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A MODEL TO EXPLAIN VARYING G AND Λ WITH CONSTANT DECELERATION PARAMETER

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We study the evolution of homogeneous and anisotropic Bianchi type-I cosmological models in the presence of perfect fluid with variable G and Λ by assuming a special law of variation for Hubble's parameter that yields a constant value of the deceleration parameter. Some physical consequences of the model are discussed in the case of Zel'dovich fluid and radiation-dominated fluid.

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1. *Introduction*

One of the most important unsolved problem in cosmology is the cosmological constant (Λ) problem. The problem is related with the extensive efforts to explain the age, formation and structure of Universe [1, 2, 3]. After the introduction of Λ by Einstein in 1917, it has been studied by various researchers [4, 5] from time to time. The results for $\Lambda \neq 0$ are favoured by recent supernovae (SNe) Ia observations [6 − 10] which are in accord with the recent anisotropy measurements of the cosmic microwave background (CMB) made by the WMAP experiment [11]. However, there is a fundamental problem related with the existence of Λ, which has been extensively discussed in the literature. The value of Λ expected from the quantum field theory-calculations is about 120 orders of magnitude higher than that estimated from the observations. A phenomenological solution to the problem is suggested by considering Λ as a function of time, so that it was large in the early Universe and became reduced with the expansion of the Universe [12 − 20].

Several modifications of general relativity have been proposed to allow for a variable gravitational constant, G, based on different arguments [21 – 23]. To consider a jointly the variation of G and Λ within the framework of general relativity has been introduced recently [24 – 28]. At the present state of evolution, the Universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But in its early stages of evolution, it could not have had such a smoothed out picture, so the forms of matter fields in the early Universe are uncertain. Friedmann-Robertson-Walker (FRW) models, being isotropic and homogeneous, represent best the large-scale structure of the present Universe. But to describe the early stages of the evolution of the Universe, models with anisotropic background are suitable. The simplest anisotropic models of the universe are Bianchi type-I homogeneous models whose spatial sections are flat, but the expansion or contraction rate are directionally dependent. For a simplification and description of the large scale structure and behaviour, for the description of the actual Universe, anisotropic Bianchi type I models have been considered by a number of authors. Researchers [29 – 32] have studied anisotropic Bianchi type-I model in different context. Saha [33 – 35] has investigated Bianchi type-I models with variables G and Λ.

In this paper, we consider the space-time to be of the Bianchi type-I with variable G and Λ, in the presence of a perfect fluid. In order to solve the field equations, we apply a law of variation for Hubble's parameter [36 – 38] that yields a constant value of the deceleration parameter. This law, together with the Einstein's field equations, leads to a number of new solutions of the Bianchi type-I space-time. The physical behaviour of the models is discussed in detail and the nature of initial singularity is clarified.

2. Model and field equations

The spatially homogeneous and anisotropic Bianchi type-I space-time is described by the line element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2(t)\mathrm{d}x^2 + B^2(t)\mathrm{d}y^2 + C^2(t)\mathrm{d}z^2 \,. \tag{1}$$

The spatial volume of this model is given by

$$V^3 = ABC \,. \tag{2}$$

We define $R^3 = (ABC)$ as the average scale factor so that the Hubble's parameters is anisotropic and may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{3}$$

where θ is the expansion scale.

Here and elsewhere a dot stands for the ordinary time-derivative of the concerned quantity.

Also, we define

$$H = \frac{1}{3}\big(H_1 + H_2 + H_3\big)\,, \tag{4}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are directional Hubble's factors in the directions of x, y and z, respectively.

We assume that the cosmic matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}\,, \tag{5}$$

where ρ and p are energy density and thermo-dynamical pressure, and v_i is the four-velocity vector of the fluid satisfying the relation $v_i v^i = -1$.

