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BIANCHI TYPE I BULK VISCOUS FLUID TILTED COSMOLOGICAL MODEL FILLED WITH DISORDERED RADIATION AND HEAT CONDUCTION

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Bianchi type I bulk viscous fluid tilted cosmological model filled with disordered radiation and heat conduction is investigated. We assume that the eigenvalue σ_1^1 of the shear tensor σ_i^j is proportional to the expansion (θ) which leads to $A = (BC)^n$ between metric potentials A, B, C with $n = constant$ and $\zeta\theta = K(constant)$, where ζ is the coefficient of bulk viscosity in the model. To get the deterministic model in terms of cosmic time t, we discuss some physically valid special models for different values of parameters. The physical and geometrical aspects of the models are discussed.

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1. *Introduction*

The distribution of matter in the universe is satisfactorily described by a perfect fluid due to the large-scale distribution of galaxies. However, the realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. In the dynamics of homogeneous cosmological models, the introduction of viscosity in the fluid content has been found to explain many physical aspects of the universe. A different picture at the initial stage of the cosmological evolution may appear due to the dissipative process caused by viscosity, as viscosity counteracts the cosmological collapse. Several researchers have attempted to find exact solutions of Einstein's field equations by considering viscous effect in isotropic as well as anisotropic models. Misner [1, 2] studied the effect of viscosity on the evolution of cosmological models. Heller and Klimek [3] obtained a viscous fluid cosmological solution without an initial singularity. They have shown that introduction of bulk viscosity effectively removes the initial singularity in certain classes of cosmological models. Murphy [4] obtained a solution for flat FRW (Friedmann-Robertson-Walker) model considering the effect of the bulk viscosity only. Belinski and Khalatnikov [5] investigated Bianchi Type I cosmological model under the influence of viscosity and found that gravitational field creates matter near the initial singularity. Roy and Prakash [6, 7] investigated some plane symmetric cosmological models for viscous fluid distribution with a free gravitational field of Petrov type I and non-degenerate Petrov type I model. Banerjee and Santos [8] have given some exact solutions for Bianchi type I viscous fluid cosmological models and discussed the role of viscosity in determining the nature of singularity. Banerjee et al. [9] also investigated spatially homogeneous and locally rotationally symmetric Bianchi type II cosmological model under the influence of shear and bulk viscosity for barotropic equation of state and considering a linear relationship between ρ, θ^2 and σ^2, where ρ is the fluid density, θ the expansion and σ the shear. Spatially homogeneous anisotropic homogeneous bulk viscous fluid cosmological model without shear viscosity was investigated by Mohanty and Pattanaik [10]. Bali and Jain [11,12] obtained two viscous fluid Bianchi type I cosmological models in general relativity. In the first model, the coefficient of shear viscosity is assumed to be constant, while in the second model, the coefficient of shear viscosity is proportional to the rate of expansion in the model. Bali and Pradhan [13] have investigated Bianchi type III time-dependent bulk viscous string cosmological model in general relativity.

In the above mentioned studies, orthogonal viscous fluid universes in which the fluid flow vector is normal to the hypersurface of homogeneity have been discussed. The tilted cosmological models in which the fluid flow vector is not normal to the hypersurface of homogeneity are more complicated than the non-tilted ones. The general dynamics of tilted models have been studied by King and Ellis [14], Ellis and King [15], Collins and Ellis [16]. Bradley and Sviestins [17] found that the heat flow is expected for tilted cosmological models. The cosmological models with heat flow were studied by a number of authors, like Ray [18], Roy and Banerjee [19], Coley and Tupper [20, 21], Coley [22]. Pradhan and Rai [23, 24], and Bali and Meena [25] investigated tilted cosmological model filled with disordered radiation of perfect fluid with heat flow. Bali and Sharma [26] obtained some tilted Bianchi type I cosmological models for barotropic perfect fluid with heat flow, using $A = t^\ell$, $B = t^m$, $C = t^n$, where A, B and C are metric potentials and ℓ, m and n are constants. Recently, Pradhan et al. [27] investigated plane symmetric viscous fluid cosmological models with variable cosmological constant (Λ). To get the deterministic model, they also assumed that the coefficient of bulk viscosity is a power function of the mass density, and the coefficient of shear viscosity is proportional to the rate of expansion in the model.

