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BIANCHI TYPE-V COSMOLOGICAL MODELS WITH CONSTANT DECELERATION PARAMETER IN SCALAR-TENSOR THEORY

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Einstein's field equations are obtained for Bianchi type-V space-time filled with perfect fluid, and are investigated within the framework of scalar-tensor theory of gravitation proposed by Saez and Ballester. By using a special law of variation for Hubble's parameter, a constant value of the deceleration parameter was obtained. Two different physically viable models of the universe are obtained and one is found to be consistent with the recent observations of type Ia supernovae. Physical and kinematical properties of the models are also discussed.

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1. *Introduction*

In cosmology, the evolution of the universe is described by Einstein's equation together with an equation of state for the matter content. In order to solve the field equations, we normally assume a form for the matter content or suppose that space-time admits killing vector symmetries [1]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman [2], that yields a constant value of deceleration parameter. Cosmological models with a constant deceleration parameter have been studied by Berman [2], Berman and Gomide [3], Beesham [4] and others.

The scalar-tensor theory of gravitation was investigated by Brans and Dicke [5]. In this theory, the scalar field has the dimension of inverse of the gravitational constant G and its role is confined to its effects on gravitational field equations. In their investigation, Saez and Ballester [6] have developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields.

In spite of the dimensionless character of the scalar field, an antigravity regime appears. This theory suggests a possible way to solve the missing-matter problem in nonflat FRW cosmologies. In earlier literature, cosmological models within the framework of Saez-Balleater scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [7, 8], Shri and Tiwari [9], Singh and Shri [10]. Mohanti and Sahu [11, 12] have studied Bianchi type-VI_0 and Bianchi typeI models in Saez-Balleater theory. Singh and Chaubey [13] have studied Bianchi type-V model with a perfect fluid and with the Λ term in general relativity. Saha [14 − 17] has investigated in a series of papers such evolution of Bianchi type-I space-time in the presence of perfect fluid as well as viscous fluid. Bianchi type cosmological models are important in the sense that they are homogenous and isotropic, from which the process of isotropization of the universe is studied through the passage of time. The simplicity of the field equations and relative ease of solution made Bianchi space-time useful in constructing models of spatially homogeneous and an isotropic cosmologies.

In this paper we consider Bianchi type-V models in a scalar-tensor theory proposed by Saez and Ballester in the presence of perfect fluid distribution. The field equations have been solved by using a special law of variation for Hubble's parameter that yields a constant value of the deceleration parameter. Two exact solutions have been obtained. The physical behaviour of the models has been discussed in details in both models.

2. Model and field equations

We consider the Bianchi type-V cosmological model in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\mathrm{d}x^2 - B^2\mathrm{e}^{-2mx}\mathrm{d}y^2 - C^2\mathrm{e}^{-2mx}\mathrm{d}z^2 \,, \tag{1}$$

where A, B and C are the metric functions of the cosmic time t.

We define $R = (ABC)^{1/3}$ as the average scale factor so that Hubble's parameter in the model is defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\theta}{3}\,, \tag{2}$$

where, an over-head dot denotes differentiation with respect to t and θ is the expansion scalar.

The field equations given by Seaz and Ballester [6] for combined scalar and tensor field are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \tag{3}$$

and the scalar field satisfies the equation

$$2\phi^n \phi_{,1}^{;i} + n\phi_{,k}\phi^{,k} = 0\,, \tag{4}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, ω and n are constants, T_{ij} is the stress-energy tensor of the matter, and comma and semicolon denote partial and covariant differentiation, respectively.

The energy-momentum tensor T_{ij} for perfect fluid distribution has the form

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}\,, \tag{5}$$

where u^i is the four-velocity vector satisfying

$$g_{ij}u^i u^j = 1 \tag{6}$$

and ρ and p, respectively, are the energy-density and pressure of the fluid.

