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NON-MINIMAL DARK ENERGY AND COSMIC ACCELERATION IN HIGHER-DERIVATIVE DUAL GRAVITY

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The role of non-minimal coupled scalar fields evolving as a power law with supergravity complex scalar potential in higher-derivative dual gravity theory is developed and its cosmological features are discussed in some details.

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Recent astronomical and astrophysical observations of the type Ia distant supernovae (SNe Ia) and of the cosmic microwave background anisotropy have led to the idea that our universe is undergoing an accelerated expansion such that $\ddot{a} > 0$ with $a(t)$ being the scale factor of the Universe, tending to a flat de-Sitter space-time as predicted by inflation theory [1−3]. These observations suggests the presence of an unknown dark energy with negative pressure obeying the state equation $w = p/\rho < -1/3$ and accounting for the missing energy, if one really believes inflation theory in all its aspects predicting $\Omega = 1$ (analysis of recent Ia Supernova data also support strongly $w < -1$) [4−6]. In other words, the dark energy contributes roughly up to two thirds of the total energy of the universe. On the other hand, investigations of recent findings of BOOMERANG experiments [7] strongly suggest that the cosmos is spatially flat. The nature of the dark energy component represents one of the most profound and important tasks for modern cosmology, astrophysics and theoretical physics. Up to now, many theoretical candidates have been postulated to fit various observations and to try to explain the physical nature of the dark component. They include the ΛCDM model [8] consisting of a mixture of cosmological constant Λ and cold dark matter (CDM) or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe. Also quintessence with a very shallow many-forms potential [9], K-essence [10], viscous fluid [11], Chaplygin gas [12,13], generalized Chaplygin gas model (GCGM) which mimics both dark matter and dark energy [14,15], Brans–Dicke (BD) pressureless solutions [16 – 18], decaying Higgs fields [19], dilaton field of string theories with gaugino condensation [20], etc. Most of these theories face many difficulties. For example, within the framework of the ΛCDM model, the vacuum energy is assumed to be constant, while the matter energy density decreases with cosmic time, their ratio must be set to a specific infinitesimally small value (10^{-120}) in the early Universe so as to nearly coincide today, i.e. there exist a huge discrepancy of about 120 orders of magnitude between the predicted and the observed values of the cosmological constant. This is called the "cosmic coincidence" problem (CCP). Note that in quantum field theory, the cosmological constant is treated as a running parameter. In the GCGM, the density perturbations in the theory exhibit large oscillations in the resulting power spectrum which do not appear in the observed spectrum of mass agglomeration [21]. Other difficulties associated with quintessence scenarios are that the couplings of the scalar field to matter can lead to observable long-range forces and time variation of fundamental constants of nature, in particular the gravitational constant and the celerity of light. Many different alternative theoretical models have been developed including the higher-derivative theory with an additional quadratic scalar curvature [22 – 25]. But in fact, the crucial feature of inflationary and quintessence models is studied when the Universe is dominated by a non-minimally coupled scalar field [26 – 28]. As for inflation theory, there are many different implementations, and most of these cannot as yet be excluded as a result of observational inconsistencies. A series of theoretical arguments imply that investigations of the inflationary theory with minimal coupling in the context of general relativity are in fact theoretically inconsistent. Nevertheless, a correct treatment of inflationary cosmology implies highly the presence of a non-minimal coupling between the inflaton and the Ricci scalar curvature.

Recently, we have investigated a particular cosmological model with a complex scalar self-interacting inflation field non-minimally coupled to gravity, based on supergravity's argument [29, 30]. It was shown that in the case of non-minimal coupling between the scalar curvature and the density of the scalar field, such as $L = -(\xi/2)\sqrt{-g}R\phi\phi^*$ (R is the scalar curvature or the Ricci scalar) and for the particular scalar complex potential field $V(\phi\phi^*) = (3m^2/4)(\omega\phi^2\phi^{*2}-1)$, ($\omega$ is a tiny parameter), ultra-light masses m are implemented naturally in the Einstein field equations (EFE), leading to a cosmological constant Λ in accord with observations. The metric tensor of the spacetime is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an external parameter, was found to be $R = 4\Lambda - 3m^2 = 4\bar{\Lambda}$ where $\bar{\Lambda} = \Lambda - (3m^2/4)$ is the effective cosmological constant. That is to say, there is another induced contribution to the vacuum cosmological constant with $\Lambda_{\mathrm{induced}} = -3m^2/4$. These ultra-light masses are in fact too low, while they may have the desirable feature for the description of the accelerated universe [31 – 34].

