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LETTER TO THE EDITOR

COSMOLOGY FROM EXTENDED GENERAL RELATIVITY

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A particular cosmology from extended general relativity where the Lagrangian is extended to include nonlinear analytic functions of the scalar curvature R up to third-order, complex scalar field and conformal coupling is explored. Important features are revealed and discussed in some details.

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The most remarkable recent astrophysical event concerns the discovery of the cosmic accelerated expansion of the Universe. In fact, recent observations of the Type SNIa distant supernovae and of the cosmic microwave background anisotropy have led to the idea that our universe is undergoing a super-expansion or accelerated expansion such that $\ddot{a} > 0$ where $a(t)$ is the scale factor of the Universe, tending to a flat de-Sitter space-time as predicted by the standard old inflation theory [1−3]. These observations suggests from theoretical point of view the presence of a mysterious dark energy with negative pressure obeying the equation of state parameter (EOSP) $w = p/\rho < -1/3$ and accounting for the missing energy if one really believes inflation theory in all its aspects predicting $\Omega = 1$. In fact, analysis of recent observational data support $w \leq -1$ strongly, violating strong or weak energy conditions [4−6]. The critical value $w = -1$ corresponds to the phantom barrier. On the other hand, investigations of recent findings of BOOMERANG experiments [7] strongly suggest that the cosmos is spatially flat in agreement with inflationary framework. Up to now, many theoretical candidates and phenomenological theories have been postulated to fit various observations and to try to explain the physical nature of the dark fluid, including the ΛCDM model [8] consisting a mixture of cosmological constant Λ and cold dark matter (CDM) or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe, quintessence with a very shallow many-forms potential [9], K-essence [10], viscous fluid [11], Chaplygin gas [12, 13], generalized Chaplygin gas model (GCGM) which mimics both dark matter and dark energy [14, 15], Brans–Dicke (BD) pressureless solutions [16 − 18], decaying Higgs fields [19], dilaton field of string theories with gaugino condensation [20], tachyon as a dark energy source [21], etc.

Most of these theories face many difficulties. For example, within the framework of the ΛCDM model, the vacuum energy is set to be constant with time while the energy density of matter decays with cosmic time. Their ratio must be set to a specific infinitesimally small value (10^{-120}) in the early Universe so as to nearly coincide today, i.e. there exists a huge of discrepancy of about 120 orders of magnitude between the predicted and the observed values of the cosmological constant. This is called the "cosmic coincidence" problem (CCP). It has been recently proved that when dark energy (DE) is driven by tachyon, non-minimally coupled with curvature, it decays lately to dark matter, a scenario giving a possible solution to CCP [21]. In fact, it is found that the ratio $\rho^{\mathrm{CDM}}/\rho^{\mathrm{DE}}$ grows with time, but keeping itself less than unity, thus provides a possible solution to CCP. Despite this interesting feature, the model is constrained by one-type potential: a self-interacting inverse cubic one. In the GCGM, the density perturbations in the theory exhibit large oscillations in the resulting power spectrum which do not appear in the observed spectrum of mass agglomeration [22]. Other difficulties associated with quintessence scenarios are that the couplings of the scalar field to matter can lead to observable long-range forces and time variation of fundamental constants of nature, in particular the gravitational constant and the celerity of light.

Many different alternative theoretical models have been developed including the higher-derivative theory with an additional quadratic scalar curvature [23 − 29]. In reality, generalizations of Einstein's theory of general relativity (EGR) are not new, it was first considered by Eddington [30]. These theories, deviating from the general relativity with small polynomial corrections in Ricci scalar curvature (for example as R^n, R^2, R^3, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $\Box R$, $R\Box R$, $f(R)$ subject to $\lim f(R)/R = 0$ when $R \to 0$, etc.)[1], may mimic the effects of DE on the Hubble flow. In addition, if non-linear term of scalar curvature (the main ingredient of the higher-derivative gravity) is added to Einstein–Hilbert part of the Lagrangian, it was theoretically found that DE and dark matter (DM) may emerge from the gravitational sector and exact appealing cosmological solutions may exist despite the fourth-order mathematically complicated field equations (early Universe singularity may be avoided in some cosmological solutions) [31 − 36]. It is worth-mentioning that such higher-order terms in curvature invariants are unavoidable if we want to construct an effective theory of gravity closed to the Planck epoch. In fact, the double role (geometrical and spinless physical field) of the Ricci curvature scalar is the key part of non-Newtonian theory of gravity, where space-time geometry has a vital role.