We assume that the matter content obeys the equation of state

$$p = \omega\rho, \qquad 0 \le \omega \le 1\,. \tag{6}$$

The Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2}g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}\,. \tag{7}$$

For the metric (1) and energy-momentum tensor (5) in co-moving system of coordinates, the field equation (7) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -8\pi G p + \Lambda\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -8\pi G p + \Lambda\,, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -8\pi G p + \Lambda\,, \tag{10}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = 8\pi G p + \Lambda\,. \tag{11}$$

In view of the vanishing divergence of the Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0\,. \tag{12}$$

The usual energy conservation equation $T^{j}_{i;j}$ yields

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0\,. \tag{13}$$

Equation (12) together with (13) give

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0\,. \tag{14}$$

The non-vanishing components of the shear tensor σ_{ij}, defined by

$$\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3} g_{ij} u^{k}_{,k}\,,$$

are obtained as

$$\sigma^{1}_{1} = \frac{4}{3}\frac{\dot{A}}{A} - \frac{2}{3}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{15}$$

$$\sigma^{2}_{2} = \frac{4}{3}\frac{\dot{B}}{B} - \frac{2}{3}\left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\right), \tag{16}$$

$$\sigma^{3}_{3} = \frac{4}{3}\frac{\dot{C}}{C} - \frac{2}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{17}$$

Thus the shear scalar σ is obtained as

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A}\right)\right]. \tag{18}$$

From Eq. (18) and equation Eq. (3), we obtain

$$\frac{\dot{\sigma}}{\sigma} = -\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -3H\,. \tag{19}$$

Einstein's field equations (8) – (11) can also be written in terms of Hubble's parameters H, shear scalar σ and deceleration parameter q as

$$H^2(2q-1) - \sigma^2 = 8\pi G p - \Lambda\,, \tag{20}$$

$$3H^2 - \sigma^2 = 8\pi G p + \Lambda\,, \tag{21}$$

where the deceleration parameter is given by

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}\,. \tag{22}$$

On integrating Eqs. (8) – (10), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC}\,, \tag{23}$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{ABC}\,. \tag{24}$$

where k_1 and k_2 are constants of integration.

3. Solution of the field equations

Equations (6), (8) – (11) together with Eq. (13) are only six independent equations in seven unknowns A, B, C, ρ, p, G and Λ. One extra equation is needed to solve the system completely, which we shall obtain in the following by using a law of variation of Hubble's parameter. Initially, this variation law was proposed by Berman [36] in FRW models. It yields a constant value of the deceleration parameter. This variation of the Hubble's parameter is consistent with observations. Recently, we used a similar type of law of variation for Hubble's parameter in Bianchi type-I space time in self-creation cosmology that yields a constant value of the deceleration parameter [38].

To determinate the solution of the field equations (8) – (11), we assume the variation of the Hubble parameter given by Berman [36]

$$H = DR^{-m} = D(ABC)^{-m/3}, \tag{25}$$

where $D > 0$ and $m \geq 0$ are constants. From Eqs. (3) and (25), we get

$$R = (mDt + c_1)^{1/m}, \qquad \text{if} \qquad m \neq 0\,, \tag{26}$$

$$R = c_2 \mathrm{e}^{Dt}, \qquad \text{if} \qquad m = 0\,, \tag{27}$$

where c_1 and c_2 are constants of integration. Substituting (26) into (22), we get

$$q = m - 1\,. \tag{28}$$

This shows that the deceleration parameter is constant.

4. *Power law cosmology ($m \neq 0$)*

When $m \neq 0$, using Eqs. (23),(24) and (26), we obtain the line-element (1) in the form

$$ds^2 = -dt^2 + (mDt + c_1)^{2/m} \exp\left[\frac{2(2k_1 + k_2)}{3D(m-3)}(mDt + c_1)^{(m-3)/m}\right] dx^2 \tag{29}$$

$$+(mDt + c_1)^{2/m} \exp\left[\frac{2(k_2 - k_1)}{3D(m-3)}(mDt + c_1)^{(m-3)/m}\right] dy^2$$

$$+(mDt + c_1)^{2/m} \exp\left[\frac{-2(k_1 + 2k_2)}{3D(m-3)}(mDt + c_1)^{(m-3)/m}\right] dz^2 \,.$$

Now we discuss the model for Zel'dovich fluid, radiation dominated case and vacuum case.