In the co-moving coordinate system, the field equations (3) and (4), for the metric (1) in case of (5), read as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -8\pi p + \frac{1}{2}\omega\phi^n\dot{\phi}^2\,, \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -8\pi p + \frac{1}{2}\omega\phi^n\dot{\phi}^2\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -8\pi p + \frac{1}{2}\omega\phi^n\dot{\phi}^2\,, \tag{9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{3m^2}{A^2} = 8\pi\rho - \frac{1}{2}\omega\phi^n\dot{\phi}^2\,, \tag{10}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0\,, \tag{11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0\,. \tag{12}$$

The energy conservation equation

$$T^{ij}_{;j} = 0 \tag{13}$$

leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0\,. \tag{14}$$

Integrating Eq. (11), we get

$$A^2 = k_1 BC\,, \tag{15}$$

where k_1 is a constant of integration, without loss of generality taken as $k_1 = 1$.

From Eqs. (7) – (9), we have

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = \frac{k_2}{ABC} \tag{16}$$

and

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = \frac{k_3}{ABC} \tag{17}$$

where k_2 and k_3 are constants of integration.

3. *Solution of the field equations*

Einstein's field equations (7) – (12) are a coupled system of highly non-linear differential equations. In order to solve these field equations, we normally assume a form for the matter content or suppose that the space-time admits killing vector symmetries [1]. Solutions to the field equations may also be generated by applying a law of variation for the Hubble's parameter, which was first proposed by Berman [2] in FRW models and that yields a constant value of the deceleration parameter. Recently, Kumar and Singh [18] have presented Bianchi type-I models with a constant deceleration parameter in scalar-tensor theory. The law for the variation of Hubble's parameter is

$$H = DR^{-\ell} = D(ABC)^{-\ell/3}\,, \tag{18}$$

where $D \geq 0$ and $\ell > 0$ are constants.

The deceleration parameter q is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2}\,. \tag{19}$$

From Eqs. (2) and (19), we get

$$\frac{\dot{R}}{R} = DR^{-\ell}\,, \tag{20}$$

which on integration leads to

$$R = (\ell\, Dt + c_1)^{1/\ell} \qquad \text{for} \qquad \ell \neq 0\,, \tag{21}$$

and

$$R = c_2 \mathrm{e}^{Dt} \qquad \text{for} \qquad \ell = 0\,, \tag{22}$$

where c_1 and c_2 are constants of integration.

Substituting (21) into (19), we get

$$q = \ell - 1\,. \tag{23}$$

This shows that the the law (18) leads to a constant value of deceleration parameter.

3.1. Cosmology for $\ell \neq 0$

Using Eq. (21) in Eqs. (15) – (17), we obtain the line-element (1) in the form

$$\begin{aligned} \mathrm{d}s^2 = {} & \mathrm{d}t^2 - (\ell\,Dt + c_1)^{2/\ell}\mathrm{d}x^2 - (\ell\,Dt + c_1)^{2/\ell}\exp\left[\frac{2k_2}{D(\ell-3)}(\ell\,Dt + c_1)^{(\ell-3)/\ell}\right]\mathrm{e}^{-2mx}\mathrm{d}y^2 \\ & -(\ell\,Dt + c_1)^{2/\ell}\exp\left[\frac{-2k_2}{D(\ell-3)}(\ell\,Dt + c_1)^{(\ell-3)/\ell}\right]\mathrm{e}^{-2mx}\mathrm{d}z^2, \end{aligned} \tag{24}$$

where $\ell \neq 3$. In this case Eq. (12) reduces to

$$\ddot{\phi} + \dot{\phi}\frac{3D}{(\ell\,Dt + c_1)} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0\,. \tag{25}$$

