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FIVE-DIMENSIONAL STRING COSMOLOGICAL MODEL IN SECOND SELF-CREATION THEORY

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In this paper we have constructed a five-dimensional string cosmological model in Barber's (1982) second self-creation theory of gravitation. When the coupling constant becomes zero, the model degenerates into two different string cosmological models in Einstein's theory corresponding to variable G and constant G. Some physical and geometrical properties of the model are discussed.

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1. *Introduction*

Recently cosmic strings have attracted many astrophysicists to try to achieve a plausible description of the early stage of the universe. The study of cosmic strings in elementary particle physics arised from the gauge theories with spontaneous broken symmetry. After the big bang, it is believed that the universe might have experienced a number of phase transitions by producing vacuum domain structures such as domain walls, strings and monopoles. Cosmic strings may act as gravitational lenses and give rise to density perturbations leading to formation of galaxies.

The study of higher-dimensional space-time is an active field of research aiming to unify gravity with other gauge interactions. The concept of extra dimension is relevant in cosmology, particularly for the early stage of universe and theoretically the present four-dimensional stage of the universe might have been preceded by a multi-dimensional stage. In fact, as time evolves, the standard dimensions expand while the extra dimensions shrink to the Planckian dimension, which is beyond our ability to detect with the currently available experimental facilities (Chatterjee [1] and Chatterjee et al. [2]). This fact has attracted many researchers to investigate the problems in the field of higher dimensions (Witten [3], Appelquist et al. [4], Chodos and Detweller [5]). Rahaman et al. [6] obtained exact solutions of the field equations for five-dimensional space-time in Lyra manifold when the source of gravitation are massive strings. Chatterjee [7] constructed massive-string cosmological model in higher-dimensional homogeneous space-time in general relativity. Krori et al. [8] constructed a Bianchi type I string cosmological model in higher dimension and obtained that matter and strings coexist throughout the evolution of the universe. Venkateswarlu and Pavan Kumar [9] constructed higher-dimensional string cosmological model in scale covariant theory of gravitation.

In 1982, Barber [10] proposed two continuous self-creation theories modifying the Brans and Dicke [11] theory and general theory of relativity. The first theory of Barber is in disagreement with experiment and also inconsistent (Brans [12]). The second self-creation theory of Barber is a simple modification of general theory of relativity to a variable-G theory. In this theory, the gravitational coupling of the Einstein's field equations is allowed to be a variable scalar on the space-time manifold. The second self-creation theory predicts the local effects that are within observational limits. The field equations in this theory given by Barber [10] (1982) are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi} T_{ij} \tag{1}$$

and

$$\Box\phi = \frac{8\pi}{3} \lambda T \,, \tag{2}$$

where $\Box\phi = \phi^{k}_{;k}$ is the invariant D'Alembertian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field matter and energy. Here λ is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of the coupling to $|\lambda| \leq 10^{-1}$. This theory leads to the Einstein's theory in every respect when $\lambda \to 0$. Some of the authors who have studied various aspects of different space-times in Barber's second self-creation theory in the presence of different gravitating fields are Venkateswarlu and Reddy [13, 14], Pimentel [15], Soleng [16 – 18], Shanti and Rao [19], Carvallho [20], Shri Ram and Singh [21], Mohanty et al. [22, 23] and recently Venkateswarlu and Pavan Kumar [24].

So far the study of higher-dimensional string cosmological model in Barber's second self-creation theory is not yet found in the literature. Therefore, in this paper we wish to study higher-dimensional string cosmological model in Barber's second self-creation theory.

2. Metric and field equations

Here we consider the five-dimensional line element in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) + A^2 \mathrm{d}\Psi^2 \,, \tag{3}$$

where R and A are functions of cosmic time t only and the fifth co-ordinate is taken to be space-like.

The energy momentum tensor for cosmic string is

$$T_{ij} = \rho\, u_i u_j - \lambda_S\, x_i x_j\,, \tag{4}$$

where $\rho = \rho_p + \lambda_S$ is the rest energy density of cloud of strings with particles attached to them, λ_S is the tension density of the string and ρ_p is the rest energy density of the particles, u' the cloud five-velocity and

$$x' = \left(0,\, 0,\, 0,\, 0,\, \frac{1}{A}\right). \tag{5}$$

The direction of the string will satisfy

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0\,. \tag{6}$$

In the co-moving co-ordinate system, we have from Eqs. (4) − (6)

$$T_{00} = \rho, \quad T_{11} = T_{22} = T_{33} = 0, \quad T_{44} = -\lambda A^2 \quad \text{and} \quad T = -(\rho + \lambda_S)\,. \tag{7}$$