[1]When $n \to 1$ or R becomes small, the EGR is recovered where Lagrangian density is $R/(2\kappa)$ and $\kappa = 8\pi G$ is the gravitational constant.

But in fact, the crucial features of inflationary and quintessence models refer to the universe dominated by a nonminimally coupled scalar field [35−37]. A series of theoretical arguments imply that the investigations of inflationary theory with minimal coupling in the framework of general relativity are in fact theoretically inconsistent. Nevertheless, a correct treatment of inflationary cosmology implies highly the presence of non-minimal coupling between the inflaton with the scalar field ϕ and the Ricci scalar curvature. Recently, we have investigated a particular cosmological model with complex scalar self-interacting inflation field non-minimally coupled to gravity, based on supergravities argument [38, 39]. It was shown that in the case of non-minimal conformal coupling between the scalar curvature and the density of the scalar field, $L = -(1/12)\sqrt{-g}R\phi\phi^*$ (ϕ^* is the complex conjugate of ϕ and g is the scalar metric), and for the particular scalar complex potential field (QPF) $\tilde{V}(\phi\phi^*) = -(3/4)m^2 + (3/4)m^2\omega\phi^2\phi^{*2}$, $\omega \ll 1$, ultra-light masses m ($|m^2| \approx H^2$, H is the Hubble parameter) are implemented naturally in the Einstein field equations (EFE), leading to a cosmological constant Λ in accord with observations. The induced (second) cosmological constant was found to be $\Lambda_m \equiv \Lambda_{\text{induced}} \approx -3m^2/4$. These ultra-light masses are in fact too low, while they may have desirable feature for the description of the accelerated universe [35, 40−46]. It is in fact required in many quintessence models based on supergravity arguments. Several alternative possible solutions have been proposed, including a time-varying energy density, dilaton from string theory, supersymmetric exotic particles, massive neutrinos, holographic dark energy, $N = 8$ and $N = 2$ supergravity, M/string theory, phantom energy etc. [23−25]. In most of these theories, we still find difficulties to solve the cosmological constant problem, to find a suitable dynamical theoretical scenario generating a small "lambda" and explain at the same time the recent astrophysical observed parameters.

We believe that higher than quadratic terms in the action are also likely to be involved in the theory, e.g. the presence of cubic terms in the action can produce important changes in the whole cosmological scenario. Motivated by these considerations, we are going to study a generalization of EGR with non-minimal coupling, where the action is taken as

$$\begin{aligned} S &= S_{\text{HE}} + S(R^\alpha) + S(R^2) + S(R^\beta) + S(R^3) + S_{\text{int}} + S(\phi\phi^*) \\ &= \int \sqrt{-g}\,\mathrm{d}^4x \left(\frac{R+2\Lambda}{2\kappa} - aR^\alpha + bR^2 - cR^\beta + dR^3 \right. \\ &\quad \left. -\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^*\right) - \tilde{V}(\phi\phi^*) - \frac{\xi R\phi\phi^*}{2} \right), \end{aligned} \tag{1}$$

