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SIGNATURE OF VOID PROBABILITY SCALING IN ^{28}Si-NUCLEUS COLLISIONS

Sh. AHMAD[a,1], A. R. KHAN[a], A. KAMAL[a], A. AHMAD[b], M. ZAFAR[a] and M. IRFAN[a]

[a]*Department of Physics, Aligarh Muslim University, Aligarh, 202002 India*

[b]*Department of Applied Physics, Aligarh Muslim University, Aligarh, 202002 India*

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Probability of detecting no particles in a limited phase space is investigated by analysing the experimental data on 14.5A GeV/c ^{28}Si-nucleus collisions. The findings are compared with the predictions of HIJING Monte-Carlo model. A scaling behaviour of the void probability is observed which confirms the linked-pair approximation for N-particle cumulant correlation functions. The analysis of the mixed events also indicates that the scaling behaviour observed in the present study is a feature of the experimental data.

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1. *Introduction*

After the observations of unexpectedly large local multiplicity fluctuations in JACEE events [1], numerous attempts have been made to investigate the non-statistical fluctuations in high-energy hadronic and nuclear collisions [2, 3]. Studies involving correlations and fluctuations in relativistic nucleus-nucleus (AA) collisions are of special interest because of the fact that the geometry of the collisions and the number of participating nucleons are believed to play important role in multiparticle production resulting in fluctuations and random emission [4, 5]. Furthermore, the presence of dynamical fluctuations in such collisions is expected to arise due to the occurrence of a phase transition from the quark-gluon plasma to the normal hadronic matter. Fluctuations and correlations in AA collisions have generally been investigated by using several different approaches [2, 3, 6 − 10], for instance, normalized factorial moments, multifractal analysis, erraticity, power spectrum analysis,

[1]Presently at Dept. of Applied Sciences, Higher College of Technology, P.O. Box 74, PC 133, Muscat, Sultanate of Oman

wavelet transform, etc. Yet another method in this context is the study of void probability in limited phase space which can lead to some conclusive remarks regarding the production dynamics [11, 12]. Such studies would provide means to examine the validity of the linked-pair ansatz [13].

It has been reported [13] that the higher-order cumulant correlation function can be expressed in terms of two-particle cumulant correlation function, defined as $C_2(\eta_1, \eta_2) = \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$, where ρ_1 and ρ_2 are referred to as the single- and two-particle densities. The two- and three-particle reduced cumulants are expressed as [11]

$$c_2(\eta_1, \eta_2) = C_2(\eta_1, \eta_2)/\rho_1(\eta_1)\rho_1(\eta_2)\,, \tag{1}$$

$$c_3(\eta_1, \eta_2, \eta_3) = A_3[c_2(\eta_1, \eta_2)c_2(\eta_2, \eta_3) + c_2(\eta_2, \eta_3)c_2(\eta_3, \eta_1) + c_2(\eta_3, \eta_1)c_2(\eta_1, \eta_2)]\,, \tag{2}$$

where, by definition, $A_1 = A_2 = 1$, while A_3 is a dimensionless coefficient to be evaluated. A_3 is referred to as the correlation amplitude of order 3. N-particle reduced cumulant may, thus, be constructed as

$$c_N(\eta_1, ..., \eta_N) = A_N \sum_{prem} \prod^{N-1} c_2(\eta_i, \eta_j)\,, \tag{3}$$

where A_N is the linking coefficient of order N.

It has been suggested by Mandelbrot [14] and by Peebles et al. [15], while studying the two-dimensional galaxy catalogs, that the above scheme can describe successfully the galaxy-galaxy correlations. Based on this scheme, several models were proposed to describe the galaxy-galaxy correlations. The difference between these models lie in the pattern of A_N coefficients which measure the amplification of the higher-order correlation [11]. In multiparticle production phenomenology, the above ansatz for N-particle cumulant correlation is called linking-pair approximation [12]. If such a relation of linked-pair ansatz holds well, the void probability is envisaged to exhibit a scaling behaviour. Scaling of void probability in limited rapidity space and validity of linked-pair approximation has been reported by Hegyi [11] in $p\overline{p}$ collisions at CERN collider energies. A few attempts have also been made to investigate the void-probability scaling in relativistic AA collisions [12, 17]. An attempt is, therefore, made to test the validity of the scaling behaviour of rapidity-gap probability by analysing the experimental data on ^{28}Si-nucleus interactions at 14.5A GeV/c. The findings are compared with the Monte Carlo models and with those obtained by analysing the mixed events in order to ensure that the observed correlations are the features of particle production phenomena.