In most of the cosmological models, the cosmological constant (and as a result the ultra-light masses) decreases as a power law, like $\Lambda = \Lambda_0 t^{-q}$, ($m^2 = m_0^2 t^{-q}$), where t is the cosmic time and $q \leq 2$, Λ_0 and m_0^2 are positive parameters [35, 36], whereas the Ricci scalar may decreases as $R = Ca^{-r}$, where a is the scale factor, C is a constant and r is a non-zero real number [37]. The effective Ricci scalar curvature yields then $a = a_0 t^{q/r}$, where a_0 is a constant. Moreover, in most of the non-minimally coupled scalar fields, the scalar field evolves as a power law $\phi = \phi_0 t^p$, p is also a real parameter and ϕ_0 is a positive parameter (the same rule holds for the complex scalar field) [38]. More recently, dark energy is obtained using dual roles of the Ricci scalar (higher-derivative dual theory (HDDT)), without any geometrical and physical input in the theory, that is without adding to the Einstein–Hilbert term in the action any non-gravitational Lagrangian density for exotic matter yielding dark energy [39].

In reality, higher-derivative gravity theories are thought to have been dominant at early epochs in the evolution of the Universe, in particular when the spacetime curvature was very high. We also believe that higher than quadratic terms in the action are also likely to be involved in the theory, e.g. the presence of cubic terms in the action can produce important changes in the whole cosmological scenario, but for simplicity, we will neglect them in this work.

Motivated by these results, we explore in this work the general case, a non-minimal coupling in HDDT. The action of the theory described in this work is written as

$$\begin{aligned} S &= S_{\mathrm{HE}} + S_{R^2} + S_{\mathrm{int}} + S_{\phi\phi^*} \\ &= \int \sqrt{-g}\,\mathrm{d}^4x \left[\left(\frac{1}{2\kappa} - \frac{\xi\phi\phi^*}{2} - \alpha(x)R \right) R \right. \\ &\quad \left. + \frac{1}{\kappa}\Lambda - \frac{1}{2} g^{\mu\nu} \left(\partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^* \right) - \tilde{V}(\phi\phi^*) \right], \end{aligned} \tag{1}$$

where g is the absolute value of the determinant of the metric tensor $g_{\mu\nu}$, $\kappa = 8\pi G$, $S_{\mathrm{g}} = (2\kappa)^{-1}\int \mathrm{d}^4x\sqrt{-g}(R + 2\Lambda)$ is the Einstein-Hilbert gravitational part of the action, $S_{\mathrm{int}} = -(\xi/2)\int \mathrm{d}^4x\sqrt{-g}R\phi\phi^*$ is the non-minimal interaction term between the gravitational and the complex scalar fields, $S_{\phi\phi^*}$ describes the material part of the action associated with the complex scalar field and $\alpha(x) \neq 0$ is a scalar depending on spacetime coordinates taken as $\alpha = Da^n$, n is a real number [37]. The action (1) yields the following gravitational field equations

$$\begin{aligned} &\left(\frac{1}{2\kappa} - \xi\phi^*\phi \right) G_{\mu\nu} + \left(\partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^* \right) - g_{\mu\nu}\partial_\lambda\phi^*\partial^\lambda\phi \\ &\quad + g_{\mu\nu}\left(\Lambda - \frac{3}{4}m^2 + \frac{3}{4}m^2\omega\phi^2\phi^{*2} \right) + \frac{1}{3}\left(g_{\mu\nu}\Box\phi^*\phi - \nabla_\nu\partial_\mu\phi^*\phi \right) \\ &\quad - \alpha\left(2\nabla_\mu\nabla_\nu R - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu} \right) \\ &\quad - 2\left(\nabla_\mu\nabla_\nu\alpha - g_{\mu\nu}\Box R \right) - 4\left(\nabla_\mu\alpha\nabla_\nu R - g_{\mu\nu}\nabla^\sigma\alpha\nabla_\sigma R \right) = 0, \end{aligned} \tag{2}$$

where the operator $\Box$ denotes the d'Alembertian, ∇_ν is the covariant derivative, $\mu, \nu = 0, 1, 2, 3$ and $g_{\mu\nu}$ are the metric tensor components. The last three terms in the bracket are due to the variations of the term $-(1/2)\sqrt{-g}\xi R\phi\phi^*$ in the action. Taking trace of Eq. (2) gives

$$\Box R + \left(\frac{1}{12\kappa\alpha(t)} + \frac{\Box\alpha}{\alpha}\right)R + \frac{2}{\alpha}\nabla^\nu\alpha\nabla_\nu R = \frac{4\Lambda - 3m^2 + 3m^2\omega\phi^2\phi^{*2}}{12(1-\xi\kappa\phi^*\phi)}. \tag{3}$$

For a flat FRW cosmological spacetime, the cosmological dynamical equation takes one of the following forms

$$\ddot{R} + \left(3\frac{\dot{a}}{a} + 2\frac{\dot{\alpha}}{\alpha}\right)\dot{R} - \left(\frac{1}{96\pi G\alpha(t)} + \frac{\ddot{\alpha}}{\alpha} + 3\frac{\dot{a}}{a}\frac{\dot{\alpha}}{\alpha}\right)R = \frac{4\Lambda_0 - 3m_0^2 + 3m_0^2\omega\phi_0^2\phi_0^{*2}t^{4p}}{12t^q\left(1-\xi\kappa\phi_0^2 t^{2p}\right)}, \tag{4}$$

