Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

CONFRONTING FRAGMENTATION FUNCTION UNIVERSALITY WITH SINGLE HADRON INCLUSIVE PRODUCTION AT HERA

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Received 23 October 2007; Accepted 20 February 2008
Online 11 July 2008

Predictions for light charged hadron production data in the current fragmentation region of deeply inelastic scattering from the H1 and ZEUS experiments are calculated using perturbative QCD at next-to-leading order, and using fragmentation functions obtained by fitting to similar data from e^+e^- reactions. Good agreement was obtained with the HERA data from years 1994 to 1997. There is a discrepancy at low Q and small scaled momentum x_p, that was reduced by incorporating mass effects of the detected hadron. We also studied the contributions from the different hadron species, as well as from the various quark flavours in the ep reaction by performing quark tagging in the calculation. Comparisons to more recent and precise data show a larger discrepancy due to various neglected effects in our calculation, such as quark masses, soft gluon resummation and higher twist.

PACS numbers: 13.60.Hb, 13.60.-r ,13.65.+i UDC 539.125

Keywords: charged hadron production, deeply inelastic scattering, H1 and ZEUS experiments, perturbative QCD

1. *Introduction*

Within the framework of the factorization theorem of Quantum Chromodynamics (QCD) at next-to-leading order (NLO) and leading twist, data for single hadron inclusive production in e^+e^- reactions have been used to constrain fragmentation functions (FFs) for light charged hadrons ($\pi^\pm$, $K^\pm$ and $p/\bar{p}$) in Refs. [1−4]. This provides a test of perturbative QCD, as well as a constraint on the strong coupling constant $\alpha_s(M_Z)$ at the Z boson mass scale M_Z. Also, since the universality principle of the factorization theorem implies that the FFs are independent of the initial state, FFs extracted in this way can be used to make predictions for other hadron production processes such as those arising from ep reactions in the current fragmentation region and from pp and $p\bar{p}$ reactions.

In this paper we confront predictions of normalized light charged hadron scaled momentum (x_p) distributions with single hadron inclusive production measurements in deeply inelastic scattering at the H1 [5, 6] and ZEUS [7 − 9] experiments at high Q in the current fragmentation region, where the detected hadron originates from the fragmentation of a parton at high scale. Experimentally, these hadrons can be reliably distinguished from those in the target fragmentation region by working in the Breit frame. Some of the results can be found in Ref. [10], but, since then, the H1 and ZEUS collaborations have released new data [6, 9] that we can now use for comparison.

The paper is organized as follows. We first present the formalism behind our calculations in Section 2. We define the observable we are studying, and give the form of the cross section in terms of the FFs. Then we discuss the modification to the cross section when the detected hadron's mass is not negligible, since this effect is important at sufficiently small x_p and low Q. Section 3 contains our comparisons with the data, and we examine the uncertainties arising from the arbitrary choice of scale, of PDF set and of FF set, as well as the importance of gluon fragmentation and of the detected hadron mass. Furthermore, although the corresponding data are absent, the contributions from the individual fragmenting parton and from the detected hadron species to the cross section are calculated to further determine differences and similarities of the FF sets. We follow the work of Ref. [10], but include comparisons with new data. In Section 4 we present our conclusions.

2. *Theoretical formalism*

We are concerned with the process $ep \to e+h+X$, where h is a detected hadron and X is the remaining unobserved part of the final state. The kinematic degrees of freedom are chosen to be the centre-of-mass (c.m.) energy $\sqrt{s}$ of the initial state electron-proton system, which is given by $s = (P + k)^2$ and which is kept fixed in the experiments, the magnitude of the hard photon's virtuality $Q^2 = -q^2$, the Bjorken scaling variable $x = Q^2/(2P\cdot q)$ and the scaled detected hadron momentum $x_p = 2p_h \cdot q/q^2$. The normalized cross section (with the s dependence omitted for brevity) takes the form