2. Experimental details

A stack of Illford G-5 emulsions, exposed to ^{28}Si ions from AGS at BNL, has been used. The events were collected by following the along-the-track method and a sample of 555 events, characterised by $n_h > 0$, were considered for the present analysis, n_h being the number of particles produced with relative velocities $\beta < 0.7$. All relevant details regarding the criteria of selection of events, classification of tracks and method of measurements may be found elsewhere [4, 18].

In order to compare the findings of the present work with QCD inspired models, a sample of 15000 events, similar to the experimental ones, were simulated using the Monte Carlo code HIJING-1.33 [19] and analysed. These events were generated according to the percentage of interaction of the incident beam with different nuclei in the emulsion [20].

3. Method of analysis

By measuring the emission angle of relativistic charged particles produced in each event, the values of pseudorapidity variable, $\eta = \ln \tan \theta/2$, are estimated for each particle. The probability of finding no particle in a given pseudorapidity space, $\Delta\eta$, has the important feature that makes it useful to investigate higher-order correlations between the emitted relativistic charged particles [11]. The probability generating function Q(λ) for the probability of finding n particles, $P_n(\Delta\eta)$, in the region $\Delta\eta$ is given as [11,12]

$$Q(\lambda) = \sum_{n=0}^{\infty} (1-\lambda)^n P_{n(}(\Delta\eta)\,. \tag{4}$$

The function $Q(\lambda)$ can be expressed in terms of reduced factorial cumulant as

$$Q(\lambda) = \exp\left(\sum_{N=1}^{\infty} \frac{(-\lambda\overline{n})^N}{N!} \int_0^{\Delta\eta} \mathrm{d}\eta_1 ... \int_0^{\Delta\eta} \mathrm{d}\eta_N \, c_N\,(\eta_1, ...\eta_N) \right), \tag{5}$$

where $\overline{n}$ is the average number of particles in $\Delta\eta$ and the N-fold integral of c_N over an N-cube of side $\Delta\eta$ defines the $\mathrm{N^{th}}$ reduced factorial cumulant moment, $\overline{K}_N$, for the bin $\Delta\eta$. Using the generating function, the probability of finding no particle in the considered η interval is given by

$$P_0(\Delta\eta) = Q(\lambda = 1)\,. \tag{6}$$

This probability, $P_0(\Delta\eta)$, in turn, may be used as the generating function for P_n as

$$P_n(\Delta\eta) = \frac{(-\overline{n})^n}{n!} \left(\frac{\partial}{\partial \overline{n}} \right) P_0(\Delta\eta)\,, \tag{7}$$

where the differentiation is carried out with the correlation function held fixed.

The void probability has been found to be symmetrically dependent on the hierarchy of the cummulant function which is noticed in the cumulant expansion of $\ln P_0(\Delta\eta)$ given as

$$\ln P_0(\Delta\eta) = \sum_{N=1}^{\infty} \frac{(-\overline{n})^N}{n!} \overline{K}_N \,. \tag{8}$$

It has been shown [23] that $P_0(\Delta\eta)$ has a fundamental relationship with all other probabilities $P_n(\Delta\eta)$.

If the cumulant correlation function satisfies the linked-pair ansatz, the reduced cumulant moment would take the form

$$\overline{K}_N = A_N \overline{K}_2^{N-1}, \tag{9}$$

while the scaled rapidity gap probability is given by

$$\chi = -\ln P_0(\Delta\eta)/\overline{n}\,. \tag{10}$$

If the linking coefficient A_N is independent of the incident beam energy and bin size in η space, χ will depend only on the single-momentum combination, $\overline{n}\overline{K}_2$ and, therefore, one may write

$$\chi = \sum_{N=1}^{\infty} \frac{1}{N!} A_N (-\overline{n}\overline{K}_2)^{N-1} = \chi(\overline{n}\overline{K}_2)\,. \tag{11}$$

It is evident from Eq. (10) that for the Poisson distribution $\overline{n} = -\ln P_0$ and $\chi = 1$. Therefore, χ measures the deviation from the uncorrelated emission of particles. The overall shape of the scaling function for $\chi < 1$ is affected only by the clustering properties of the produced particles involving correlations of all orders [11]. This feature of the scaling function, therefore, suggests that the void probability scaling would help to investigate the particle production and their behaviour in the rapidity windows of varying widths.

