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RENORMALIZATION OF TENSOR SELF-ENERGY IN RESONANCE CHIRAL THEORY

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We study the problems related to the renormalization of propagators in resonance chiral theory, concentrating on the case of vector resonances in the antisymmetric tensor formalism. The general form of the propagators for antisymmetric tensor fields contains not only the resonance states but also the states that are ghosts or tachyons which decouple in the free-field limit. However, when the interaction terms are taking into account they are dynamically generated through the renormalization procedure.

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1. Introduction

The use of effective field theories for description of dynamics of hadrons has made a considerable progress in recent years. In the low-energy region $E < \Lambda_H = 1\mathrm{GeV}$, the dynamics of the lowest lying states (the pseudoscalar mesons) is effectively described by chiral perturbation theory (χPT) [1–4] based on the spontaneous symmetry breaking of chiral symmetry of QCD.

In the intermediate energy region ($1\,\mathrm{GeV} \leq E \leq 2\,\mathrm{GeV}$), one uses the resonance chiral theory (RχT) [5, 6, 9, 10, 11, 7, 8]. RχT is based on large N_C QCD which partially shares a lot of interesting properties with $N_C = 3$ case. In the leading order of the $1/N_C$ expansion, the QCD spectrum contains infinite towers of meson resonances with residual interaction suppressed by powers of $1/\sqrt{N_C}$. Their dynamics at the leading order in $1/N_C$ can in principle be described in terms of the tree level diagrams within an effective theory with an infinite number of fields. Such a theory is not known from the first principles; however, it can be constructed basically on symmetry grounds, and its free parameters can be fixed by the phenomenology. The flavour group of large N_C QCD is $U(N_f)_L \times U(N_f)_R$ (because of the absence of axial anomaly in the large N_C limit) that is spontaneously broken to $U(N_f)_V$.

RχT is the approximation of the large N_C QCD when only finite number of resonances in each channel are included. This approach is well-founded, for example because of the successful prediction of $\mathcal{O}(p^4)$ coupling constants of χPT Lagrangian[5], some developments in saturation of $\mathcal{O}(p^6)$ coupling constants [13] and of many phenomenological consequences (see, e.g. [10, 11, 14]).

In this paper we want to show that RχT might contain some problems and features of inner inconsistency when we go beyond the leading order (quantum loops in RχT were already studied in Refs. [15, 16]). A more detailed treatise of the discussed problem will be published in Ref. [17].

In the following we are interested only in the sector of vector resonances 1^{--} but the results of a more general discussion do not differ from this special case.

2. *Antisymmetric tensor formulation of RχT*

The standard description of vector resonances is provided by the vector or antisymmetric tensor fields. It was shown that Lagrangians of these two formulations are not equivalent unless some contact terms are included (it was also proved in Ref. [7] that in the general case an infinite number of such terms are necessary). Another possibility of description of spin-1 resonances is the so-called first-order formalism investigated in Ref. [7], where both types of fields are used. For illustrative purposes, we restrict in the following to the antisymmetric tensor case.

The nonet of vector resonances 1^{--} can be represented by the antisymmetric tensor fields collected in 3×3 matrix $R_{\mu\nu}$

$$R_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu} . \qquad (1)$$

These fields transform in the nonlinear realization of the $U(3)_L \times U(3)_R$.

Let us start with the following Lagrangian

$$\mathcal{L}_0 = -\frac{1}{2}\langle \partial_\mu R^{\mu\nu} \partial^\rho R_{\rho\nu} \rangle + \frac{1}{4} M^2 \langle R_{\mu\nu} R^{\mu\nu} \rangle + \mathcal{L}_{\text{int}} \,, \qquad (2)$$

where the brackets denote the trace over group indices. For a general $\mathcal{L}_{\text{int}}$, the complete two-point 1PI Green function has the form

$$\Gamma^{(2)}_{\mu\nu\alpha\beta}(p) = \frac{1}{2}(M^2 + \Sigma^T(p^2))\Pi^T_{\mu\nu\alpha\beta} + \frac{1}{2}(M^2 - p^2 + \Sigma^L(p^2))\Pi^L_{\mu\nu\alpha\beta}\,, \qquad (3)$$

with the projectors

$$\Pi^T_{\mu\nu\alpha\beta} = \frac{1}{2}\left(P^T_{\mu\alpha}P^T_{\nu\beta} - P^T_{\nu\alpha}P^T_{\mu\beta}\right), \qquad (4)$$