where $S_{\text{g}} = (2\kappa)^{-1}\int \mathrm{d}^4x\,\sqrt{-g}(R+2\Lambda)$ is the Einstein–Hilbert gravitational part of the action, $S_{\text{int}} = -(1/12)\int \mathrm{d}^4x\,\sqrt{-g}R\phi\phi^*$ is the non-minimal interaction term between the gravitational and the complex scalar fields, ξ is the non-minimal coupling term ($= 1/6$ for conformal coupling case), $S(\phi\phi^*)$ describes the material part of the action associated with the complex scalar field, α and β are real numbers such that $1 < \alpha < 2$ and $2 < \beta < 3$ (of course α and β may be larger than 2 and 3 respectively), a, b, c and d are dimensionless coupling constants different from zero to avoid the ghost problem. It is worth-mentioning that the current accelerated universe could be produced by modified gravitational dynamics, in particular when the curvature of the universe reaches nearly zero values. Thus, one expects that the dimensionless coupling parameter "a" is much bigger in comparison to the other dimensionless parameters in the theory (asymptotically, the curvature is small). One expects that order larger than three are mathematically difficult and may display singular or particular perturbation behaviour. In fact, one may consider also terms evolving as R^{-1}, $R^{-\gamma}$, etc, where γ is a positive number. In other words, the extended Lagrangian is of the form $L = R + R^m + R^{-n} + \ldots$, where m and n are positive parameters. However, the solutions obtained are piecewise and are treated separately, i.e. large and small values of the scalar curvature, corresponding to early and late time behaviour of the model, respectively [23]. There is in addition a criticism against negative power of the scalar curvature due to the occurrence of many stability problems, unsuitable for local astrophysics. The present work shows that a combination of nonlinear contributions from the Ricci curvature scalar in the Lagrangian can drive a late time acceleration of expansion of the universe without involving any kind of phantom or tachyon dark energies.

The action (1) yields the following gravitational field equations [36]

$$\begin{aligned}
&\left(\frac{1}{2\kappa} - \xi\phi^*\phi\right) G_{\mu\nu} + (\partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^*) - g_{\mu\nu}\partial_\lambda\phi^*\partial^\lambda\phi \\
&\quad + g_{\mu\nu}\left(\Lambda - \frac{3}{4}m^2 + \frac{3}{4}m^2\omega\phi^2\phi^{*2}\right) + \frac{1}{3}\left(g_{\mu\nu}\Box\phi^*\phi - \nabla_\nu\partial_\mu\phi^*\phi\right) \\
&\qquad - a\left[\alpha\left\{\nabla_\mu\nabla_\nu R^{(\alpha-1)} - 2g_{\mu\nu}\Box R^{(\alpha-1)} + \alpha R^{(\alpha-1)}R_{\mu\nu}\right\} - \frac{1}{2}g_{\mu\nu}R^{(\alpha)}\right] \\
&\qquad\quad + 2b[R_{;\mu\nu} - g_{\mu\nu}\Box R + RR_{\mu\nu}] - \frac{1}{2}g_{\mu\nu}R^2 \\
&\qquad\qquad - c\left[\beta\left\{\nabla_\mu\nabla_\nu R^{(\beta-1)} - 2g_{\mu\nu}\Box R^{(\beta-1)} + \alpha R^{(\beta-1)}R_{\mu\nu}\right\} - \frac{1}{2}g_{\mu\nu}R^{(\beta)}\right] \\
&\qquad\qquad\quad + d\left[\left\{3\nabla_\mu\nabla_\nu R^2 - 3g_{\mu\nu}\Box R^2 + 3R^2R_{\mu\nu}\right\} - \frac{1}{2}g_{\mu\nu}R^3\right] = 0, \qquad (2)
\end{aligned}$$

where the operator

$$\Box = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right)$$

denotes the d'Alembertien, ∇_ν is the covariant derivative, $\mu, \nu = 0, 1, 2, 3$ and $g_{\mu\nu}$