On integrating (25), we obtain

$$\phi = \left[\frac{n+2}{2}\left\{\frac{k_5}{D(\ell-3)}(\ell\,Dt + c_1)^{(\ell-3)/\ell} + k_6\right\}\right]^{2/(n+2)}, \tag{26}$$

where k_5 and k_6 are constants of integration. The energy density and pressure of the model (24) read as

$$\rho = \frac{1}{8\pi}\left[\frac{3D^2}{(\ell\,Dt + c_1)^2} - \frac{3m^2}{(\ell\,Dt + c_1)^{2/\ell}} + \left(\frac{\omega}{2}k_5^2 - k_2^2\right)\frac{1}{(\ell\,Dt + c_1)^{6/\ell}}\right], \tag{27}$$

$$p = \frac{1}{8\pi}\left[\frac{(2\ell-3)D^2}{(\ell\,Dt + c_1)^2} + \frac{m^2}{(\ell\,Dt + c_1)^{2/\ell}} + \left(\frac{\omega}{2}k_5^2 - k_2^2\right)\frac{1}{(\ell\,Dt + c_1)^{6/\ell}}\right]. \tag{28}$$

Now we find expressions for some other cosmological parameters of the model. The spatial volume V, expansion scalar θ and the shear scalar σ for the model are given by

$$V = (\ell\,Dt + c_1)^{3/\ell}\,\mathrm{e}^{-2mx}, \tag{29}$$

$$\theta = \frac{3D}{(\ell\, Dt + c_1)}\,, \tag{30}$$

and

$$\sigma = \frac{k_2}{(\ell\, Dt + c_1)^{3/\ell}}\,. \tag{31}$$

In the model, we observe that the spatial volume V is zero at $t = -c_1/(\ell D) = t_0$ (say) and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = t_0$, with an infinite rate of expansion. The scalar factors also vanish at $t = t_0$ and, hence, the model has a "*point type*" singularity at the initial epoch. The pressure p, energy density ρ and shear scalar σ diverge at the initial singularity. The scalar field also tends to infinity at the initial epoch provided $\ell < 3$.

The universe exhibits the power law expansion after the big-bang impulse. As t increases, the scalar factors and spatial volume increase, but the expansion scalar decreases. Thus the rate of expansion slows down with the increase in time. Also, ϕ, ρ and p decrease as the time t increases. As $t \to \infty$, the scalar factor and volume become infinite, whereas ϕ, ρ, p, θ and σ^2 tend to zero. Therefore, the model would essentially give an empty universe for large time t. The ratio σ/θ^2 tends to zero as $t \to \infty$ provided $\ell < 3$, so the model approaches isotropy for large values for t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big-bang start approaching to isotropy at late times.

Further, it is observed that the above solution is not valid for $\ell = 3$. For $\ell = 3$ the spatial volume grows linearly with cosmic time. For $\ell > 1$, one obtains $q > 0$; therefore, the model represents a decelerating model of the universe. For $\ell \leq 1$, we get $-1 \leq q \leq 0$, which implies an accelerating model of the universe. Also recent observations of type Ia supernovae (Refs. [19], [20], [21], [22], [23], [24], [25] and [26]) reveal that the present universe is accelerating and the value of deceleration parameter lies in the range $-1 \leq q \leq 0$. It shows that the solutions obtained in this model are consistent with the observations.

3.2. Cosmology for $\ell = 0$

Using (22) in Eqs. (15) – (17), we obtain the line element (1) in the form

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2Dt}\mathrm{d}x^2 - \exp\left\{2Dt - \frac{2k_2}{3Dc_2^2}\mathrm{e}^{-3Dt}\right\}\mathrm{e}^{-2mx}\mathrm{d}y^2 \\ - \exp\left\{2Dt + \frac{2k_2}{3Dc_2^2}\mathrm{e}^{-3Dt}\right\}\mathrm{e}^{-2mx}\mathrm{d}z^2. \end{aligned} \tag{32}$$

In this case the scalar field ϕ is given by

$$\phi(t) = \left[\frac{n+2}{n}\left\{\frac{-k_5}{3Dc_2^3}\mathrm{e}^{-3Dt} + k_7\right\}\right]^{2/(n+2)}, \tag{33}$$

where k_7 is a constant of integration.

