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LETTER TO THE EDITOR

LOW-ENERGY EFFECTIVE STRING COSMOLOGY: TRANSITION FROM INFLATION TO DEFLATION TO ACCELERATED EXPANSION

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Low energy effective string cosmology in higher dimensions with a specific type of quantum-like potential is investigated and some features are explored in some details. It is shown that in four and five dimensions, the universe may exist from inflation gracefully with no reheating, deflates in the radiation dominated epoch and finally expands acceleratedly in the dust epoch while the scalar field decays in time.

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In modern days, following the theoretical progress in mathematical physics, including the Kaluza-Klein (KK) theories, supersymmetry, supergravities in various spacetime dimensions and their relevance to string dualities, superstrings and M-theory, it is widely believed that extra-dimensions play a crucial and leading role in high energy physics and in particular in the unification of all fundamental forces. This leads to many new insights in their geometrical properties which are inaccessible in the dimensionally reduced theories themselves. We believe also that extra-dimensions play an important role at the cosmological level because the present four-dimensional stage could have been preceded by a higher-dimensional one if the standard hot Big Bang scenario is admitted as the standard model to describe the evolution and dynamics of our universe [1]. Further, extra dimensions are considered as the most interesting attempt to solve some of the major problems in fundamental physics, including the initial inevitable singularity in the cosmological past and the hierarchy problem in particle physics [2]. In this work, we discuss multidimensional spacetime Friedmann-Robertson-Walker (FRW) model within the framework of Low-Energy String Theory Action (LESTA) with a coupling parameter ω. Actually, there are many reasons to believe that LESTA plays an important role in the description of the universe. It has been argued that the coupling term ω is to be expected whenever the spacetime curvature is large.

Further, it is well-known that the cosmological constant plays a leading role in primordial inflation theory based on the very early universe dominance of inflaton scalar field representing the vacuum energy density producing the quasi-de-Sitter spacetime with equation of state $p_\Lambda = w\rho_\Lambda, w = -1$ [3]. From the theoretical point of view, the cosmological constant has been identified to the dark exotic form of energy that is smoothly distributed and which contributes 2/3 to the critical density of the present universe. Several phenomenological theories have been proposed including the ΛCDM model consisting a mixture of cosmological constant Λ and cold dark matter (CDM) or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe [4], quintessence [5], K-essence [6], viscous fluid [7], Chaplygin gas [8], generalized Chaplygin gas model [9], Brans-Dicke (BD) pressureless solutions [10], decaying Higgs fields ϕ with hidden Higgs condition and potential-like (kinetic) $V(\phi) = \varepsilon - \dot{\phi}^2/2$ (ε is a negligible constant), where the dark energy field was assumed to be a decayed scalar component of a supermultiplet field in the early Universe that creates inertial mass through spontaneous symmetry breaking, e.g. a Higgs field [11], dual role of the Ricci scalar [12], etc. In most of these theories, it has been observed that the cosmological constant decays as $\Lambda \approx t^{-2}$[13]. In fact, some of the models proposed were $\Lambda \propto a^{-2}$ [14], $\Lambda \propto \dot{a}^2/a^2$ [15], $\Lambda \propto \ddot{a}/a$ [16], $\Lambda \propto \dot{a}/at$ [17], where $a(t)$ is the scale factor.