are the metric tensor components. Taking trace of Eq. (2) gives

$$\Box R-\frac{2-\alpha}{R}\nabla^{\mu}R\nabla_{\mu}R-\frac{1}{3\alpha(\alpha-1)}\left[\frac{R^{(3-\alpha)}}{2\kappa a}-(2-\alpha)R^{2}\right]+\frac{R}{12\kappa b}$$
$$-\frac{2-\beta}{R}\nabla^{\nu}R\nabla_{\nu}R-\frac{1}{3\beta(\beta-1)}\left[\frac{R^{(\beta-1)}}{2\kappa c}+(\beta-2)R^{2}\right]$$
$$+\frac{1}{R}\nabla^{\mu}R\nabla_{\mu}R+\frac{1}{36\kappa d}-\frac{R^{2}}{18}$$
$$=\frac{4\Lambda-3m^{2}+3m^{2}\omega\phi^{2}\phi^{*2}}{12(1-\xi\kappa\phi^{*}\phi)}\left(\frac{R^{2-\alpha}}{2a}+\frac{R^{\beta-2}}{2c}-\frac{1}{36dR}-\frac{1}{2a}\right). \quad (3)$$

In most of the cosmological models, the cosmological constant, and as a result the ultra-light masses, decrease as a power law like $\Lambda=\Lambda_0 t^{-q}$ ($m^2=m_0^2t^{-q}$), where t is the cosmic time and $q\le 2$, Λ_0 and m_0^2 are positive parameters [33,34], whereas the Ricci scalar may decreases as $R=C\tilde{a}^{-r}$, where $\tilde{a}$ is the scale factor, C is a constant and r is a non-zero real number but less than unity [31,32]. The effective Ricci scalar curvature yields then $\tilde{a}=\tilde{a}_0t^{q/r}$, $\tilde{a}_0$ is some constant. Moreover, in most of the non-minimally coupled scalar fields, the scalar field evolves as a power law $\phi=\phi_0t^p$, p is also a real parameter and ϕ_0 is a positive parameter similar to the phantom field used to describe the inflationary phase of the Universe (the same rule holds for the complex scalar field). For a flat FRW cosmological spacetime, the cosmological dynamical equation takes one of the following forms:

$$\ddot{R}+3\frac{\dot{\tilde{a}}}{\tilde{a}}\dot{R}-(2-\alpha)\frac{\dot{R}^{2}}{R}-\frac{1}{12\kappa b}-\frac{1}{3\alpha(\alpha-1)}\left[\frac{R^{(3-\alpha)}}{2\kappa a}-(2-\alpha)R^{2}\right]$$
$$-\frac{1}{3\beta(\beta-1)}\left[\frac{R^{(\beta-1)}}{2\kappa c}+(\beta-2)R^{2}\right]+\frac{1}{36\kappa d}-\frac{R^{2}}{18}$$
$$=\frac{4\Lambda_0-3m_0^{2}+3m_0^{2}\omega\phi_0^{2}\phi_0^{*2}t^{4p}}{12t^{q}(1-\xi\kappa\phi_0^{2}t^{2p})}\left(\frac{R^{2-\alpha}}{2a}+\frac{R^{\beta-2}}{2c}-\frac{1}{36dR}-\frac{1}{2a}\right)$$
$$\approx-\frac{m_0^{2}\omega\phi_0^{2}\phi_0^{*2}}{4\xi\kappa\phi_0^{2}t^{q-2p}}\left(\frac{R^{2-\alpha}}{2a}+\frac{R^{\beta-2}}{2c}-\frac{1}{36dR}\right), \quad (4)$$

$$\frac{\ddot{\tilde{a}}}{\tilde{a}}+(2-r(\alpha+\beta+1))\left(\frac{\dot{\tilde{a}}}{\tilde{a}}\right)^{2}$$
$$=\frac{1}{3\alpha(\alpha-1)}\left[-\frac{\tilde{a}^{r(2-\alpha)}}{a\kappa r}+(2-\alpha)\frac{C}{r\tilde{a}^{r}}\right]$$
$$+\frac{1}{3\beta(\beta-1)}\left[-\frac{\tilde{a}^{r(\beta-2)}}{c\kappa r}+(\beta-2)\frac{C}{r\tilde{a}^{r}}\right]-\frac{1}{6b\kappa r}+\frac{1}{18}\left[\frac{\tilde{a}^{r}}{d\kappa r}+\frac{C}{r\tilde{a}^{r}}\right]$$