$$\Pi^L_{\mu\nu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}\right) - \Pi^T_{\mu\nu\alpha\beta}\,, \qquad (5)$$

where $P^T_{\mu\nu} = g_{\mu\nu} - p_\mu p_\nu/p^2$. The poles $p^2 = M^2_{V,A}$ of the corresponding propagator

$$\Delta_{\mu\nu\alpha\beta}(p) = -\frac{2}{p^2 - M^2 - \Sigma^L(p^2)}\Pi^L_{\mu\nu\alpha\beta} + \frac{2}{M^2 + \Sigma^T(p^2)}\Pi^T_{\mu\nu\alpha\beta}$$

satisfy the equations

$$M_V^2 - M^2 - \Sigma^L(M_V^2) = 0\,, \qquad (6)$$

$$M^2 + \Sigma^T(M_A^2) = 0\,. \qquad (7)$$

Assume that $M_V^2 > 0$. Such a pole corresponds to the spin-one state $|p,\lambda,V\rangle$ which couples to $R_{\mu\nu}$ as

$$\langle 0|R_{\mu\nu}(0)|p,\lambda,V\rangle = |Z_V|^{1/2}u^{(\lambda)}_{\mu\nu}(p)\,. \qquad (8)$$

with

$$Z_V = \frac{1}{1 - \Sigma'^L(M_V^2)},\;\; u^{(\lambda)}_{\mu\nu}(p) = \frac{i}{M_V}\left(p_\mu\epsilon^{(\lambda)}_\nu(p) - p_\nu\epsilon^{(\lambda)}_\mu(p)\right). \qquad (9)$$

Eq. (6) can be solved perturbatively with the result

$$M_V^2 = M^2 + \delta M_V^2\,, \qquad Z_V = 1 + \delta Z_V\,,$$

where δM_V^2 and δZ_V are small corrections vanishing in the free-field limit. This solution corresponds to the original degree of freedom described by the free part of the Lagrangian $\mathcal{L}_0$. The other possible solutions of (6) decouple in the limit of vanishing interaction.

The second possible type of poles with $M_A^2 > 0$ corresponds to the spin-one particle states $|p,\lambda,A\rangle$ with opposite intrinsic parity which couple to the antisymmetric tensor field as

$$\langle 0|R_{\mu\nu}(0)|p,\lambda,A\rangle = |Z_A|^{1/2}w^{(\lambda)}_{\mu\nu}(p)\,, \qquad (10)$$

with

$$Z_A = \frac{1}{\Sigma'^T(M_A^2)},\;\; w^{(\lambda)}_{\mu\nu}(p) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}u^{(\lambda)\alpha\beta}(p)\,. \qquad (11)$$

In the free-field limit, $\Sigma^T(p^2) = 0$ and the additional degrees of freedom are frozen.

The previous discussion suggests that the general form of the interaction Lagrangian can cause the dynamical generating of additional degrees of freedom on the one-loop level. However, these states might correspond to ghosts or tachyons (cf. Ref. [17]).

As a toy example, let us assume the interaction Lagrangian of the form

$$\mathcal{L}_{\rm int} = \frac{\alpha}{4}\langle \partial_\alpha R^{\mu\nu}\partial^\alpha R_{\mu\nu}\rangle\,, \tag{12}$$

which represents actually another type of kinetic term. It generates contribution to both self-energies $\Sigma^{L,T}(p^2)$

$$\Sigma^T(p^2) = \Sigma^L(p^2) = \alpha p^2\,. \tag{13}$$

For $\alpha > 0$, the ghost states are propagated (tachyons for the case $\alpha < 0$). This Lagrangian term is not present on the tree level, but it is dynamically generated in the renormalization procedure as we will see in the next section.

3. *One loop contribution*

In order to avoid lengthy expressions, let us concentrate on the effect of just one special term of the interaction Lagrangian[1] with two resonance fields

$$\mathcal{L}_{\rm int} = d_1\epsilon_{\mu\nu\alpha\sigma}\langle D_\beta u^\sigma\{R^{\mu\nu}, R^{\alpha\beta}\}\rangle + \ldots\,. \tag{14}$$

The most general result will be published in Ref. [17] but it does not differ in essence from what follows.