$$F^{\mathrm{proton}\,h}(\mathrm{cuts}, x_{pA}, x_{pB}) = \frac{\int\limits_{\mathrm{cuts}} \mathrm{d}Q^2 \mathrm{d}x \int\limits_{x_{pA}}^{x_{pB}} \mathrm{d}x_p \frac{\mathrm{d}\mathcal{O}^{\mathrm{proton}\ h}}{\mathrm{d}x_p}(x, x_p, Q^2)}{\int\limits_{\mathrm{cuts}} \mathrm{d}Q^2 \mathrm{d}x \mathcal{O}^{\mathrm{proton}}(x, Q^2)}\,, \tag{1}$$

where, for convenience later, we use the shorthand $\mathcal{O}$ for $\mathrm{d}^2\sigma/(\mathrm{d}x\mathrm{d}Q^2)$, where "cuts" refers to a specified region in the (x, Q^2) plane (see Refs. [5 − 9] for the various cuts used by H1 and ZEUS), and where $x_{pA(B)}$ is the lower (upper) edge of the x_p bin. The cross section and the kinematic variables are frame invariant, and are measured in the Breit frame.

The factorization theorem dictates that the leading twist component of the factorized cross section in the numerator of Eq. (1) takes the form

$$\begin{aligned}\frac{\mathrm{d}\mathcal{O}^{\mathrm{proton}\ h}}{\mathrm{d}x_p}(x,x_p,Q^2) &= \int\limits_x^1 \frac{\mathrm{d}y}{y}\int\limits_{x_p}^1 \frac{\mathrm{d}z}{z}\sum_{ij}\frac{\mathrm{d}\widehat{\mathcal{O}}^{ij}}{\mathrm{d}z}\left(y,z,\frac{Q^2}{\mu^2},a_s(\mu^2)\right) \\ &\times\frac{x}{y}f_i^{\mathrm{proton}}\left(\frac{x}{y},\mu^2\right)\frac{x_p}{z}D_j^h\left(\frac{x_p}{z},\mu^2\right).\end{aligned} \tag{2}$$

In this framework, the incoming parton i has momentum $p = (x/y)P$ and the outgoing parton j has momentum $p' = (z/x_p)p_h$. f_i^{proton} is the PDF of parton i in the proton, D_j^h is the FF of parton j to the hadron h, $\widehat{\mathcal{O}}^{ij}$ is the equivalent factorized partonic observable, which is given to NLO in Ref. [11], μ is the factorization/renormalization scale which distinguishes the soft from the hard subprocesses and $a_s(\mu^2) = \alpha_s(\mu)/(2\pi)$. Using the momentum sum rule, the integration over x_p from 0 to 1 and the sum over h of Eq. (2) yields the factorized cross section in the denominator of Eq. (1), viz.

$$\mathcal{O}^{\mathrm{proton}}(x,Q^2) = \int\limits_x^1 \frac{\mathrm{d}y}{y}\sum_i \widehat{\mathcal{O}}^i\left(y,\frac{Q^2}{\mu^2},a_s(\mu^2)\right)\frac{x}{y}f_i^{\mathrm{proton}}\left(\frac{x}{y},\mu^2\right). \tag{3}$$

Now we introduce briefly the procedure to include the hadron mass effect. The production rate of the detected hadron falls as its mass m_h increases due to the reduction in the size of the available phase space. This effect is particularly pronounced at small x_p and low Q, where m_h cannot be neglected relative to the hadron's spatial momentum. To account for this effect we note that in general, the scaling variables of the factorization theorem are given by ratios of the light cone momenta. To find the general relation between the true scaling variable of fragmentation and the measured variable x_p in the presence of hadron mass, we work in the class of frames in which the spatial momenta of the virtual photon and the detected hadron are parallel, but which is otherwise completely general. The deduction of the formula can be found in Ref. [10], where it is found that $\mathrm{d}\mathcal{O}^{\mathrm{proton}\,h}/\mathrm{d}\xi_p$ is related to the experimentally measured quantity $\mathrm{d}\mathcal{O}^{\mathrm{proton}\,h}/\mathrm{d}x_p$ by

$$\frac{\mathrm{d}\mathcal{O}^{\mathrm{proton}\,h}}{\mathrm{d}x_p}(x,x_p,Q^2) = \frac{1}{1+\dfrac{m_h^2}{Q^2\xi_p^2(x_p)}}\,\frac{\mathrm{d}\mathcal{O}^h}{\mathrm{d}\xi_p}(x,\xi_p(x_p),Q^2)\,. \tag{4}$$