$$
\begin{aligned}
&-\left(\frac{4\Lambda_0 - 3m_0^2}{12qC(t/a)^q(1-\xi\kappa\phi_0^2 t^{2p})} + \frac{3m_0^2\omega\phi_0^2\phi_0^{*2}t^{4p}}{12qC(t/a)^q(1-\xi\kappa\phi_0^2 t^{2p})}\right) \\
&\quad\times\left(\frac{E\tilde{a}^{r(2-\alpha)}}{2a} + \frac{F\tilde{a}^{r(\beta-2)}}{2c} - \frac{\tilde{a}^r}{36dC} - \frac{1}{2a}\right) \\
&\approx \frac{1}{3\alpha(\alpha-1)}\left[-\frac{\tilde{a}^{r(2-\alpha)}}{a\kappa r}\right] + \frac{1}{3\beta(\beta-1)}\left[-\frac{\tilde{a}^{r(\beta-2)}}{c\kappa r}\right] - \frac{1}{6b\kappa r} + \frac{1}{18}\left[\frac{\tilde{a}^r}{d\kappa r}\right] \\
&\quad+\frac{1}{12qC\xi\kappa\phi_0^2}\left(\frac{R_0 a^q}{t^{2p+q}} + \frac{3m_0^2\omega\phi_0^2\phi_0^{*2}a^q}{t^{q-2p}}\right)\left(\frac{E\tilde{a}^{r(2-\alpha)}}{2a} + \frac{F\tilde{a}^{r(\beta-2)}}{2c} - \frac{\tilde{a}^r}{36dC}\right),
\end{aligned}
\tag{5}
$$

where E and F are constants and $R_0 = 4\Lambda_0 - 3m_0^2$. In Eqs. (4) and (5), we have assumed that $4\Lambda_0 \approx 3m_0^2$ for later convenience and $a \gg 1$ for reasons mentioned above. Equation (5) integrates easily to the following equation

$$
\begin{aligned}
\frac{\dot{a}^2}{a^2} &= -\frac{A}{a^{[3-r(\alpha+\beta+1)]}} + \frac{1}{3\alpha(\alpha-1)}\Bigg[-\frac{\tilde{a}^{r(2-\alpha)}}{a\kappa r\big(r(2-\alpha)+2[3-r(\alpha+\beta+1)]\big)C^{2-\alpha}} \\
&+\frac{(2-\alpha)C}{r\big(-r+2[3-r(\alpha+\beta+1)]\big)\tilde{a}^r}\Bigg] + \frac{1}{3\beta(\beta-1)}\Bigg[\frac{(\beta-2)C}{3r\big(-r+2\,[3-r(\alpha+\beta+1)]\,\big)\tilde{a}^r} \\
&+\frac{\tilde{a}^{r(\beta-2)}}{c\kappa r\big(r(\beta-2)+2\,[3-r(\alpha+\beta+1)]\,\big)C^{\beta-2}}\Bigg] + \frac{1}{72b\kappa r[3-r(\beta+3)]} \\
&+\frac{1}{18}\left[\frac{\tilde{a}^r}{c\kappa r\big(r+2[3-r(\alpha+4)]\big)C} + \frac{C}{3r\big(-r+2[3-r(\alpha+4)]\big)\tilde{a}^r}\right] \\
&+\frac{R_0 r\tilde{a}_0^{(2p+q)q/r}\left(\tilde{E}\tilde{a}^{-[(2p+q)q-qr-r^2(2-\alpha)]/r} + \tilde{F}\tilde{a}^{-[(2p+q)q-qr-r^2(\beta-2)]/r}\right)}{24aqC\xi\kappa\phi_0^2\big((2p+q)q-qr-r^2(2-\alpha)+2r[3-r(\alpha+\beta+1]\big)} \\
&+\frac{m_0^2 r\omega\phi_0^{*2}\tilde{a}_0^{(q-2p)q/r}a\left(\tilde{E}\tilde{a}^{-[(q-2p)q-qr-r^2(2-\alpha)]/r} + \tilde{F}\tilde{a}^{-[(q-2p)q-qr-r^2(\beta-2)]/r}\right)}{8qaC\xi\kappa\big((q-2p)q-qr-r^2(2-\alpha)+2r\,[3-r(\alpha+\beta+1)]\,\big)} \\
&-\frac{R_0\tilde{a}_0^{(2p+q)q/r}\,\tilde{a}^{-[(2p+q)q-qr-r^2]/r}}{432dqC^2\xi\kappa\phi_0^2\left(\dfrac{(2p+q)q-qr-r^2}{r}+2\,[3-r(\alpha+\beta+1)]\right)} \\
&-\frac{m_0^2\omega\phi_0^{*2}\,\tilde{a}_0^{(q-2p)q/r}\,\tilde{a}^{-[(q-2p)q-qr-r^2]/r}}{144dqC^2\xi\kappa\left(\dfrac{(q-2p)q-qr-r^2}{r}+2\,[3-r(\alpha+\beta+1)]\right)},
\end{aligned}
\tag{6}
$$