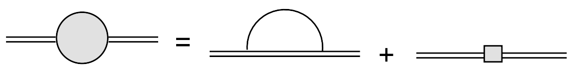

Fig. 1. The one loop correction to tensor self-energy. The double line stands for resonance fields, the single line stands for Goldstone bosons.

The explicit calculation of the Feynman diagram depicted in Fig. 1, with vertices corresponding to the interaction term (14), gives for the infinite part of the self-energies $\Sigma^T(p^2)$ and $\Sigma^L(p^2)$,

$$\Sigma^T_{\rm loop}(p^2) = \Sigma^L_{\rm loop}(p^2) = -\frac{5d_1^2M^2(p^2+M^2)}{6\pi^2F^2}\lambda_\infty + \ldots\,, \tag{15}$$

where

$$\lambda_\infty = \frac{2\mu^{d-4}}{d-4} + \gamma_E - \ln 4\pi - 1\,.$$

[1] The complete list of terms in the even intrinsic parity sector can be found in Ref. [11], the part of the basis for the odd intrinsic parity sector is provided in Ref. [10].

In order to cancel these UV divergences, it is necessary to add to (14) the following counterterms

$$\mathcal{L}_{\rm ct} = \frac{1}{4}\delta M^2 \langle R_{\mu\nu} R^{\mu\nu}\rangle + \frac{\alpha}{4}\langle D^\alpha R_{\mu\nu} D_\alpha R^{\mu\nu}\rangle + \frac{\beta}{2}\langle D^\alpha R_{\alpha\mu} D_\beta R^{\beta\mu}\rangle + \ldots, \qquad (16)$$

i.e., a mass term and two kinetic terms, one of which was not present in the original leading-order Lagrangian. These couterterms contribute to $\Sigma^T(p^2)$ and $\Sigma^L(p^2)$ as (cf. (3) and (13))

$$\Sigma^T_{ct}(p^2) = \delta M^2 + \alpha p^2 , \qquad (17)$$

$$\Sigma^L_{ct}(p^2) = \delta M^2 + (\alpha+\beta) p^2 , \qquad (18)$$

and infinite parts of which are fixed as

$$\delta M^2 = \frac{5 d_1^2 M^4}{6\pi^2 F^2}\lambda_\infty + (\delta M^2)^r(\mu) + \ldots, \quad \alpha = \frac{5 d_1^2 M^2}{24\pi^2 F^2}\lambda_\infty + \alpha^r(\mu), \quad \beta = \beta^r(\mu) + \ldots$$

We see that the chosen interaction Lagrangian has lead to the nontrivial momentum dependence of $\Sigma^T(p^2)$, and therefore to the possible presence of ghost or tachyon states in the antisymmetric formulation of RχT.

It can be shown that not only the antisymmetric formalism but also the vector formalism (the ghost or tachyon states are scalar modes) and the first-order formalism (the structure of states is much richer) suffer from this feature. In Ref. [17], the complete calculation in all three formalisms will be published with complete Lagrangians up to $\mathcal{O}(p^6)$.

4. Conclusion

In this article we have illustrated the problems connected with one-loop renormalization of the propagators of spin-1 resonances within the antisymmetric tensor formulation of RχT. As we have shown by means of explicit calculation, the renormalization of the theory at one-loop level needs counterterms, including new type of kinetic term responsible for possible propagation of additional degrees of freedom which could correspond to negative norm states or tachyons. Analogous feature can be seen also in alternative formulations of RχT with spin-1 resonances described by vector fields or by the first-order formalism [17] and can be understood as a manifestation of the well known fact that, without gauge symmetry and Higgs mechanism, the quantum field theory of massive spin-1 particles suffers from internal inconsistencies.

In all cases, in order to vindicate RχT as a useful effective quantum field theory, we have to take into account this phenomenon.

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RENORMALIZACIJA TENZORSKE SVOJSTVENE ENERGIJE U REZONANTNOJ KIRALNOJ TEORIJI

Proučavamo probleme oko renormalizacije propagatora u rezonantnoj kiralnoj teoriji, usredotočivši se na vektorske rezonancije u formalizmu antisimetričnih tenzora. Opći oblik propagatora antisimetričnih tenzorskih polja sadrži pored rezonantnih stanja i duhove i tahione koji se odvajaju u granici slobodnog polja. Međutim, ako se članovi međudjelovanja uzmu u obzir, oni se stvaraju dinamički renormalizacijskim postupkom.