$$\frac{\ddot{a}}{a} + \left(2+n-q\right)\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{96\pi GD(q-n)a^n} = -\frac{4\Lambda_0 - 3m_0^2}{12qC(t/a)^q\left(1-\xi\kappa\phi_0^2t^{2p}\right)} - \frac{3m_0^2\omega\phi_0^2\phi_0^{*2}t^{4p}}{12qC(t/a)^q\left(1-\xi\kappa\phi_0^2t^{2p}\right)}, \tag{5}$$

$$\approx \frac{Q_1}{96\pi GCa^{(2pr+q(r-q))/q}} + \frac{Q_2}{32\pi GCa^{((r-p)q-2pr)/q}}, \tag{6}$$

where $Q_1 = R_0 a_0^{(2p+q)r/q}/xi\phi_0^2$, $R_0 = 4\Lambda_0 - 3m_0^2$ and $Q_2 = m_0^2\omega\phi_0^{*2}a_0^{r(q-2p/q)}/q\xi$. If we neglect the non-minimal coupling and we set α constant, we fall into the empty de-Sitter spacetime. Eq. (6) integrates to yield approximately the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\rho_{\mathrm{DE}} + \rho_{\mathrm{M}} + \rho_{1,Q_1}(\phi\phi^*) + \rho_{2,Q_2}(\phi\phi^*)\right), \tag{7}$$

where the densities are

$$\rho_{\mathrm{DE}} \propto \frac{3}{8\pi Ga^{2(3-(r-n))}}, \qquad \rho_M \propto \frac{3}{8\pi Ga^n},$$
$$\rho_{1,Q_1}(\phi\phi^*) \propto \frac{3}{8\pi Ga^{(2pr+q(r-q))/q}}, \qquad \rho_{2,Q_2}(\phi\phi^*) \propto \frac{3}{8\pi Ga^{((r-p)q-2pr)/q}}.$$

In summary, four different densities arise: the dark energy (DE) density ρ_{DE} and the matter density ρ_{M} from dual roles of the Ricci scalar and the scalar-field densities $\rho_{1,Q_1}(\phi\phi^*)$ and $\rho_{2,Q_2}(\phi\phi^*)$ from non-minimal coupling that depend on the minimal coupling parameter ξ. The four densities behave as power law of the scaling factor. Note that the scalar density is independent of n ($n = 3$ is for matter density and $n = 4$ corresponds for radiation density), while the dark energy (DE) density arises independently of the non-minimal coupling and from any other source. As for the scalar densities, they arise due to the non-minimally-coupled scalar field. Being interested in $q = 2$ ($\Lambda, m^2 \propto t^{-2}$), it is easy to check that the most dominant density in Eq. (7) at the late age of the Universe is the effective non-minimal phantom density $\rho_{1,Q_1}(\phi\phi^*)$, while the other densities decreases more rapidly in time. If $\rho_{2,Q_2}(\phi\phi^*)$ dominates, then should have $p = 0$ or $r = -1$. The first case corresponds to static scalar field and the second one to a decelerating universe. Both are unrealistic and in conflict with observations. So they are of little physical interest. In fact, the dominant term $\rho_{1,Q_1}(\phi\phi^*)$ corresponds to the static part of our supergravity potential implemented in the theory, e.g. independent of the complex scalar field. In this way, the scale factor evolves as $a \propto t^{2/(pr+r-2)}$ with the constraint $pr = 2$. It can be viewed as a second type of dark energy appearing in the theory. Acceleration occurs if $r < 2$ or $p > 1$. In other words, the scalar factor and the scalar field are both accelerating with time [40]. In the absence of the non-minimal coupling scalar field, the DE density arising from HDDT dominates [41]. This shows the role of the non-minimal coupling term in dual gravity theory and its dominance on the DE density arising from HDDT. The DE density is extremely strongly suppressed by the presence of $\rho_{1,Q_1}(\phi\phi^*)$. One can always consider the tracker case where the scalar-field density scales in the same way as the DE component, but our interest is to show the role of the non-minimal coupling theory in the problem and that superacceleration of the Universe may occur from massless, quartic non-minimally coupled $\phi^2\phi^{*2}$ theory. If the complex scalar potential is taken as $\lambda\phi^2\phi^{*2}$ with $V(0) = 0$, λ is a positive parameter, then the DE component originating from the HDDT dominates. The model described has somewhat interesting features which are somewhat promising for the description of quintessence (dark energy) as non-minimally coupled scalar field and the role of the ultra-light tiny masses and the vacuum cosmological constant. It is important in the future to generalize the theory to include higher than quadratic term in the action with the presence of the Chaplygin gas. Further details and analysis are in progress.

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NEMINIMALNA TAMNA ENERGIJA I KOZMIČKO ZRAČENJE U DUALNOJ GRAVITACIJI S VIŠIM DERIVACIJAMA

Razvijamo dualnu teoriju gravitacije s višim derivacijama, neminimalnim skalarnim poljima u vidu potencija i s kompleksnim supergravitacijskim skalarnim potencijalom, i podrobno raspravljamo njene kozmološke značajke.