In this paper we investigate the Bianchi type I bulk viscous fluid tilted cosmological model filled with disordered radiation and heat conduction. To get the deterministic model, we assume the conditions $A = (BC)^n$ where A, B and C are metric potentials, n is a constant and $\zeta\theta = constant$, where ζ is the coefficient of bulk viscosity and θ the expansion in the model. We also investigate a special model. The physical and geometrical aspects of the model are also discussed.

2. Metric and field equations

We consider the Bianchi type I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2\,, \tag{1}$$

where A, B and C are functions of t alone.

The energy-momentum tensor for heat conduction given by Ellis [28] and for bulk viscosity given by Landau and Lifshitz [29] is given by

$$T_i^j = (\epsilon + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j - \zeta\theta(g_i^j + v_i v^j)\,, \tag{2}$$

together with

$$g_{ij}v^i v^j = -1\,, \tag{3}$$

$$q_i v^i = 0\,, \tag{4}$$

$$q_i q^i > 0\,, \tag{5}$$

where p is the isotropic pressure, ϵ the matter density and q_i the heat conduction vector orthogonal to v_i. The fluid flow vector v^i has the components

$$\left(\frac{\sinh\lambda}{A},\, 0,\, 0,\, \cosh\lambda\right),$$

satisfying (3), λ being the tilt angle.

The Einstein's field equation

$$R_i^j - \frac{R}{2}g_i^j = -8\pi T_i^j\,,\ \text{in the geometrized unit, where } c=1, G=1 \text{ and taking } \Lambda = 0,$$

for the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\left[(\epsilon + p)\sinh^2\lambda + p + 2q_1\frac{\sinh\lambda}{A} - K\cosh^2\lambda\right], \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi(p - K)\,, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi(p - K)\,, \tag{8}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}=-8\pi\left[-(\epsilon+p)\cosh^2\lambda+p-2q_1\frac{\sinh\lambda}{A}+K\sinh^2\lambda\right], \quad (9)$$

$$(\epsilon+p)A\sinh\lambda\cosh\lambda+q_1\cosh\lambda+q_1\frac{\sinh^2\lambda}{\cosh\lambda}-KA\sinh\lambda\cosh\lambda=0\,, \quad (10)$$

where the subscript '4' denotes the ordinary differentiation with respect to 't'.

3. Solution of the field equations

Equations from (6) – (10) are five equations in seven unknowns, A, B, C, ϵ, p, λ and q_1. For the complete determination of these quantities, we assume that universe is filled with disordered radiation, which leads to

$$3p=\epsilon\,. \quad (11)$$

To get the deterministic solution, we assume a special relation between the metric potentials A, B and C as follows

$$(i)\ A=(BC)^n \quad \text{and} \quad (ii)\ \zeta\theta=K\,, \quad (12)$$

where n and K are constants.

The motive behind assuming the condition (i) $A=(BC)^n$ is explained as follows: Referring to Thorne [30], the observations of velocity-redshift relation for extra-galactic sources suggest that the Hubble expansion of the universe is isotropic to within 30% [31, 32]. More precisely, the redshift studies place the limit $\sigma/H \leq 0.30$ where σ is the shear and H the Hubble constant. Collins et al. [33] have pointed out that for a spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition $\sigma/\theta = constant$. The condition $\sigma_1^1/\theta = constant$ leads to $A=(BC)^n$ when the tilt angle is $\lambda=0$. Here σ_1^1 is the eigenvalue of shear tensor σ_i^j.

The condition $\zeta\theta = constant$ is due to the peculiar characteristic of the bulk viscosity. It acts like a negative energy field in an expanding universe (Johri and Sudarshan [34]), i.e. $\zeta\theta = constant$. In other words, when expansion increases, then bulk viscosity decreases.

Equations (6) and (9) lead to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}=8\pi\big[(\epsilon-p)+K\big]\,. \quad (13)$$

Using the disordered radiation condition $3p=\epsilon$ given by (11) in (13), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=2(8\pi p)+8\pi K\,. \quad (14)$$

Using (8) in (14), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=-2\left[\left(\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}\right)-12\pi K\right]. \tag{15}$$

Equations (7) and (8) lead to

$$\left(\frac{B_{44}}{B}-\frac{C_{44}}{C}\right)+\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=0\,. \tag{16}$$