4.1. *Zel'dovich fluid distribution ($\omega = 1$)*

It corresponds to the equation of state $\rho = p$. This equation of state has been widely used in general relativity [39]. We find that the model in this case is described by

$$V^3 = (mDt + c_1)^{3/m}, \tag{30}$$

$$\rho = p = \frac{k_3}{(mDt + c_1)^{6/m}}\,, \tag{31}$$

$$\sigma = \sqrt{\frac{(k_1^2 + k_1 k_2 + k_2^2)}{3}}\,\frac{1}{(mDt + c_1)^{3/m}}\,, \tag{32}$$

$$G(t) = \frac{1}{8\pi k_3}\left[\frac{mD^2}{(mDt + c_1)^{2(m-3)/m}} - \frac{(k_1^2 + k_1 k_2 + k_2^2)}{3}\right], \tag{33}$$

$$\Lambda(t) = \frac{2D^2(3-m)}{(mDt + c_1)^2}\,, \tag{34}$$

$$\theta = \frac{3D}{(mDt + c_1)}\,. \tag{35}$$

The anisotropy parameter is defined by

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right), \tag{36}$$

where $\Delta H_i = H_i - H,\ (i = 1, 2, 3)$. The directional Hubble's factors, as defined in (4), are given by

$$H_1 = \frac{D}{(mDt + c_1)} + \frac{2k_1 + k_2}{3(mDt + c_1)^{3/m}}\,, \tag{37}$$

$$H_2 = \frac{D}{(mDt + c_1)} + \frac{k_2 - k_1}{3(mDt + c_1)^{3/m}}\,, \tag{38}$$

$$H_3 = \frac{D}{(mDt + c_1)} - \frac{k_1 + 2k_2}{3(mDt + c_1)^{3/m}}\,. \tag{39}$$

Using Eqs. (35), (37), (38) and (39) in (36), we get

$$\bar{A} = \frac{2}{9}\ \frac{(k_2^2 + k_1k_2 + k_2^2)}{D^2(mDt + c_1)^{(6-2m)/m}}\,. \tag{40}$$

We observe that the spatial volume V is zero at $t = -c_1/(mD) = t_0$ (say), and expansion scalar θ is infinite at $t = t_0$ which shows that the universe starts evolving with zero volume and infinite rate of expansion at $t = t_0$. Initially, at $t = t_0$, the spacetime exhibits a 'point type' singularity. At $t = t_0$, ρ, p, Λ, θ, σ, G and $\bar{A}$ are all infinite. As t increases the spatial volume increases, but the expansion scalar decreases. Thus, the expansion rate decreases as the time increases.

As $t \to \infty$, the spatial volume V becomes infinitely large. All parameters, ρ, p, Λ, θ, σ, H_1, H_2 and H_3 tend to zero, and G is constant at late times. Therefore, the model essentially gives an empty universe for large t. The ratio $\sigma/\theta \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for large value of t. We see that Λ is positive and also $\Lambda \sim 1/t^2$, i.e., Λ is a decreasing function of time [25]. This supports the results obtained from recent supernova Ia observations [40]. Also, G is a decreasing function of time and becomes negligible for large t. Therefore, the model represents shearing, non-rotating and expanding universe with a big-bang start.

4.2. Radiation dominated solution ($\omega = \frac{1}{3}$)

Disordered radiation corresponds to the equation of state $\rho = 3p$. In this case, the model is described by

$$V^3 = (mDt + c_1)^{3/m}, \tag{41}$$

$$\rho = 3p = \frac{k_3}{(mDt + c_1)^{4/m}}\,. \tag{42}$$

Other cosmological parameters are:

$$\theta = \frac{3D}{(mDt + c_1)}\,, \tag{43}$$

$$\sigma = \sqrt{\frac{k_1^2 + k_1 k_2 + k_2^2}{\sqrt{3}\,(mDt + c_1)^{3/m}}}\,, \tag{44}$$

$$\bar{A} = \frac{2(k_1^2 + k_1 k_2 + k_2^2)}{9D^2(mDt + c_1)^{(6-2m)/m}}\,, \tag{45}$$

$$G(t) = \frac{1}{16\pi k_3}\left[\frac{3mD^2}{(mDt + c_1)^{(2m-4)/m}} - \frac{k_1^2 + k_1 k_2 + k_2^2}{(mDt + c_1)^{2/m}}\right], \tag{46}$$

$$\Lambda(t) = \frac{2D^2(2-m)}{(mDt + c_1)^2} + \frac{k_1^2 + k_1 k_2 + k_2^2}{6(mDt + c_1)^{6/m}}\,. \tag{47}$$