The pressure and energy density are given by

$$p = \frac{1}{8\pi}\left[-3D^2 + \frac{m^2}{c_2^2}\mathrm{e}^{-2Dt} + \left(\frac{\omega}{2}k_5^2 - k_2^2\right)\frac{1}{c_2^6}\mathrm{e}^{-6Dt}\right], \tag{34}$$

$$\rho = \frac{1}{8\pi}\left[3D^2 - \frac{3m^2}{c_2^2}\mathrm{e}^{-2Dt} + \left(\frac{\omega}{2}k_5^2 - k_2^2\right)\frac{1}{c_2^6}\mathrm{e}^{-6Dt}\right]. \tag{35}$$

The other cosmological parameters of the model are given by the following expressions:

$$\theta = 3H = 3D, \tag{36}$$

$$V^3 = \mathrm{e}^{3Dt}, \tag{37}$$

$$\sigma = \frac{k_2}{c_2^3}\,\mathrm{e}^{-3Dt}. \tag{38}$$

In the model (32), we observe that the model has no initial singularity. The spatial volume, the scalar factors, scalar field, pressure, energy density and other cosmological parameters are constant at $t = 0$. Thus the universe starts evolving with a constant volume and expands with exponential rate. As t increases, the scalar factors and spatial volume increase exponentially, while the scalar field $\phi(t)$, pressure, energy density, and shear scalar decrease. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and, therefore, the universe exhibits a uniform exponential expansion in the model. As $t \to \infty$, the scalar factors and volume of the universe become infinitely large, whereas the scalar field and shear scalar tend to zero provided $k_7 = 0$. The pressure and energy density become constant such that $p = -\rho$. This shows that the universe is dominated by vacuum energy at late times, which drives the expansion of the universe. The model approaches isotropy for large time t. Thus the model represents a shearing, non-rotating and expanding universe with a finite start approaching to isotropy at late times.

It has also been observed that $\lim(\rho/\theta^2)$ turns out to be constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin; this is a similar result to that already given by Collins [27]. Recent observations of type Ia supernovae (Refs. [19], [20], [21], [22], [23], [24], [25] and [26]) suggest that the universe is accelerating in its present state of evolution. It is believed that the way universe is accelerating presently, it will expand at the fastest possible rate in the future and forever. For $\ell = 0$, we get $q = -1$: this value of the deceleration parameter leads to $\mathrm{d}H/\mathrm{d}t = 0$, which implies the greatest value of the Hubble's parameter and the fastest rate of expansion of the universe. Therefore, the solutions presented in this model are consistent with the observation.

4. Conclusion

Here we have presented a spatially homogeneous Bianchi type-V cosmological model in the scalar-tensor theory proposed by Saez and Ballester [6] in the presence of a perfect fluid. The field equations have been solved exactly by using a special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Two exact physically viable models have been obtained in Secs. 3.1 and 3.2. Expressions for the cosmological parameters have been obtained for both models and behaviour of the models is discussed in detail. For $\ell \neq 0$, the matter and radiation are concentrated at the big-bang epoch and the cosmic expansion is driven by the big-bang impulse. The model has a "point type singularity" at initial epoch as the scalar factor and volume vanish at this moment.

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BIANCHIJEV KOZMOLOŠKI MODEL TIPA V SA STALNIM PARAMETROM USPORAVANJA U SKALARNO-TENZORSKOJ TEORIJI

Izvode se Einsteinove jenadžbe polja za Bianchijev prostor-vrijeme tipa V ispunjen perfektnom tekućinom i one se istražuju u okviru skalarno-tenzorske teorije gravitacije koju su predložili Saez i Ballester. Pretpostavljajući poseban izraz za Hubbleov parametar postignuta je stalnost parametra usporavanja. Izvedena su dva fizički moguća modela svemira, a jedan od njih je u skladu s nedavnim opažnjima supernova tipa Ia. Raspravljaju se također fizička i kinematička svojstva modela.