Equation (12) leads to

$$\frac{A_4}{A}=n\left(\frac{B_4}{B}+\frac{C_4}{C}\right). \tag{17}$$

Thus equation (16) becomes

$$\frac{CB_{44}-BC_{44}}{CB_4-BC_4}+n\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=0\,, \tag{18}$$

which on integration leads to

$$C^2\left(\frac{B}{C}\right)_4=b(BC)^{-n}\,, \tag{19}$$

where b is a constant of integration. Let

$$BC=\mu \quad \text{and} \quad \frac{B}{C}=\nu\,. \tag{20}$$

Using (20) in (19), we have

$$\frac{\nu_4}{\nu}=b\mu^{-(n+1)}\,. \tag{21}$$

Using the assumptions (20) and (12) in (15), we have

$$\frac{\mu_{44}}{\mu}+n\frac{\mu_4^2}{\mu^2}=-2\left[\left(\frac{2n+1}{2}\right)\frac{\mu_{44}}{\mu}+\frac{\nu_{44}}{\nu}+\left(n^2-\frac{n}{2}-\frac{1}{4}\right)\frac{\mu_4^2}{\mu^2}-\frac{1}{4}\frac{\nu_4^2}{\nu^2}+\left(\frac{n+1}{2}\right)\frac{\mu_4\nu_4}{\mu\nu}-12\pi K\right]. \tag{22}$$

Equations (21) and (22) lead to

$$2\mu_{44}+\frac{(4n^2-1)\mu_4^2}{2(n+1)\mu}=-\frac{b^2}{2(n+1)}\mu^{-2n-1}+\frac{24\pi K}{(n+1)}\mu\,. \tag{23}$$

Let us assume that

$$\mu_4=f(\mu)\,. \tag{24}$$

Thus

$$\mu_{44} = \frac{\mathrm{d}\mu_4}{\mathrm{d}t} = ff' \quad \text{and} \quad f' = \frac{\mathrm{d}f}{\mathrm{d}\mu}\,. \tag{25}$$

Therefore, Eq. (23) leads to

$$\frac{\mathrm{d}f^2}{\mathrm{d}\mu} + \frac{(4n^2-1)}{2(n+1)}\frac{f^2}{\mu} = -\frac{b^2}{2(n+1)}\mu^{-2n-1} + \frac{24\pi K}{(n+1)}\mu\,. \tag{26}$$

From Eq. (26), we have

$$f^2 = \frac{b^2}{(4n+1)}\mu^{-2n} + \frac{48\pi K}{(4n^2+4n+3)}\mu^2 + L\mu^{-(4n^2-1)/[2(n+1)]}\,. \tag{27}$$

Equation (21) leads to

$$\frac{\mathrm{d}\nu}{\nu} = \frac{b}{\mu^{(n+1)}}\frac{\mathrm{d}t}{\mathrm{d}\mu}\mathrm{d}\mu\,.$$

Thus, we have

$$\log\nu = \int \frac{b}{\mu^{(n+1)}} \frac{\mathrm{d}\mu}{\sqrt{\frac{b^2}{(4n+1)}\mu^{-2n} + \frac{48\pi K}{(4n^2+4n+3)}\mu^2 + L\mu^{-(4n^2-1)/[2(n+1)]}}}\,. \tag{28}$$

Hence the metric (1) reduces to the form

$$\mathrm{d}s^2 = -\left(\frac{\mathrm{d}t}{\mathrm{d}\mu^2}\right)\mathrm{d}\mu^2 + \mu^{2n}\mathrm{d}x^2 + \mu\left(\nu\mathrm{d}y^2 + \nu^{-1}\mathrm{d}z^2\right), \tag{29}$$

which leads to

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{\frac{b^2}{(4n+1)}T^{-2n} + \frac{48\pi K}{(4n^2+4n+3)}T^2 + LT^{-(4n^2-1)/[2(n+1)]}} + T^{2n}\mathrm{d}X^2 + T(\nu\mathrm{d}Y^2 + \nu^{-1}\mathrm{d}Z^2)\,, \tag{30}$$

where ν is determined by (28) and $\mu = T$.

In the absence of bulk viscosity, i.e. when $K \to 0$, the metric (30) reduces to

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{\frac{b^2}{(4n+1)}T^{-2n} + LT^{-(4n^2-1)/[2(n+1)]}} + T^{2n}\mathrm{d}X^2 + T(\nu\mathrm{d}Y^2 + \nu^{-1}\mathrm{d}Z^2)\,, \tag{31}$$

where ν is determined by (28) when $K = 0$.