The field equations (1) and (2) with the help of Eqs. (4) − (7) yield the following system of equations:

$$3\left(\frac{R'}{R}\right)^2 + 3\frac{R'A'}{RA} = \frac{8\pi\rho}{\phi}\,, \tag{8}$$

$$2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 + \frac{A''}{A} + 2\frac{R'A'}{RA} = 0\,, \tag{9}$$

$$3\frac{R''}{R} + 3\left(\frac{R'}{R}\right)^2 = \frac{8\pi\lambda_S}{\phi}\,, \tag{10}$$

$$\phi'' + \phi'\left(3\frac{R'}{R} + \frac{A'}{A}\right) = \frac{8\pi}{3}\lambda(\rho + \lambda_S)\,, \tag{11}$$

where dash denotes differentiation with respect to t.

3. Solutions of the field equations

Here we have four independent field equations (8) − (11) connecting five unknowns, viz. R, A, λ_S, ρ and ϕ. In order to get a determinate solution, we assume an analytic relation between the metric coefficients, i.e.

$$A = R^n \qquad \text{(Chakraborty and Chakraborty [25])}\,, \tag{12}$$

where n is a constant.

Using Eq. (12) in Eq. (9), we obtain

$$\frac{R''}{R} + \alpha \left(\frac{R'}{R}\right)^2 = 0\,, \tag{13}$$

where $\alpha = (n^2 + n + 1)/(n + 2)$, $n \neq -2$.

Now for the sake of convenience we take $R = \mathrm{e}^{B(t)}$. Hence Eq. (13) yields

$$B'' + (\alpha + 1){B'}^2 = 0\,. \tag{14}$$

Further, using $B' = f(B)$ in Eq. (14), we obtain

$$R = c_1 \big[(\alpha + 1)t + c\big]^{1/(\alpha+1)}. \tag{15}$$

Substituting equation (15) in equation (12) we get

$$A = c_1^n \big[(\alpha + 1)t + c\big]^{n/(\alpha+1)}, \tag{16}$$

where c_1 ($\neq 0$) and c are constants of integration.

Now using Eqs. (15) and (16) in the field Eqs. (8) and (10) we find

$$\frac{3(n+1)}{\big[(\alpha + 1)t + c\big]^2} = \frac{8\pi\rho}{\phi} \tag{17}$$

and

$$\frac{3(1-\alpha)}{\big[(\alpha + 1)t + c\big]^2} = \frac{8\pi\lambda_S}{\phi}\,. \tag{18}$$

Equation (11) together with (17) and (18) yields

$$\phi = k_1 t^{\left[(l_1-1)+\sqrt{(l_1-1)^2+4l_2}\right]/2} + k_2 t^{\left[(l_1-1)-\sqrt{(l_1-1)^2+4l_2}\right]/2}. \tag{19}$$

Substituting (19) in (17) and (18), we obtain

$$\rho = \frac{1}{8\pi} \frac{3(n+1)}{\big[(\alpha + 1)t + c\big]^2} \left[k_1 t^{\left[(l_1-1)+\sqrt{(l_1-1)^2+4l_2}\right]/2} + k_2 t^{\left[(l_1-1)-\sqrt{(l_1-1)^2+4l_2}\right]/2}\right] \tag{20}$$

and

$$\lambda_S = \frac{1}{8\pi} \frac{3(1-\alpha)}{\big[(\alpha + 1)t + c\big]^2} \left[k_1 t^{\left[(l_1-1)+\sqrt{(l_1-1)^2+4l_2}\right]/2} + k_2 t^{\left[(l_1-1)-\sqrt{(l_1-1)^2+4l_2}\right]/2}\right], \tag{21}$$

where k_1 and k_2 are nonzero constants,

$$l_1 = \frac{n+3}{\alpha+1} \quad \text{and} \quad l_2 = \frac{n+2-\alpha}{(\alpha+1)^2}\,\lambda\,.$$

The particle density is obtained as

$$\rho_p = \rho - \lambda_S$$
$$= \frac{1}{8\pi}\frac{3(n+\alpha)}{\left[(\alpha+1)t+c\right]^2}\left[k_1 t^{\left[(l_1-1)+\sqrt{(l_1-1)^2+4l_2}\,\right]/2} + k_2 t^{\left[(l_1-1)-\sqrt{(l_1-1)^2+4l_2}\,\right]/2}\right]. \tag{22}$$

Thus, the five-dimensional string cosmological model corresponding to the above solution is written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + c_1^2\left[(\alpha+1)t+c\right]^{2/(\alpha+1)}(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2) + c_1^{2n}\left[(\alpha+1)t+c\right]^{2n/(\alpha+1)}\mathrm{d}\Psi^2. \tag{23}$$