where $\tilde{E}$ and $\tilde{F}$ are other constants in the theory function of C. Setting $r = 3$ [31, 32] this equation is the modified Friedman equation which can be written at late times as

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_{DE} + \rho_1 + \rho_2 + \ldots + \rho_{13}) \tag{7}$$

where

$$\begin{aligned}
&\rho_{\mathrm{DE}} \propto A_0\, a^{-3(\alpha+\beta)}, \quad \rho_1 \propto A_1\, \tilde{a}^{6-3\alpha}, \quad \rho_2 \propto A_2\, \tilde{a}^{-3}, \quad \rho_3 \propto A_3\, \tilde{a}^{3\beta-6},\\
&\rho_4 \propto A_4\, \tilde{a}^{-3}, \qquad \rho_5 \propto A_5\, \tilde{a}^0 = cte, \quad \rho_6 \propto A_6\, \tilde{a}^3, \quad \rho_7 \propto A_7\, \tilde{a}^{-3},\\
&\rho_8 \propto A_8\, \tilde{a}^{-[(2p+q)q-3q-9(2-\alpha)]3}, \qquad \rho_9 \propto A_9\, \tilde{a}^{-([2p+q)q-3q-9(\beta-2)]/3},\\
&\rho_{10} \propto A_{10}\, \tilde{a}^{-[(q-2p)q-3q-9(2-\alpha)]/3}, \qquad \rho_{11} \propto A_{11}\, \tilde{a}^{-[(q-2p)q-3q-9(\beta-2)]/3},\\
&\rho_{12} \propto A_{12}\, \tilde{a}^{-[(2p+q)q-3q-9]/3}, \qquad \rho_{13} \propto A_{13}\, \tilde{a}^{-[(q-2p)q-3q-9]/3}.
\end{aligned}$$

A_i, $i = 0, 1, 2, ..., 13$ are constants which are functions of the dimensionless parameters in the theory.