4. Some physical and geometrical features

The isotropic pressure (p), the matter density (ϵ), the expansion (θ), $\cosh\lambda$, v^1, v^4, q^1, q^4, σ_{11} and σ_{14} are given by

$$8\pi p = \frac{4\pi K(-4n^2+2n+3)}{4n^2+4n+3} + \frac{4n+1}{8(n+1)}\frac{L}{T^\alpha}\,, \tag{32}$$

where

$$\alpha = \frac{4n^2+4n+3}{2(n+1)}\,, \tag{33}$$

$$8\pi\epsilon = 12\pi K\frac{-4n^2+2n+3}{4n^2+4n+3} + \frac{3(4n+1)}{8(n+1)}\frac{L}{T^\alpha}\,, \tag{34}$$

$$\cosh\lambda = \frac{\big(-\beta n + a_1/T^\alpha\big)^{1/2}}{\big(-\beta + a_2/T^\alpha\big)^{1/2}}\,, \tag{35}$$

where

$$\beta = \frac{24\pi K(2n-1)}{4n^2+4n+3}\,, \tag{36}$$

$$a_1 = \frac{8n^2+14n+3}{8(n+1)}\,L\,, \tag{37}$$

$$a_2 = \frac{8n^2+6n+1}{4(n+1)}\,L\,, \tag{38}$$

$$\theta = v^i_{;i} = \frac{1}{\sqrt{-g}}\,\frac{\partial}{\partial x^i}(v^i\sqrt{-g})$$

$$= \frac{1}{ABC}\left[\frac{\partial}{\partial x}(v^1\sqrt{-g}) + \frac{\partial}{\partial t}(v^4\sqrt{-g})\right]$$

$$= \frac{1}{ABC}\left[\frac{\partial}{\partial x}\left\{\frac{\sinh\lambda}{A}(ABC)\right\} + \frac{\partial}{\partial t}\big\{\cosh\lambda(ABC)\big\}\right]$$

$$= \frac{1}{ABC}\left[0 + (ABC)\frac{\partial}{\partial t}\cosh\lambda + \cosh\lambda(A_4BC + AB_4C + ABC_4)\right]$$

($\because$ λ is a function of t alone)

$$= \frac{\partial}{\partial t}(v^4\sqrt{-g})$$

$$= \frac{\partial}{\partial t}\big[\cosh\lambda\,(ABC)\big]$$

$$= \frac{1}{ABC}\left[(ABC)\frac{\partial}{\partial t}\cosh\lambda + \cosh\lambda(A_4BC + AB_4C + ABC_4)\right]$$

$$= \frac{\partial}{\partial t}\cosh\lambda + \cosh\lambda\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right).$$

We now introduce the following abbreviations for the expressions which repeatedly occur in subsequent equations:

$$E_1 = \frac{b^2}{4n+1}\,\frac{1}{T^{2n+2}} + \frac{48\pi K}{4n^2+4n+3} + \frac{L}{T^\alpha}\,,$$

$$E_2 = -n\beta + \frac{a_1}{T^\alpha}\,,$$

$$E_3 = (a_1 + na_2)\frac{6\pi K}{(n+1)T^\alpha}\,,$$

$$E_4 = \beta + \frac{a_2}{T^\alpha}\,,$$

and

$$E_5 = -(n+1)\beta + \frac{a_1 - a_2}{T^\alpha}\,.$$

With these abbreviations, one obtains

$$\theta = \frac{E_1^{1/2}\big[-E_3(2n-1) + (n+1)E_2E_4\big]}{E_2^{1/2}E_4^{3/2}}\,, \tag{39}$$

$$v^1 = \frac{E_5^{1/2}}{T^n E_4^{1/2}}\,, \tag{40}$$

$$v^4 = \frac{E_2^{1/2}}{E_4^{1/2}}\,, \tag{41}$$

$$\sigma_{11} = \frac{(2n-1)T^{2n}E_1^{1/2}E_2^{1/2}\big[E_4E_2 - 2E_3\big]}{3E_4^{5/2}}\,, \tag{42}$$

$$\sigma_{14} = \frac{-(2n-1)T^n E_1^{1/2} E_5^{1/2} \big[E_4 E_2 - 2E_3\big]}{3E_4^{5/2}}\,, \tag{43}$$

$$\sigma_{11}v^1 + \sigma_{14}v^4 = \frac{(2n-1)T^n E_1^{1/2} E_2^{1/2} E_5^{1/2} \big[E_4 E_2 - 2E_3\big](1-1)}{3E_4^3} = 0\,. \tag{44}$$