This normalization of the theoretical cross section agrees with one which has already been proposed [13], and applied in analyses of experimental data [14, 15], up to terms of $O((m_h^2/(\xi_p^2 Q^2))^2)$.

3. Comparisons with HERA data

In this section we present our numerical results for the single hadron inclusive production measurements from H1 and ZEUS. Cross sections are calculated to NLO in the $\overline{\mathrm{MS}}$ scheme using the CYCLOPS software [17]. We set the number of active quark flavours $n_f = 5$. To account for the initial state proton, we use the CTEQ6M PDF set of Ref. [18] unless otherwise stated. We use their value $\Lambda^{(5)}_{\mathrm{QCD}} = 226$ MeV. The factorization/renormalization scale is chosen as $\mu = Q$ unless stated otherwise. The detected hadron's mass m_h is set to zero unless otherwise stated.

3.1. Scaled momentum distributions

In this subsection we compare theoretical predictions with single hadron inclusive production x_p distributions measured by H1 [5] and ZEUS [7]. The predictions generally agree well with the ZEUS and H1 data (Fig. 1). For both the H1 and ZEUS data, the predictions using the KKP FF set are the most gradual in x_p, while the Kretzer predictions are the steepest. The predictions from the AKK and Kretzer sets are quite similar, particularly at large x_p and for all x_p values of the high Q H1 data. The predictions for the low Q H1 data show an undershoot at large x_p. This behaviour may result from unresummed logarithms at large x_p in the partonic cross section, since resummation tends to enhance the cross section. The overshoot from the low Q H1 data at small x_p may be due to the theoretical errors in the calculation of ep reaction data discussed above. Indeed, better agreement is found at all values of x_p with the high Q H1 data, where resummation is less necessary and where higher twist and mass effects are significantly reduced.

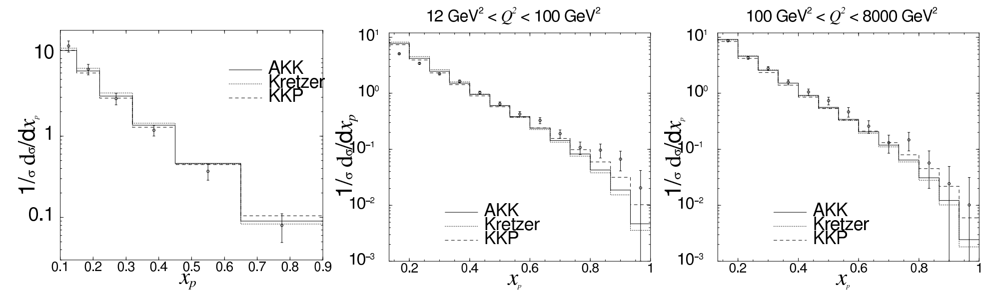

Fig. 1. Comparisons of theoretical predictions using the AKK, Kretzer and KKP FF sets with the x_p distribution from ZEUS [7] and H1 [5].