In order to satisfy the reality condition of energy density and pressure, we require $k_3 > 0$. The model has a singularity at $t = -c_1/(mD) = t_0$. The model starts from a big-bang with ρ, p, A, $\bar{A}$, θ, σ and G all infinite. The space-time exhibits a 'point type' singularity at $t = t_0$. As t increases, spatial volume V increases, but the rate of expansion slows down. All physical parameters decrease with time. Spatial volume V becomes infinitely large as $t \to \infty$, while ρ, p, $\bar{A}$, Λ, θ, σ and G vanish asymptotically. Since $\lim_{t\to\infty} \sigma/\theta \to 0$, the model approaches isotropy for large value of t. Therefore, the model represents non-rotating, shearing and expanding universe with a big-bang start.

4.3. Vacuum solution: ($\omega = 0$)

In this case, cosmological parameters are:

$$\theta = \frac{3D}{(mDt + c_1)}\,, \tag{48}$$

$$\sigma = \frac{\sqrt{(k_1^2 + k_1 k_2 + k_2^2)}}{\sqrt{3}\,(mDt + c_1)^{3/m}}\,, \tag{49}$$

$$\bar{A} = \frac{2(k_1^2 + k_1 k_2 + k_2^2)}{9D^2(mDt + c_1)^{(6-2m)/m}}\,, \tag{50}$$

$$\Lambda(t) = \frac{D^2(3-2m)}{(mDt + c_1)^2} + \frac{k_1^2 + k_1 k_2 + k_2^2}{3(mDt + c_1)^{6/m}}\,, \tag{51}$$

$$G(t) = \frac{1}{4\pi k_3}\left[\frac{mD^2}{(mDt + c_1)^{(2m-3)/m}} - \frac{k_1^2 + k_1 k_2 + k_2^2}{3(mDt + c_1)^{3/m}}\right]. \tag{52}$$

Clearly, the spatial volume is zero and expansion scalar is infinite at initial singularity $t = -c_1/(mD) = t_0$. The universe starts expanding with zero volume and at infinite rate of expansion. At $t = t_0$, the space-time exhibits a 'point type' singularity. $\bar{A}$, σ, Λ and G tend to infinity at initial singularity. As t increases, spatial volume increases, but the rate of expansion slows down. $\bar{A}$, σ, Λ and G vanish asymptotically. The ratio σ/θ tends to zero when $t \to \infty$, thus the model approaches isotropy.

5. *Exponential cosmology ($m = 0$)*

When $m = 0$, using Eqs. (23), (24) and (27), we obtain the line-element (1) in the form :

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \exp\left[2Dt + \frac{2(2k_1 + k_2)}{9DC_2^3}\mathrm{e}^{-3Dt}\right]\mathrm{d}x^2 \tag{53}$$

$$+ \exp\left[2Dt + \frac{2(k_2 - k_1)}{9DC_2^3}\mathrm{e}^{-3Dt}\right]\mathrm{d}y^2 + \exp\left[2Dt - \frac{2(k_1 + 2k_2)}{9DC_2^3}\mathrm{e}^{-3Dt}\right]\mathrm{d}z^2 .$$

We analyze the model for Zel'dovich fluid, radiation dominated case and vacuum case.

5.1. *Zel'dovich fluid distribution ($\omega = 1$)*

It corresponds to the equation of state $\rho = p$. In this case, the physical parameters are given by the expressions:

$$\rho = p = \frac{k_3}{c_2^6\,\mathrm{e}^{6Dt}}\,, \tag{54}$$

$$\theta = 3D\,, \tag{55}$$

$$\bar{A} = \frac{2}{9D^2\,c_2^6}\,(k_1^2 + k_1k_2 + k_2^2)\mathrm{e}^{-6Dt}, \tag{56}$$

$$\sigma^2 = \frac{1}{3c_2^3}\,(k_1^2 + k_1k_2 + k_2^2)\mathrm{e}^{-6Dt}, \tag{57}$$