Similarly

$$\omega_{11}v^1 + \omega_{14}v^4 = 0\,, \tag{45}$$

$$q^1 = -\frac{1}{8\pi}\frac{E_5^{1/2} E_2}{E_4^{1/2} T^n}\,, \tag{46}$$

$$q^4 = -\frac{1}{8\pi}\frac{E_5 E_2^{1/2}}{E_4^{1/2}}\,. \tag{47}$$

In the absence of bulk viscosity, the above mentioned quantities lead to

$$8\pi\epsilon = \frac{3(4n+1)}{8(n+1)}\frac{L}{T^\alpha}\,, \tag{48}$$

$$8\pi p = \frac{(4n+1)}{8(n+1)}\frac{L}{T^\alpha}\,, \tag{49}$$

where

$$\alpha = \frac{4n^2+4n+3}{2(n+1)}\,,$$

$$\cosh\lambda = \sqrt{\frac{a_1}{a_2}}\,, \tag{50}$$

$$\theta = \left(\frac{b^2}{4n+1}\frac{1}{T^{2n+2}} + \frac{L}{T^\alpha}\right)^{1/2}(n+1)\sqrt{\frac{a_1}{a_2}}\,, \tag{51}$$

$$v^1 = \frac{\sqrt{a_1 - a_2}}{T^n\sqrt{a_2}}\,, \tag{52}$$

$$v^4 = \sqrt{\frac{a_1}{a_2}}\,, \tag{53}$$

$$\sigma_{11} = \frac{(2n-1)T^{2n}\left[\dfrac{b^2}{4n+1}\dfrac{1}{T^{2n+2}} + \dfrac{L}{T^\alpha}\right]^{1/2}\sqrt{a_1}\,(a_1a_2)}{3a_2^{5/2}}\,, \tag{54}$$

$$\sigma_{14} = \frac{-(2n-1)T^{n}\left[\dfrac{b^2}{4n+1}\dfrac{1}{T^{2n+2}} + \dfrac{L}{T^\alpha}\right]^{1/2}\sqrt{a_1-a_2}\,(a_1a_2)}{3a_2^{5/2}}\,. \tag{55}$$

Thus

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0\,. \tag{56}$$

Similarly

$$\omega_{11}v^1 + \omega_{14}v^4 = 0\,, \tag{57}$$

The physical significance of conditions (44), (45), (46) and (57) are explained by Ellis [35]: The shear tensor (σ_{ij}) determines the distortion arising in the fluid flow, leaving the volume invariant. The direction of principal axis is unchanged by the distortion, but all other directions are changed. Thus we have

$$\sigma_{ij}v^j = 0\,,$$

which leads to

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0 \qquad (\because v_1 \neq 0, v_4 \neq 0)\,.$$

Shear (σ) is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}\,.$$

Thus $\sigma^2 \geq 0$ and $\sigma = 0 \Leftrightarrow \sigma_{ij} = 0$.

The vorticity tensor (ω_{ij}) determines a rigid rotation of cluster of galaxies with respect to a local inertial rest frame. Thus, we have

$$\omega_{ij} = \eta_{ijk\ell}\omega^k\omega^\ell\,,$$

where $\eta_{ijk\ell}$ is pseudo tensor and

$$\omega^i = \frac{1}{2}\eta_{ijk\ell}v_j\omega_{k\ell}\,.$$

Thus

$$\omega_{ij}v^j = 0\,.$$

This leads to

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0 \qquad (\because v_1 \neq 0, v_4 \neq 0)\,.$$

The magnitude of ω_{ij} is ω and is defined as

$$\omega^2 = \frac{1}{2}\omega_{ij}\omega^{ij} \,,$$

$$\omega = 0 \quad \Leftrightarrow \quad \omega_{ij} = 0 \,,$$

$$q^1 = -\frac{1}{8\pi} \, \frac{(a_1 - a_2)^{1/2} \, a_1}{\sqrt{a_2} T^{\alpha+n}} \,, \tag{58}$$

$$q^4 = -\frac{1}{8\pi} \, \frac{(a_1 - a_2) \sqrt{a_1}}{\sqrt{a_2} T^{\alpha}} \,, \tag{59}$$

4.1. Discussion

The model (30) represents a tilted model, $\epsilon \to \infty$ when $T \to 0$ and $\alpha > 0$. The model (30) starts with a big-bang at $T = 0$, and the expansion in the model decreases as time increases. $\sigma_{ij}v^j = 0$ and $\omega_{ij}v^j = 0$ are satisfied as $\sigma_{11}v^1 + \sigma_{14}v^4 = 0$ and $\omega_{11}v^1 + \omega_{14}v^4 = 0$. Since $\lim_{T\to\infty} \sigma/\theta \neq 0$. Hence the model does not approach isotropy for large values of T. The model (30) has the point-type singularity at $T = 0$ when $n > 0$, and it has the cigar-type singularity at $T = 0$ [36] when $n < 0$.