In this short communication, we will treat the problem from a different approach: we will deal in fact with a specific type of potential following the Zel'dovich arguments within the framework of LESTA. Recently, there has been some particular interest on low energy four-dimensional effective action for the higher-dimension theory, where a matter fluid, having a dissipative pressure over and above its positive equilibrium pressure, is present. The cold dark matter is in general considered to be a perfect fluid having a zero positive equilibrium pressure. These models are of interest because they can be extended to meet the constraints from the big-bang nucleosynthesis. For this, we consider the vacuum state of a particle field as containing itself virtual pairs of particles with mass m assuming here to be of the same order of ultra-light particles ($\approx H \approx 10^{-33}$ eV) (and not of the proton as considered by Zel'dovich) and with an effective density $n \propto 1/\lambda_C$, where $\lambda_C = 1/m$ is the Compton wavelength ($\hbar = c = G = 1$) [18]. Then one can consider the gravitational interaction energy of these virtual pairs, which is λ_C/m^2 for one pair, thus leading a contribution to an effective energy density in the vacuum of order of magnitude $\rho_{\text{vacuum}} \propto m^6$. It is worth-mentioning that such virtual particles exist only for a brief moment, but their energies combine to provide each cubic centimeter of space (physical vacuum) with a nonzero energy. This gives rise to a vacuum energy-momentum tensor $\langle T_{ik}\rangle_{\text{vacuum}} \propto m^6 g_{ik}$. Motivated by this fact, we consider in this work the quantum-like exponential potential $V(\phi) = m^6\phi^2 \exp(-\alpha H^{-2}\ddot{\phi}/\phi)$ (de-Broglie-Bohm quantum-like potential). Note that $V(\phi) \to m^6\phi^2$ in the classical limit regime [19]. α is a positive free tiny parameter in the theory which may vary with time. Our motivation came in fact from de-Broglie comment that the quantum theory of motion for relativistic spinless particles is very similar to the classical theory of motion in a conformally flat space-time, where the conformal factor is related to the Bohm's quantum potential. The presence of this kind of potential in the theory is equivalent to a geometrization of the quantum aspects of matter and there exist many theoretical indications that there is a dual aspect to the role of geometry in physics: gravity$\longleftrightarrow$quantum. In other words, the spacetime geometry sometimes looks like gravity and sometimes reveals quantum behaviour. The curvature due to the quantum potential may have a huge influence on the classical contribution to the curvature of spacetime. In addition, one expects that a local variation of the cosmic matter field distribution will change the quantum-like potential acting on the spacetime geometry and alter it globally; the non-local character is forced by the quantum-like potential. On the other hand, it has been proved that using two scalar fields, one can relax this pre-assumption and for the equations of motion, the correct form of quantum potential will be achieved. In other words, the presence of the quantum potential is equivalent to a curved space-time with its corresponding metric. Since the equations governing the geometry are non-linear, the curvature due to the quantum potential may have a large influence on the classical contribution to the curvature of the space-time. In order that the above assumptions contribute to the dark energy problem, we have considered a quantum potential-like related to the scalar field ϕ exponentially like $V(\phi) = m^6\phi^2 \exp(-\alpha Q)$, where $Q = \lambda^2\Box\phi/\phi = \lambda^2\ddot{\phi}/\phi$ for a time-dependent scalar field, where λ is a scale factor assumed to be of the order of the inverse of the Hubble parameter and $\Box$ represents the d'Alembertian. Further, from the cosmological point of view, exponential potentials were used frequently in literature to model power-law inflation theory with self-similar and scaling solutions [20]. Moreover, we expect that the scalar field ϕ decreases with time and consequently at very late times ($t \to \infty$), the dominant minimum of the potential $V(\phi)$ is positive. The ultra-light masses (ULM) are introduced based on our recent work on non-minimal coupling theories, where it was proved they have important cosmological consequences [21]. In fact, these ultra-light masses are part of the dark energy hidden sector and have the desirable feature for the description of the accelerated universe. Their presence signals that the corresponding potentials are very shallow and they may decay with time as $\propto t^{-1}$ (inverse of time). In extended supergravity theories, ultra light fields necessarily come in a package with too small Λ. In this paper, we will analyze the cosmological consequences of such a model. We expect the appearance of a natural positive cosmological constant decaying like t^{-2}. Despite the fact that particle physics predicts many exotic particles that should have been created in the early universe and that spontaneous symmetry breaking predicts an abundance of magnetic monopoles and other types of higher-dimensional objects that would be created from topological defects, e.g., cosmic strings, domain walls and textures, as well as gravitinos and moduli fields predicted by supergravity and superstring theories, respectively, we will neglect their presence in this work. In fact, the hot big-bang picture is not able to explain why today we can not observe these relics.