We now study various modifications to the predictions in order to understand the effect of increasing Q on the theoretical and propagated experimental errors. First we modify our theoretical approach to incorporate the detected hadron mass according to the method described in Ref. [10]. Since the hadron sample is dominated by pions, the "average" hadron mass is expected to be around $m_h = 0.2 - 0.3$ GeV. However, to exaggerate the effect of hadron mass for illustration, we choose the larger value $m_h = 0.5$ GeV. At small x_p, this effect improves the description

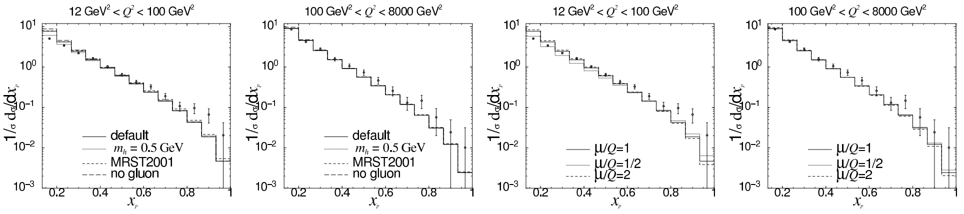

Fig. 2. As in Fig. 1, for the H1 data, using only the AKK FF set. The modifications to the default predictions (solid line) arising from the replacement of the CTEQ6M PDF set by the MRST2001 PDF set of Ref. [12], from the removal of the evolved gluon, and from the incorporation of the hadron mass effect are shown.

of the low Q H1 data (Fig. 2), while making negligible difference to the high Q H1 data over the whole x_p range. However, this improvement should not be taken too seriously, since other low Q, small x_p effects may also be relevant. In addition, the FFs from the various sets are artificially suppressed at small x_p since the hadron mass effect was not accounted for in the analyses of Refs. [1, 2, 4].

The error due to the freedom in the choice of PDF set, which we determine by calculating the predictions using the MRST2001 PDF set [12], is rather small, particularly for intermediate x_p values and for the high Q data.

The gluon contribution (also shown in Fig. 2) is clearly negative, although the evolved gluon FF is positive. In general, the gluon fragmentation is unimportant, particularly away from the smaller x_p range and for the high Q H1 measurements.

For the low Q H1 data, the uncertainty from the freedom in the scale choice is largest at the smaller and larger x_p values (Fig. 2). In general, increasing the scale steepens the drop in the cross section with increasing x_p.

To determine the importances of the fragmentations from the various quark flavours, we study the quark flavour tagged components of the cross section in Fig. 3 (the low Q predictions are not considered since, as we have just seen, the theoretical errors are larger). As anticipated from the valence structure of the initial state proton, the contribution to the overall fragmentation from the u quark fragmentation constitutes a significant amount (50% or more) of the H1 and ZEUS data, followed by the contribution from the c and d quark. Finally the contribution from the s quark is rather small and fragmentation arising from the b quark is neglible.

The relative importances of the fragmentations into the various light charged hadrons in ep reactions can be determined from the composition of the detected hadron sample with respect to the hadron species (Fig. 4b). The uncertainty in the different yields is estimated by the spread of the results for the different FF sets, and is largest at large x_p and smallest at intermediate x_p. The AKK and Kretzer sets give rather similar descriptions of the $\pi^{\pm}$ and $K^{\pm}$ yields for all x_p values shown, while the KKP set gives larger yields at large x_p.