4. *Physical and geometrical properties.*

In this section we study the physical and geometrical properties of the model obtained in the preceding section. The metric (23) represents the string cosmological model in Barber's second self-creation theory. At the initial epoch $t = 0$, this space-time becomes flat. Hence it is interesting to note that the model is free from initial singularity. As time increases, the model expands indefinitely along x, y and z axes. The extra dimension becomes insignificant as time proceeds after the creation and we are left with real four dimensional world when $-2 < n < 0$. The parameters involved in the model behave as follows:

(a) The rest energy density satisfies the reality condition $\rho > 0$ if $c = 0$, $-1 < n < 0$ and $k_1,\ k_2 > 0$. Further, $\rho \to \infty$ as $t \to 0$.

(b) The value of expansion scalar θ for the model (23) is obtained as

$$\theta = \frac{n+3}{(\alpha+1)t+c}\,,$$

which indicates that the universe is expanding with increase of time but the rate of expansion becomes slow as the time increases.

(c) The shear scalar σ^2 for the model (23) is given by

$$\sigma^2 = \frac{4[(\alpha+1)t+c]^2 + 3[3+(\alpha+1)t+c]^2 + [3n+(\alpha+1)t+c]^2}{18[(\alpha+1)t+c]^2}\,,$$

(d) The spatial volume of the model is

$$V = c_1^{n+3}\left[(\alpha+1)t+c\right]^{(n+3)/(\alpha+1)}.$$

At the initial epoch $t = 0$, the volume of the universe is zero (if $c = 0$). The volume of the universe increases with the increase of time t and $V \to \infty$ as $t \to \infty$. Hence Eq. (23) represents an expanding model of the universe.

(e) The deceleration parameter of the model is obtained as

$$q = \frac{-3(n+1)}{(n+3)(n+2)} \,.$$

If $-1 < n < 0$, then $q < 0$, which indicates inflation in the model.

(f) If $\lambda \to 0$, we get Einstein string cosmological model given by equation (23) where

$$\rho = \frac{1}{8\pi} \frac{3(n+1)}{\big[(\alpha+1)t + c\big]^2} (k_1 + k_2)\, t^{\big[(l_1-1) \pm (l_1-1)\big]/2},$$

$$\lambda_S = \frac{1}{8\pi} \frac{3(1-\alpha)}{\big[(\alpha+1)t + c\big]^2} (k_1 + k_2)\, t^{\big[(l_1-1) \pm (l_1-1)\big]/2}$$

and

$$\phi = \frac{1}{G} = (k_1 + k_2)\, t^{\big[(l_1-1) \pm (l_1-1)\big]/2} \,,$$

which yields the following two sets of values for ρ, λ and G:

Set 1:

$$\rho = \frac{3(n+1)}{8\pi\big[(\alpha+1)t + c\big]^2} (k_1 + k_2)\, t^{(l_1-1)},$$

$$\lambda_S = \frac{3(1-\alpha)}{8\pi\big[(\alpha+1)t + c\big]^2} (k_1 + k_2)\, t^{(l_1-1)}$$

and

$$G = \frac{1}{(k_1 + k_2)}\, t^{(l_1-1)}.$$

Set 2:

$$\rho = \frac{3(n+1)(k_1 + k_2)}{8\pi\big[(\alpha+1)t + c\big]^2},$$

$$\lambda_S = \frac{3(1-\alpha)(k_1 + k_2)}{8\pi\big[(\alpha+1)t + c\big]^2}$$

and

$$G = \frac{1}{k_1 + k_2} \,.$$

Here Eq. (23), together with set 1 and set 2, represents Einstein string cosmological models with variable G and constant G, respectively.

5. Conclusion

In this paper we obtained an inflationary string cosmological model of Barber's second self-creation theory in a five-dimensional space-time by restricting the constant n, i.e. $-1 < n < 0$. Morever when $\lambda \to 0$, the model degenerates into two different string cosmological models in Einstein's theory, corresponding to variable G and constant G.

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PETERODIMENZIJSKI KOZMOLOŠKI MODEL STRUNA U DRUGOJ SAMOTVORNOJ TEORIJI

U ovom smo radu razvili peterodimenzijski kozmološki model struna u okviru druge Barberove samotvorne teorije gravitacije. Kada konstanta vezanja postane jednaka nuli, model se pretvara u dva različita kozmološka modela struna u Einsteinovoj teoriji koji odgovaraju promjenljivom G i stalnom G. Raspravljaju se neka fizička i geometrijska svojstva modela.