It is well known that the nonminimal coupling of fields to gravity have non-trivial implications on astrophysical and cosmological issues, in particular the variation of constants of nature. In here, we shall consider the low-energy effective string as a prototype of a non-minimally coupled theory described in the Jordan-Fierz conformal frame by the four-dimensional action

$$S = \frac{1}{16\pi} \int \mathrm{d}^4 x \sqrt{-\widetilde{g}} \left(\phi \widetilde{R} - \phi^{-1} \omega \widetilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + S_{NG} \tag{1}$$

where

$$\widetilde{R} = \widetilde{g}^{\mu\nu} \widetilde{R}_{\mu\nu}$$

denotes the curvature scalar arising from the physical spacetime metric $\widetilde{g} = \det \widetilde{g}_{\mu\nu}$, ω is the Jordan-dimensionless coupling constant between the scalar field and gravity, ϕ is the scalar field which is a dynamical quantity, $\phi \approx G^{-1}$ where G is Newton's gravitational constant (*the gravitational constant is determined by the total amount of matter in the universe through the auxiliary scalar field*), $V(\phi)$ is a potential function between the scalar field and gravity, and S_{NG} is the action for the non-gravitational matter. It is worth-mentioning that one can pass to the Einstein's frame by means of the conformal transformation

$$g_{\mu\nu} \to \widetilde{g}_{\mu\nu} = C^2(\phi) g_{\mu\nu}$$

where $\widetilde{g}_{\mu\nu}$ denotes the Einstein's metric tensor. In reality, there has been a debate about scalar field theories regarding which frame is the physical and realistic one, as both are problematic; the equivalence principle being violated in one frame and the scalar field energy being negative in the other. One expects that one realistic class of such models may lead to significant Newton's law corrections at large cosmological scales. Nevertheless, these corrections are small at solar system as well as at the future universe [22]. The explicit field equations are [23]

$$R_{ab} - \frac{1}{2} g_{ab} R = \phi^{-1} T_{\mu\nu} + \omega \phi^{-2} \left(\phi_{,a} \phi_{,b} - \frac{1}{2} g_{ab} \phi_{,c} \phi^{,c} \right) + \phi^{-1} (\phi_{,a;b} - g_{ab} \Box) - \frac{1}{2} g_{ab} \phi^{-1} V(\phi) \tag{2}$$

$$\Box \phi = \frac{1}{2\omega + 3} \left(T - \phi \frac{\mathrm{d}V}{\mathrm{d}\phi} + 2V(\phi) \right) \tag{3}$$

where $T_{ab}(\text{matter}) = (p + \rho) u_a u_b + p g_{ab}$, p and ρ are, respectively, the pressure and density of the cosmic fluid, u^a is the four-velocity of the fluid such that $u^a u_a = 1$ for $a, b = 1, 2, 3, ..., n$. $T = g^{ab} T_{ab}$ is the trace of the stress-energy tensor. We consider in what follows a spatially flat Friedmann-Robertson-Walker (FRW) metric extended as follows [24]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t) \left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 (\mathrm{d}\theta_1^2 + \sin^2\theta_1 \mathrm{d}\theta_2^2 + ... \sin^2\theta_1 \sin^2\theta_2 ... \sin^2\theta_{n-1} \mathrm{d}\theta_n^2) \right] \tag{4}$$

The field equations look like:

$$\frac{n\,(n+1)}{2}\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)+\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}=\frac{\rho}{\phi}+\frac{m^6\phi}{2}\left(1-\frac{\alpha H^{-2}\ddot{\phi}}{\phi}\right) \tag{5}$$

$$n\frac{\ddot{a}}{a}+\frac{n(n-1)}{2}\left[\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right]+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}=-\frac{p}{\phi}+\frac{m^6\phi}{2}\left(1-\frac{\alpha H^{-2}\ddot{\phi}}{\phi}\right) \tag{6}$$

$$\ddot{\phi}+\frac{n(n+1)}{2}H\dot{\phi}\approx\frac{\rho-3p}{2\omega+3}-\frac{\alpha m^6H^{-2}\phi\ddot{\phi}}{2\omega+3} \tag{7}$$