4. *Results and discussion*

The values of the pseudorapidity gap probability, $P_0(\Delta\eta)$, were calculated for the window size, $\Delta\eta = 0.5$ with center at η_c, η_c being the center of mass hadron-nucleon rapidity. The window size is then increased in steps of 0.25 up to $\Delta\eta = 2.5$. Similar calculations were performed by shifting the η-centres on either side of η_c in steps of 0.5. In order to estimate the values of single-momentum combination, $\overline{n}\overline{K}_2$ corresponding to various $P_0(\Delta\eta)$ values, $\overline{K}_2$ is evaluated from the relation $\overline{K}_2 =< F_2 > -1$, where $F_2 =< n(n-1) >$ is referred to as the second-order factorial moment, n being the number of relativistic charged particles in the selected η region. The quantities within $<>$ or bearing a bar over denote their average values for the entire event sample. The upper and the lower limits of $\chi^{\mathrm{obs}} = -\ln P_0(\Delta\eta)/\overline{n}$ may be determined from the predictions of two hierarchical models, the negative binomial (NB) model and the minimal or Poisson model. According to the predictions of these models, the reduced factorial cumulants $\overline{K}_N$ should satisfy Eq. (8). In the

NB model, the A_N coefficients increase as $(N-1)!$, whereas the scaled rapidity gap probability is given by

$$\chi = \ln(1+\overline{n}\overline{K}_2)/\overline{n}\overline{K}_2\,. \tag{12}$$

In the minimal or Poisson model, which was proposed for the galaxy clustering studies, $A_N = 1$ for all N values. According to this model, the rapidity gap probability is given by

$$\chi = (1-\mathrm{e}^{-\overline{n}\overline{K}_2})/\overline{n}\overline{K}_2\,. \tag{13}$$

Eq.(12 and 13) provide the upper and the lower limits for the χ^{obs} corresponding to the different values of single momentum combination $\overline{n}\overline{K}_2$.

In Fig. 1, the values of χ^{obs} are plotted against $\overline{n}\overline{K}_2$ for various bin centres obtained in 14.5A GeV/c ^{28}Si-nucleus interactions. The errors shown in the figure are purely statistical. The solid and the broken curves, respectively, represent the predictions of the NB and Poisson models. Similar plots for the HIJING data are displayed in Fig. 2. It is interesting to note in these figures that almost all data points lie in the region bounded by the two curves. These observations, therefore, indicate that the experimental and the HIJING data obey the scaling law given by Eq. (11). In order to confirm that the observed scaling behaviour is a feature of the data, a sample of mixed events equivalent to the experimental one is generated. For this purpose, the rapidity values of all events are collected and then randomly re-assigned to various particles of each event. The results of the analysis of the mixed event sample are presented in Fig. 3. It is observed from the figure that

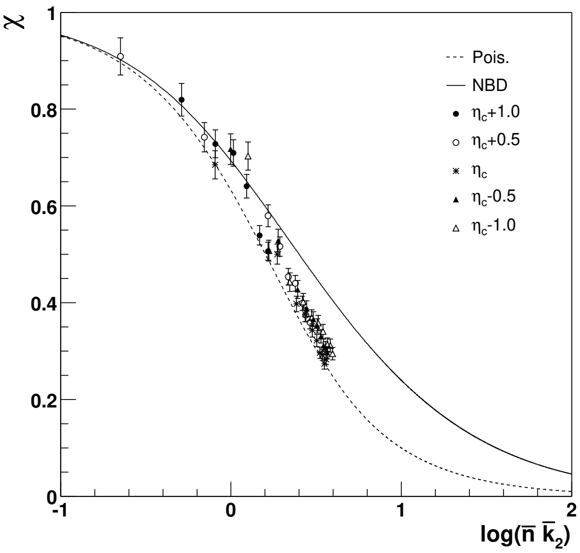

Fig. 1. Variations of χ^{obs} against $\log(\overline{n}\overline{k}_2)$ for the experimental events. The solid and broken curves represent the predictions of NB and minimal models, respectively.

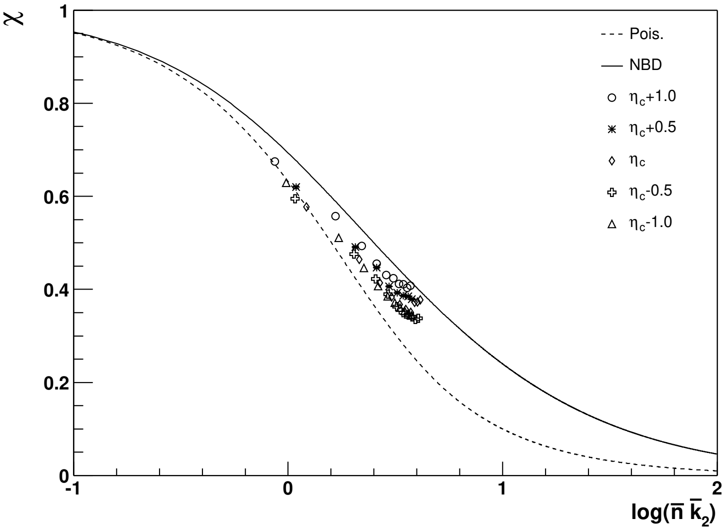

Fig. 2. Dependence of $\chi^{\rm obs}$ on $\log(\overline{n}\overline{k}_2)$ for the HIJING *events. The solid and the broken curves are results of Eqs. (11) and (12), respectively.*