$$\Lambda = 3D^2, \tag{58}$$

$$G = -\frac{(k_1^2 + k_1k_2 + k_2^2)}{24\pi}\,. \tag{59}$$

The model has no initial singularity. ρ, p, θ, $\bar{A}$ and σ are constant at $t = 0$. The universe starts evolving with a constant volume and expands with an exponential rate. The energy density and pressure decrease while spatial volume increases as the cosmic time increases. As $t \to \infty$, ρ, p, $\bar{A}$ and σ tend to zero. The cosmological constant Λ is finite and gravitational constant G becomes negative during the whole span of evolution. The negative gravitational constant has been discussed by Vishwakarma [18]. It is interesting to note that the expansion scalar is a constant for $0 \leq t \leq \infty$ and, therefore, the model represents uniform expansion. The ratio $\sigma/\theta \to 0$ as $t \to \infty$, and the model approaches isotropy for large value of t. Also, $\rho/\theta^2 = constant$. Thus the model approaches homogeneity and matter is dynamically negligible near the origin. This is similar to the result given by Collins [41]. The model represents shearing non-rotating and expanding universe with a finite start.

5.2. Radiation dominated solution ($\omega = \frac{1}{3}$)

In this case the physical parameters are:

$$\rho = 3p = \frac{k_3}{c_2^4\, e^{4Dt}}\,, \tag{60}$$

$$\theta = 3D\,, \tag{61}$$

$$\bar{A} = \frac{2}{9D^2\, c_2^6}\,(k_1^2 + k_1 k_2 + k_2^2) e^{-6Dt}, \tag{62}$$

$$\sigma^2 = \frac{1}{3c_2^3}\,(k_1^2 + k_1 k_2 + k_2^2) e^{-6Dt}, \tag{63}$$

$$\Lambda = 3D^2 + \frac{1}{6\, c_2^6}\,(k_1^2 + k_1 k_2 + k_2^2) e^{-6Dt}, \tag{64}$$

$$G = -\frac{(k_1^2 + k_1 k_2 + k_2^2)}{16\pi c_2^2 k_3} e^{-2Dt}. \tag{65}$$

The model has no initial singularity. Initially, ρ, p, $\bar{A}$, σ, Λ and G are constant. The universe starts evolving with a constant volume and expands with an exponential rate. When t increases, ρ, p, $\bar{A}$, σ decrease. As $t \to \infty$, ρ, p, $\bar{A}$, σ and G tend to zero, whenever Λ is constant. The model expands uniformly and approaches isotropy for large value of t.

5.3. Vacuum solution ($\omega = 0$)

In this case, the behaviour of the model is the same as in Sec. 5.2 above.

6. Conclusion

In this paper we present a class of solutions to the Einstein's field equations for the orthogonal Bianchi type-I space-time in the presence of a perfect fluid with variable G and Λ. Cosmological models with a constant deceleration parameter are presented for $m \neq 0$ and $m = 0$ cosmologies. There are two solutions: one is the power-law solution and the other is the exponential solution. We discuss both solutions for Zel'dovich fluid, radiation and vacuum cases. We discuss geometrical and kinematical properties of different parameters in detail for each phase. The nature of singularities of the models is clarified and explicit forms of scalar factors are obtained in each case. For $m \neq 3$, the spatial volume V grows linearly with cosmic time. It has been observed that the model represents shearing, non-rotating and expanding universe with a big-bang start. For $m \neq 0$, cosmological constant $\Lambda \propto 1/t^2$, which is thought to be fundamental [25]. Recent cosmological observations ([6], [7], [9], [10], [40]) suggest the existence of a positive constant Λ of magnitude $\Lambda(Gh/c^3) \sim 10^{-123}$. These observations on magnitude and redshift of type Ia supernovae suggest that our Universe may be an accelerating one including cosmological density through the cosmological term Λ. Thus our models are are in agreement with the results of recent observations.

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MODEL ZA OBJAŠNJENJE PROMJENLJIVIH G I Λ UZ STALAN PARAMETAR USPORAVANJA

Proučavamo razvoj za homogene i anizotropne Bianchijeve kozmološke modele tipa I u prisustvu perfektne tekućine s promjenljivim G i Λ, pretpostavljajući posebnu ovisnost Hubbleovog parametra koja daje stalnu vrijednost parametra usporavanja. Raspravljaju se neki fizički ishodi modela za slučaj Zel'dovicheve tekućine i tekućinu u kojoj prevladava zračenje.