost netermičkog faznog prijelaza, ali smo našli različite specifične topline u pretincima.