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PARTICLE RATIOS IN HEAVY- ION COLLISIONS

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We calculated different particle ratios in a wide range of energies $3.5 < \sqrt{s_{NN}} \leq 200$ GeV using the hadron resonance gas model, which was successfully applied in previous work to describe lattice QCD thermodynamics and characterize the phase transition and phase diagram lines. In this work, we assume that the particle production occurs along the freeze-out line, which has been defined by a constant entropy density normalized by temperature s/T^3 at all collision energies. The quark phase space occupancy parameters, γ_i, have been allowed to take values other than one. Doing this, we are able to describe K^+/π^+ ratio over the whole range of energies, including the sharp peak observed at top SPS energies. The parameters resulting from this analysis have been applied in calculating K^-/π^-, Λ/π^+ and $\Lambda/\langle\pi\rangle$ particle ratios and we found that within the statistical acceptance, the peaks in all these particle ratios occur in a very narrow range of the center-of-mass energy, $\sqrt{s_{NN}} \simeq 7.5$ GeV, which might be related to the baryo-chemical potential $\mu_B \simeq 0.43$ GeV. We also find that the entropy per particle becomes maximum in this energy-region refering to fluctuations at QCD tricritical endpoint.

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1. Introduction

Statistical models have been successfully used to describe the particle production in various heavy-ion collisions. Studying the ratios of particle yields [1, 2] is of a great interest, not only because it can be used to determine two intensive thermodynamic parameters, $T_{\rm ch}$ and μ_B, at the freeze-out stage, but also to avoid the volume fluctuations and therefore the dependence of freeze-out on the initial conditions. On the other hand, serious challenges arise when we try to bring together results from different heavy-ion collisions experiments. The two intensive parameters, $T_{\rm ch}$ and μ_B, are used to be calculated by means of statistical models. In doing this, we combine various particle ratios calculated by statistical models with the experimental ones. Nevertheless, it is still debated about the methods or models suggested to bring order to $T_{\rm ch}$ vs. μ_B [3−8] by the statistical models themselves.

The K^+/π^+ particle ratio represents another challenge to the statistical models in many aspects. One of them is the interplay between strange and light quarks. The second one is the prediction of sharp peak at the top SPS energies [9, 10]. Up to date, no-one has succeeded to interpret it statistically [11, 12]. Using statistical models, one merely gets a mild maximum. Nevertheless, there are many attempts to interpret this phenomenon [13, 14]. One might think about the tricritical endpoint observed in QCD phase diagram. In the present work, we find indications supporting the phase transition as a potential source for the K^+/π^+ peak.

We study ratios of strangeness to non-strangeness particle yields. The strangeness enhancement, which can be reflected in the particle yields, has been conjectured as a signature of phase transition to quark-gluon plasma [13]. This is why strangeness to non-strangeness ratios are analyzed in this work. We start with kaon-to-pion ratios and then apply the resulting parameters on lambda-to-pion ratios and so on.

The experimental data show that while there is a sharp peak in the K^+/π^+ ratio, the peak in K^-/π^- ratios is very smooth. This is an indication about the strangeness asymmetry, which reflects itself also in the strangeness hyperon production. We notice a sharp peak in Λ/π^+ ratios, but a smooth one in $\Lambda/\langle\pi\rangle$. Here we denote by $\langle\pi\rangle$ the value $1.5\,(\pi^+ + \pi^-)$.

The experimental results on K^+/π^+ ratio at different collision energies are studied by hadron resonance gas model. We allow γ_i, the quark phase space occupancy parameters, to take values other than unity, the equilibrium value. In that a way, we can reproduce the particle ratios almost perfectly. The subscript i refers to light and strange quark flavors [13, 15]. The readers are advised to notice the differences between this work and that of Refs. [13] and [15]. Here we explicitly assume that the particle production occurs along the freeze-out line, which is characterized by $s/T^3 = 7$ [6−8], where s stands for the entropy density. This means that the system freezes out when it possesses a certain value of energy which is required to add or remove one particle, i.e. the entropy. Thus, this value varies with T^3.