In summary, thirteen different densities arise: $\rho_{2,4,7}$ represent the density of pressureless matter. ρ_5 is negative and can be viewed as a new negative cosmological constant emerging in the theory. The sign of this cosmological constant can explain why the present cosmological constant is too small. ρ_{DE} is the dark energy density and is proportional to $\tilde{a}^{3(\alpha+\beta)}$. It is interesting to note that the dark energy density increases with time. Note that $-3 < 3\alpha - 6 < 0$ and $-3 < 6 - 3\beta < 0$, and consequently ρ_1 and ρ_3 will contribute to the formation of more dark energy in the Universe. The density ρ_6 arises due to the curvature of order three. It is recognized also as DE component. As for $\rho_{8,9,10,11,12,13}$, they arise from the critical power-law behavior of the complex scalar field, the decaying ultra-light masses and the evolution of the scale factor of the Universe. If we set $\alpha = \beta = 10/3$, then ρ_1 and ρ_3 will contribute to the radiation matter. They may also contribute to the DE or to the cosmological constants if the scale factor exponents are well-adjusted. There exist in literature some indications that $q = 4$ and $p = 1$ are favorable values, i.e. $\Lambda, m^2 \propto t^{-4}$ and $\phi \propto t$ [40 – 48]. Then, one get easily

$$\begin{aligned}
\rho_8 \propto A_8\, \tilde{a}^{-(9\alpha-6)/3} &\qquad \big(-4 < (6-9\alpha)/3 < -1\big),\\
\rho_9 \propto A_9\, \tilde{a}^{-(30-9\beta)/3} &\qquad \big(-4 < (9\beta-30)/3 < -1\big),\\
\rho_{10} \propto A_{10}\, \tilde{a}^{-(9\alpha-22)/3} &\qquad \big(4/3 < (22-9\alpha)/3 < 13/3\big),\\
\rho_{11} \propto A_{11}\, \tilde{a}^{-(14-9\beta)/3} &\qquad \big(4/3 < (9\beta-14)/3 < 13/3\big),\\
\rho_{12} \propto A_{12}\, \tilde{a}^{-1}, &\qquad \rho_{13} \propto A_{13}\, \tilde{a}^{13/3}.
\end{aligned} \tag{8}$$

That is to say that $\rho_{10,11,13}$ will contribute certainly to the DE, while $\rho_{8,9,12}$ will contribute to matter and radiation (combined effect of the power-curvature exponents and non-minimal coupling). It is worth-mentioning that for $q = 2$, $p = 1$ and $r = 3$, that is for $\tilde{a} \propto t^{2/3}$, the Universe is not accelerating with time, whereas the cosmological constant is reduced to a value less than that of the standard FRW cosmology with more dark energy. This seems also interesting since neither the weak energy nor the strong energy conditions are violated as in phantom cosmologies. In fact, note that accelerated cosmology is not favored by 100% of astrophysicists. Many claimed that recent astrophysical observations, in particular the SNIa distant supernovae must be carefully reexamined.

This shows the role of the higher-order terms of the Ricci scalar as well as the role of the non-minimal coupling term in dual gravity theory and its dominance in the DE density arising from higher derivative dual theory. The higher curvature order terms (HCOT) in R model described in this work have some interesting features and are somewhat promising: the contribution of the HCOT on dark energy problem, the role played by the non-minimally coupled scalar field, and the role of the ultra-light tiny masses and the vacuum cosmological constant. The model introduces a third cosmological constant $\Lambda_3 \propto -\rho_5$ in the theory which is negative and consequently explains the smallness of the cosmological constant. The effective cosmological constant is then

$$\Lambda_{\text{effective}} = \Lambda_{\text{Einstein}} - \Lambda_m - \Lambda_3. \tag{9}$$

These results emphasize the need for careful and critical examination of models with nonlinear contribution from the Ricci scalar curvature in differential geometry in general and cosmology in particular. Further details and analysis, in particular the presences of negative and exponential powers of curvature [49 – 52] in different dimensions [53] are in progress.

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KOZMOLOGIJA ZASNOVANA NA PROŠIRENOJ OPĆOJ RELATIVNOSTI

Istražujemo posebnu kozmologiju zasnovanu na proširenoj općoj relativnosti u kojoj je lagranžijan proširen nelinearnim analitičkim funkcijama skalarne zakrivljenosti R do trećeg reda, kompleksno skalarno polje i konformno vezanje. Nalazimo i raspravljamo važne značajke te teorije.