$H=\dot{a}/a$ is the Hubble parameter. In this situation, we can make some assumptions as we have more unknowns with lesser numbers of equations to determine them. For this, we choose the equation of state $p=(\gamma-1)\rho$, γ is a real parameter and we propose the following phenomenological decay law for the cosmic fluid density $\rho=\xi\phi\dot{a}^2/a^2$ where ξ is a positive parameter. Namely, the theory employs the viewpoint in which Newton's constant is allowed to vary with space and time and can be written in terms of the inverse of the scalar field like $\phi\propto G^{-1}$. Here $\dot{a}^2/a^2\propto H^2$ and consequently $\rho\propto H^2/G$, as deduced from the standard model. With $\alpha\ll 1$, we expand the exponential parts in series up to the second order and we neglect higher-order terms. Consequently, Eqs. (5) - (7) are written like

$$\frac{n\,(n+1)}{2}\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)+\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}=\xi\frac{\dot{a}^2}{a^2}+\frac{m^6\phi}{2}\left(1-\frac{\alpha H^{-2}\ddot{\phi}}{\phi}\right) \tag{8}$$

$$n\frac{\ddot{a}}{a}+\frac{n(n-1)}{2}\left[\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right]+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}=-\frac{(\gamma-1)\xi}{\phi}\frac{\dot{a}^2}{a^2}+\frac{m^6\phi}{2}\left(1-\frac{\alpha H^{-2}\ddot{\phi}}{\phi}\right) \tag{9}$$

$$\ddot{\phi}+\frac{n(n+1)}{2}H\dot{\phi}\approx\frac{(4-3\gamma)\rho}{2\omega+3}-\frac{\alpha m^6H^{-2}\phi\ddot{\phi}}{2\omega+3} \tag{10}$$

In fact, we have started with a non-closed set of field equations which give us the freedom to assume the functional behaviour for the scalar-dependent functions used in the theory. As we have the freedom to assume a functional form for $a(t)$ and $\phi(t)$, we assume that (late-time behaviour) the scalar field and the scale factor evolve both as power-law functions of time as $\phi=\phi_0(t/t_0)^s$ and $a=a_0(t/t_0)^q$, where s and q are real parameters, while ϕ_0 and a_0 denote the values of the scalar field and the scale factor at the origin of time, i.e. at $t=t_0$ (beginning of time). Eqs. (9) and (10) give, consequently, for $k=0$ (flat spacetime) at any positive time

$$\left(\frac{n\,(n+1)\,q^2}{2}-sq-\frac{\omega s^2}{2}-\xi q^2\right)\frac{1}{t^2}=\frac{m^6\phi_0}{2\,t_0^s\,t^{-s}}\left(1-\frac{\alpha s(s-1)}{q^2}\right) \tag{11}$$

$$\left(nq(q-1)+\frac{n(n-1)q^2}{2}+s(s-1)+2qs+\frac{\omega s^2}{2}+(\gamma-1)\xi q^2\right)\frac{1}{t^2}=\frac{m^6\phi_0}{2t_0^s t^{-s}}\left(1-\frac{\alpha s(s-1)}{q^2}\right) \tag{12}$$

A consistent and possible solution is obtained in particular for $m^6 \ll 1$ if we assume $s = -2$ and at late times $q^2 = \alpha s(s-1)$, yielding $q = \pm\sqrt{6\alpha}$, independent of the number of dimensions n. The cosmic fluid density decays in our scenario as inverse of the quartic of time (radiation-like), i.e. $\rho \propto t^{-4}$. The negative (positive) value of q corresponds to a decelerating (accelerating) universe. We admit in what follows the positive value for q to be compatible with the present accelerated expanding universe. Thus, for the vanishing pressure epoch, the universe is in the stage of accelerated expansion, while the scalar field decays with time (increasing gravitational constant) and where the cosmic fluid density decays like $\rho \propto t^{-4}$. From Eqs. (11) and (12), one obtains

$$\gamma = -\frac{6+(n^2+n)q^2-q(2+n)}{\xi q^2}+2 \tag{13}$$

In addition to Eq. (13), the continuity equation in $(n+2)$-dimensional spacetime $\dot\rho + 3(n+1)(p+\rho)H = 0$ gives $4 = \gamma(n+1)q$. The results are presented in Table 1 and Table 2 which correspond to the four-, five- and six-dimensional spacetime.