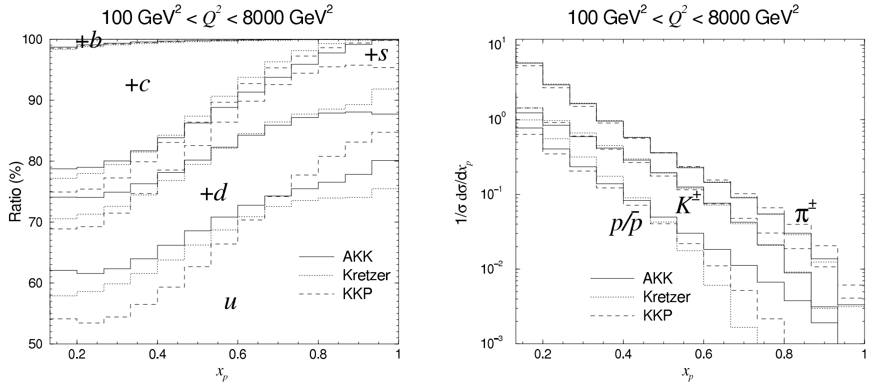

Fig. 3. (a) The ratios of the quark tagged components of the cross section to the untagged cross section for the high Q H1 data, using the AKK, Kretzer and KKP FF sets. The lowest three curves show the contribution from the u quark tagged component only, the next three curves above the sum of the u and d components, the next three u, d and s etc. (b) The individual hadron species constituting the sample for the high Q H1 data, using the AKK, Kretzer and KKP FF sets (each line is for a single hadron, not a summation of hadrons).

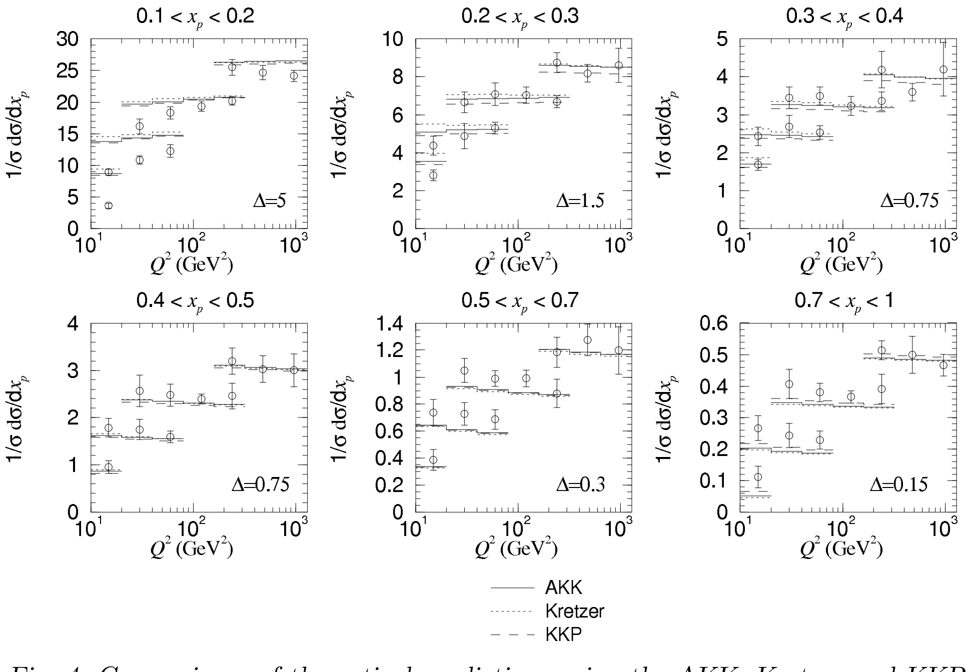

Fig. 4. Comparisons of theoretical predictions using the AKK, Kretzer and KKP FF sets with the ZEUS data [8]. Each data set is measured in a specific x-bin and, together with its predictions, is shifted upwards relative to the one below by the indicated value for Δ.

3.2. Distributions in photon virtuality

Next we compare theoretical predictions with the single hadron inclusive production measurements at various Q values from H1 [6] and ZEUS [8, 9]. The predictions agree well with the ZEUS data in Ref. [8] (Fig. 4), except for, at low Q, the overshoot at small x_p and the undershoot at large x_p. Note that the theoretical