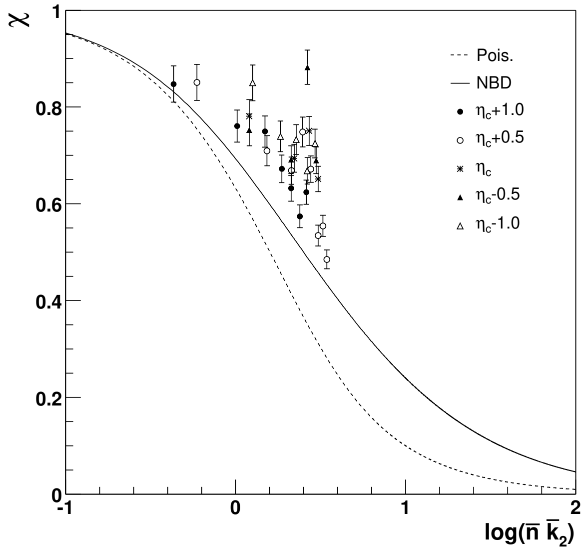

Fig. 3. The same plot as in Fig. 1, but for the mixed events.

the data points lie far beyond the region bounded by the two theoretical curves. These observations, therefore, suggest that the scaling behaviour observed in the case of experimental and HIJING data might be due the underlying dynamics of the multiparticle production and not because of the manifestation of the statistics alone.

In order to test whether such a scaling holds for the interactions with targets of different masses, the experimental data were divided into four distinct groups according to their n_h values, *(i)* $n_h < 8$ representing the interactions due to HCNO group of nuclei or due to the peripheral collisions with AgBr nuclei, *(ii)* $n_h > 8$, collisions exclusively with AgBr nuclei, *(iii)* $n_h < 15$, the semi-central and *(iv)* $n_h > 15$, the central AgBr collisions. These events are separately analysed taking the bin centres at η_c. Variations of $\chi^{\rm obs}$ with $\overline{n}\overline{K}_2$ for these events are displayed in Fig. 4. It may be noticed in the figure that the data points corresponding to the various categories of events overlap and lie within the region bounded by the NB and Poisson curves. This suggests that the void-probability scaling law is satisfied by all categories of events, whether produced in peripheral or in central collisions.

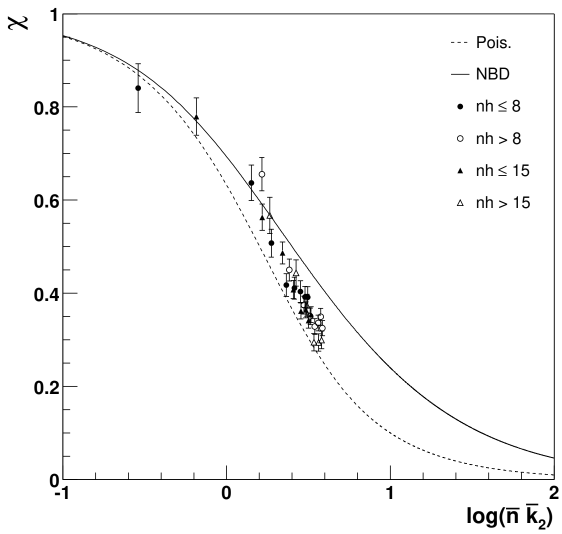

Fig. 4. $\chi^{\rm obs}$ *vs.* $\log(\overline{n}\overline{k}_2)$ *for various categories of events, as explained in the text.*

5. Conclusions

The findings of the present study give a positive indication of the presence of void-probability scaling law. The analysis of the mixed events further confirms that the observed scaling is not only due to the statistical reasons but might have some dynamical origin, too. The observed scaling behaviour is found to be exhibited by all kinds of events, whether produced in the interactions with light or heavy target nuclei. The validity of linked-pair approximation for N-particle cumulant correlation function in the case of multiparticle production in AA collisions is, therefore, confirmed by the present study.

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ZNAK SUMJERLJIVOSTI VJEROJATNOSTI PRAZNINA U SUDARIMA ^{28}Si-JEZGRA

Analizirajući eksperimentalne podatke za sudare ^{28}Si-jezgre na 14.5A GeV/c, istražujemo vjerojatnosti nenalaženja čestica u dijelovima faznog prostora. Ishode uspoređujemo s predviđanjima Monte-Carlo modela HIJING. Nalazimo svojstvo sumjerljivosti vjerojatnosti praznina koje potvrđuje približenje vezanih parova za N-čestične kumulativne korelacijske funkcije. Analiza miješanih događaja također ukazuje da je sumjerljivost svojstvo ovih eksperimentalnih podataka.