In the absence of bulk viscosity $\epsilon \to \infty$, when $T \to 0$ and $\alpha > 0$, and $\epsilon \to 0$ when $T \to \infty$ and $\alpha > 0$. The reality conditions $\epsilon + p > 0$, $\epsilon + 3p > 0$ given by Ellis [37] are satisfied when $4n + 1 > 0$. The model (31) represents a tilted model. The model (31) starts with a big-bang at $T = 0$ when $\alpha > 0$ and $n + 1 > 0$. The expansion in the model decreases as time increases. The model (31) has point-type singularity at $T = 0$ when $n > 0$ and it has cigar-type singularity at $T = 0$ when $n < 0$. $\sigma/\theta \neq 0$. Hence the model (31) represents an anisotropic universe.

5. Special cases

To get the deterministic model in terms of cosmic time t, we assume $b = 0$ and $n = \frac{1}{2}$. Thus Eq. (21) leads to

$$v = constant = M \,. \tag{60}$$

Now using $n = \frac{1}{2}$ in equation (27), we have

$$\frac{\mathrm{d}\mu}{\sqrt{\mu^2 + \alpha^2}} = \sqrt{8\pi K}\, \mathrm{d}t \,, \tag{61}$$

where

$$\alpha^2 = \frac{L}{8\pi K} \,, \tag{62}$$

which leads to

$$\mu = \frac{\sqrt{L}}{\sqrt{8\pi K}} \sinh(\sqrt{8\pi K}\, t) \,. \tag{63}$$

Hence the metric (1) reduces to the form

$$\mathrm{d}S^2 = -\mathrm{d}t^2 + \frac{\sqrt{L}}{\sqrt{8\pi K}} \sinh(\sqrt{8\pi K}\, t) \left(\mathrm{d}x^2 + M\mathrm{d}y^2 + \frac{\mathrm{d}z^2}{M} \right) . \tag{64}$$

In the absence of bulk viscosity, i.e. when $K \to 0$, the metric (64) leads to

$$\mathrm{d}S^2 = -\mathrm{d}T^2 + T\big(\mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2\big) \,, \tag{65}$$

where $L^{1/4}x = X$, $L^{1/4}\sqrt{M}y = Y$, $L^{1/4}\frac{1}{\sqrt{M}}z = Z$ and t=T.

5.1. *Physical and geometrical features*

The isotropic pressure (p),the matter density (ϵ), the expansion (θ), $\cosh\lambda$, v^1, v^4, q^1, q^4, σ_{11} and σ_{44} are given by

$$p = \frac{8\pi K}{32\pi} \coth^2(\sqrt{8\pi K}\, t) \,, \tag{66}$$

$$\epsilon = 3\frac{8\pi K}{32\pi} \coth^2(\sqrt{8\pi K}\, t) \,, \tag{67}$$

$$\cosh\lambda = 1 \,, \tag{68}$$

$$\theta = \frac{3}{2}\sqrt{8\pi K} \coth(\sqrt{8\pi K}\, t) \,, \tag{69}$$

$$v^1 = 0 \,, \tag{70}$$

$$v^4 = 0 \,, \tag{71}$$

$$\sigma_{11} = 0 \,, \tag{72}$$

$$\sigma_{14} = 0 \,. \tag{73}$$

Thus

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0 \,. \tag{74}$$

Similarly

$$\omega_{11}v^1 + \omega_{14}v^4 = 0 \,, \tag{75}$$

$$\sigma_{22} = 0, \quad \sigma_{33} = 0, \quad \sigma_{44} = 0\,, \tag{76}$$

$$q^1 = 0, \quad q^4 = 0\,. \tag{77}$$

The deceleration parameter (q) is given by

$$q = -\frac{2 - \coth^2(\sqrt{8\pi K}\,t)}{\coth^2(\sqrt{8\pi K}\,t)}\,. \tag{78}$$