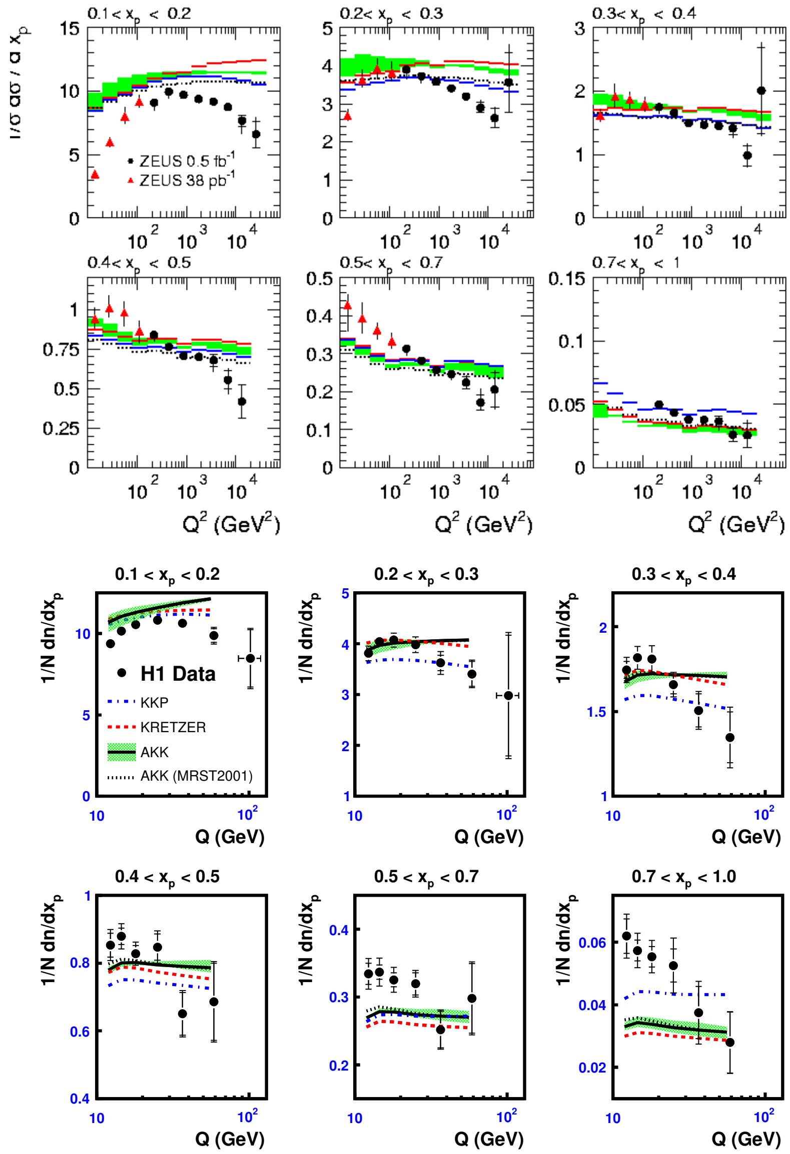

Fig. 5. As in Fig. 4 for the ZEUS (top) and H1 (bottom) most recent data [6, 9].

predictions are rather constant over the whole Q range of both data sets. Except at the lower Q and smaller x_p region, the AKK predictions tend to be closer to the Kretzer predictions than to the KKP ones.

In the case of the recent results obtained by the ZEUS and H1 collaborations, some deviation from the data is found. These data are more precise, and therefore it may be necessary to include some effects neglected in our calculations such as masses of the heavy quarks, unresummed logarithms and higher twist, in order to obtain agreement as in the earlier data. Our results are shown in Fig. 5.

4. *Conclusions*

We have performed a comprehensive analysis of single hadron inclusive production data at HERA, by calculating the theoretical predictions using FF sets that were obtained by fitting to accurate e^+e^- data. In general, good agreement was found using the AKK, Kretzer and KKP FF sets. However, at low Q and small x_p the predictions overshoot the data, a problem which is partially remedied by including the detected hadron mass effect.

Fragmentation from the u quark gives the largest contribution to the overall fragmentation in ep reactions, followed by c and d quark fragmentation and finally from the s quark. Fragmentation from the b quark is negligible.

The fractional yields of each of the light charged hadron species in ep reactions depend strongly on x_p, but to a much lesser extent on Q.

Relative to the experimental accuracy of the data sets, the AKK and Kretzer predictions, as well as their quark tagged components and $\pi^\pm$ and $K^\pm$ yields are very similar for all data considered.