It is clear from Table 1 that the universe passes three consecutives stages: inflation ($p = -\rho$, vacuum epoch) $\rightarrow$ deflation or power law inflation ($p = \rho/3$, radiation epoch) $\rightarrow$ acceleration ($p = 0$, dust epoch). While in six dimensions, the universe at the dust epoch is not accelerating but there is a deflationary epoch followed by an increase of expansion. The same scenario occurs for with $n > 6$. The universe in our scenario expands with different constant power-law fashions and thus doesn't upset the primeval nucleosynthesis and the structure formation scenario. The available supernova data provide sufficient evidence for the fact that the universe was decelerating in the near past. The numerical results obtained in the three tables agree with recent available astronomical observations [25]. As for the coupling parameter, making use of Eq. (13) and the previous tables, it can be derived from Eq. (10), assuming $m^6 \ll 1$ and, in particular for the radiation and dust epochs.

TABLE 1. The results for q, $a(t)$ and α for different vaules of γ for the four-, five- and six-dimensional spacetime.

	$n=2$			$n=3$			$n=4$		
γ	q	$a(t)$	α	q	$a(t)$	α	q	$a(t)$	α
1	4/3	$\propto t^{4/3}$	8/27	1	$\propto t$	1/6	4/5	$\propto t^{4/5}$	8/75
4/3	1	$\propto t$	1/6	3/4	$\propto t^{3/4}$	3/32	3/5	$\propto t^{3/5}$	1/25
0	∞	$\propto e^{Ht}$	∞	∞	$\propto e^{Ht}$	∞	∞	$\propto e^{Ht}$	∞

TABLE 2. The results for ξ and ω for different vaules of γ for the four-, five- and six-dimensional spacetime.

	$n=2$		$n=3$		$n=4$	
γ	ξ	ω	ξ	ω	ξ	ω
1	6.375	-4.3325	13	-5.83	21.875	-4.3
4/3	12	-3/2	24	-3/2	40	-3/2

Thus, in four, five and six dimensions, we require ω to be in the range $-6 < \omega < -1.5$. Although this range is not compatible with the solar system bound $\omega > 600$, there exists much evidence in the literature where small ω is supported [26].

In conclusion, under the assumptions based on the specific type of potential $V(\phi) = m^6\phi^2 \exp(-\alpha H^{-2}\ddot{\phi}/\phi)$, the simple model described in this paper has interesting features and consequences on the description of the evolution of the Universe. It has been observed that such higher-dimensional model is compatible with the result of recent observations. The universe exits from inflation gracefully and with no reheating, enters a phase of power-law inflation (deflation) in the radiation epoch and finally undergoes a phase of accelerated expansion in the dust epoch, while the scalar field decays like $\phi(t) \propto t^{-2}$. In this model, the cosmic fluid density varies as the inverse quartic of time, thus behaving as radiation matter. The coupling parameter is negative and lies in the range $-6 < \omega < -1.5$. The low-energy effective action used in this work, augmented by the dBPL and ZQV, contains many interesting features. The dynamics found has rich properties and may have important consequences in string and brane cosmology, scalar gravitation and quintessence theory in general, and scalar-tensor black holes. The cosmological implications that such a scenario entails are examined and shown to be consistent with a universe expanding with the power-law acceleration. We hope that in the near future, with the new generation of telescopes and cosmological satellites, the present model could be observed.

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NISKOENERGIJSKA DJELATNA KOZMOLOGIJA STRUNA: PRIJELAZ OD NAPUHAVANJA K SKUPLJANJU I UBRZANOM ŠIRENJU

Proučava se niskoenergijska kozmologija struna u višim dimenzijama s posebnim kvantnim potencijalom i neke se odlike podrobno istražuju. Pokazuje se kako svemir u pet i šest dimenzija može postojati skladno, od napuhavanja bez pregrijavanja, preko skupljanja u dobi prevladavanja zračenja do konačnog ubrzanog širenja u dobi prašine, dok skalarno polje opada s vremenom.