In the absence of viscosity, the above quantities lead to

$$\epsilon = \frac{3}{32\pi}\frac{1}{t^2}\,, \tag{79}$$

$$p = \frac{1}{32\pi}\frac{1}{t^2}\,, \tag{80}$$

$$\cosh\lambda = 1\,, \tag{81}$$

$$\theta = \frac{3}{2t}\,. \tag{82}$$

5.2. *Discussion*

In the presence of bulk viscosity, the energy density $\epsilon \to \infty$ at $t = 0$. The reality conditions $\epsilon + p > 0$ and $\epsilon + 3p > 0$, given by [37], lead to $\coth(\sqrt{8\pi K}\,t) > 0$. The model (64) starts with a big-bang at $t = 0$ and the expansion in the model decreases as time increases. The model (64) has a point-type singularity at $t = 0$ [36]. The deceleration parameter $q < 0$ if $\coth^2(\sqrt{8\pi K}\,t) < 2$ and, hence, the model represents an accelerating universe. $q > 0$ if $\coth^2(\sqrt{8\pi K}\,t) > 2$. Therefore, the model in this condition represents a decelerating universe. Since $\cosh\lambda = 1$. This leads to $\lambda = 0$. Hence the model in the presence of bulk viscosity represents a non-tilted model in the special case.

It is possible to discuss entropy for the special model (64) which is obtained in terms of cosmic time t in the presence of bulk viscosity. To solve the entropy problem of the standard model, it is necessary to have $\mathrm{d}S > 0$ for at least a part of the evolution of the universe. In Riemannian geometry without a cosmological constant, we have

$$T\mathrm{d}S = \mathrm{d}(\epsilon S^3) + p\mathrm{d}(S^3)\,, \tag{83}$$

where S is the scale factor. Equation (83) leads to

$$T\mathrm{d}S = \dot{\epsilon} + (\epsilon + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0\,.$$

This again leads to

$$T\mathrm{d}S = \dot{\epsilon} + \frac{4}{3}\epsilon\theta\,,$$

since $\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$ for the metric (1) and for the disordered radiation $\epsilon = 3p$. For entropy, we have

$$S > 0.$$

This leads to

$$\frac{\dot{\epsilon}}{\epsilon} + \frac{4}{3}\theta > 0\,, \tag{84}$$

which for the metric (64) in the presence of bulk viscosity leads to

$$2\sqrt{8\pi K}\tanh(\sqrt{8\pi K}\,t) > 0\,.$$

Thus entropy is possible in the presence of bulk viscosity for the special model (64).

In the absence of bulk viscosity $\epsilon \to \infty$ when $t \to 0$. The reality condition $\epsilon > 0$ is satisfied. The model (65), in the absence of bulk viscosity, starts with a big-bang at $t = 0$, and the expansion in the model decreases as time increases. Since $\sigma = 0$. Hence the model (65) represents an isotropic model. The model (65) has point-type singularity at $T = 0$ [36]. The deceleration parameter $q = 1$ in the absence of bulk viscosity. The special model, discussed above represents non-tilted, non-shearing, non-rotating and decelearating universe.

5.3. *Some other cases*

We also considered the following cases, but these do not give physically valid results:

(i) For $b = 0$, $n = -1/2$, a solution is possible but the matter density is negative and the tilt angle is not determined in the absence of bulk viscosity.

(ii) For $L = 0$, the solution is possible but the tilt angle is not determined.

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BIANCHIJEV VOLUMNO VISKOZAN ZAKRENUT KOZMOLOŠKI MODEL TIPA I ISPUNJEN NEREDNIM ZRAČENJEM I VOĐENJEM TOPLINE

Istražujemo Bianchijev volumno viskozan zakrenut kozmološki model s nerednim zračenjem i vođenjem topline. Da bi postigli određenost modela, pretpostavili smo uvjet: svojstvena vrijednost σ_1^1 smičnog tenzora σ_i^j razmjerna je širenju (θ), što vodi na relaciju $A = (BC)^n$ među metričkim potencijalima A, B i C, gdje je n stalan a $\zeta\theta = K(const)$. ζ je koeficijent volumne viskoznosti u modelu. Radi postizanja određenosti modela za kozmičko vrijeme t, raspravljamo neke posebne fizikalno ispravne modele za različite vrijednosti parametara. Također raspravljamo fizičke i geometrijske značajke tih modela.