Finally, we compared our results to the most recent H1 and ZEUS data, which use improved triggering and higher luminosity to obtain more precise results. In this case our predictions tend to deviate from the data. This discrepancy suggests phenomena unaccounted for, such as unresummed logarithms, higher twist and quark masses, or some other phenomena not mentioned here.

Acknowledgements

This work was done in collaboration with S. Albino, B. Kniehl and G. Kramer.

References

[1] B. A. Kniehl, G. Kramer, and B. Pötter, Nucl. Phys. B **582** (2000) 514.

[2] S. Kretzer, Phys. Rev. D **62** (2000) 054001.

[3] L. Bourhis, M. Fontannaz, J. P. Guillet and M. Werlen, Eur. Phys. J. C **19** (2001) 89. Since the FF set of this reference does not distinguish between hadron species, we will not consider it in this paper.

[4] S. Albino, B. A. Kniehl and G. Kramer, Nucl. Phys. B **725** (2005) 181.

[5] C. Adloff et al. [H1 Collaboration], Nucl. Phys. B **504** (1997) 3.

[6] H1 Collaboration, Phys. Lett. B **654** (2007) 148; e-Print: arXiv:0706.2456 [hep-ex].

[7] M. Derrick et al. [ZEUS Collaboration], Z. Phys. C **70** (1996) 1.

[8] J. Breitweg et al. [ZEUS Collaboration], Phys. Lett. B **414** (1997) 428.

[9] ZEUS Collaboration, ZEUS-prel-07-006.

[10] S. Albino, B. A. Kniehl, G. Kramer and C. Sandoval, Phys.Rev. D **75** (2007) 034018.

[11] G. Altarelli, R. K. Ellis, G. Martinelli and S. Y. Pi, Nucl. Phys. B **160** (1979) 301.

[12] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **23** (2002) 73.

[13] Yu. Dokshitzer and B. R. Webber, discussion at *Third UK Phenomenology Workshop on HERA Physics*, Durham, UK, 20-25 September 1998.

[14] J. Breitweg et al. [ZEUS Collaboration], Eur. Phys. J. C **11** (1999) 251.

[15] P. Dixon, D. Kant and G. Thompson, J. Phys. G **25** (1999) 1453.

[16] D. Kant, *A study of the fragmentation of quarks in e^-p collisions at HERA using the H1 detector*, PhD thesis, University of London (1995); P. Dixon, *A study of the fragmentation of quarks in ep collisions at HERA*, PhD thesis, University of Lancaster (1997).

[17] D. Graudenz, Fortsch. Phys. **45** (1997) 629.

[18] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky and W. K. Tung, JHEP **0207** (2002) 012.

[19] G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C **16** (2000) 407.

USPOREDBA OPĆENITOSTI FUNKCIJE LOMLJENJA S INKLUZIVNOM TVORBOM POJEDINAČNIH HADRONA NA HERI

Primjenom teorije smetnje u QCD-u u prvom redu do vodećeg, i rabeći takod ¯er funkcije lomljenja izvedene prilagodbom podacima iz e^+e^- reakcija, izračunali smo približne vrijednosti za tvorbu lakih nabijenih hadrona u području trenutnog lomljenja u duboko-neelastičnom raspršenju mjerenja H1 i ZEUS. Postigli smo dobar sklad s podacima iz HERE od 1994 do 1997 godine. Nalazimo odstupanja za male Q i malen razmjereni impuls x_p, što se je smanjilo uvođenjem učinka mase opažanog hadrona. Proučavali smo također doprinose drugih vrsta hadrona, kao i raznih okusa kvarkova iz *ep* reakcija uključivši u račun obilježavanje kvarkova. Usporedbe s nedavnijim i točnijim podacima pokazuju veća odstupanja zbog više učinaka koje smo zanemarili u našem računu, kao što su mase kvarkova, ponovno zbrajanje mekih gluona i učinci višeg tvista.