Using the resulting parameters, we calculate other particle ratios. We find that the peak in K^+/π^+ ratio is not the only one [9, 10]. Its height and sharpness are obviously greater than the peaks in other particle ratios, like K^-/π^-, Λ/π^+ and $\Lambda/\langle\pi\rangle$. Another worthwhile finding is that all peaks are located at almost one energy value. This could be understood as an indication about a critical phenomenon. Contrary to the statistical models with $\gamma_i = 1$, our calculations with varying γ_i result in an excellent agreement with all particle ratios. Also, we find that the entropy density per particle density, s/n, which measures the averaged phase space density [16−20], has a maximum value located in a very narrow range of center-of-mass energy as the peaks of particle ratios do. This can be interpreted as a manifestation of the QCD tricritical endpoint, which connects the first-order transition line with the region of cross-over. According to the recent lattice QCD simulations, the tricritical endpoint might be located at $\mu_B \sim 0.42$ GeV. Our estimation for s/n at RHIC energy is qualitatively consistent with the results given in Refs. [18, 21].

2. *The model*

The pressure in the hadron phase is given by contributions from all resonances (of masses up to 2 GeV) treated as a free gas [22 − 25]. At finite temperature T, baryo-chemical μ_B, strangeness-chemical μ_S and iso-spin chemical potential μ_{I_3}, the pressure reads

$$p(T, \mu_B, \mu_S, \mu_{I_3}) \quad = \quad \frac{g}{2\pi^2} T \int_0^\infty k^2 \mathrm{d}k \ln \left[1 \pm \gamma\, e^{(\mu_B + \mu_S + \mu_{I_3} - \varepsilon(k))/T}\right], \qquad (1)$$

where $\varepsilon(k) = \sqrt{(k^2 + m^2)}$ is the single-particle energy, $\pm$ stands for bosons and fermions, respectively, g is the spin-isospin degeneracy factor and γ are the quark phase-space occupancy parameters.

The quark chemistry is given by relating the chemical potentials μ and γ to the quark constituents. $\gamma \equiv \gamma_q^n \gamma_s^m$, where n and m are the number of light and strange quarks, respectively. $\mu_B = 3\mu_q$ and $\mu_S = \mu_q - \mu_s$, where q and s are the light and strange quark quantum numbers, respectively. The baryo-chemical potential μ_q for light quarks is $\mu_q = (\mu_u + \mu_d)/2$. μ_S is calculated as a function of T and μ_B under the condition of strangeness conservation. The iso-spin chemical potential is $\mu_{I_3} = (\mu_u - \mu_d)/2$. In our calculations, we use full the grand-canonical statistical set of all intensive thermodynamic parameters. Corrections due to the van der Waals repulsive interactions have not been taken into account [25].

Before discussing the results, we like to study the dependence of particle ratios on energies. For simplicity, we assume Stefan-Boltzmann approximation in deriving the following equations, although we do not apply the Stefan-Boltzmann approximation in our calculations. With these equations, we like to show on which parameters our particle ratios depend. For finite iso-spin fugacity λ_{I_3} we get

$$\frac{n_{K^+}}{n_{\pi^+}} \equiv \frac{K^+}{\pi^+} \quad \propto \quad \lambda_s^{-1} \left(\frac{\lambda_q}{\lambda_{I_3}}\right) \frac{\gamma_q}{\gamma_s} \qquad (2)$$

$$\frac{n_{K^-}}{n_{\pi^-}} \equiv \frac{K^-}{\pi^-} \quad \propto \quad \lambda_s \left(\frac{\lambda_{I_3}}{\lambda_q}\right) \frac{\gamma_s}{\gamma_q} \qquad (3)$$

$$\frac{n_\Lambda}{n_{\pi^+}} \equiv \frac{\Lambda}{\pi^+} \quad \propto \quad \lambda_s \left(\frac{\lambda_q}{\lambda_{I_3}^2}\right)^2 \gamma_q^2 \gamma_s \qquad (4)$$

The particle numbers, which can directly be obtained from Eq. (1), read $n_j(T) \simeq T m_j^2 K_2(m_j/T)$. They represent the proportionality factors in Eqs. (2), (3) and (4). It is clear that $n_j(T)$ and the fugacity factors λ are smooth functions of T. Thus a monotonic increase in the particle ratios is expected with increasing energy, unless some critical phenomenon takes place.

In Fig. 1, we depict the experimental results of all particle ratios together with our calculations at $\gamma_i = 1$. Apparently, no horn or sharp peak is observed. As discussed above, the quark phase-space occupancy factor $\gamma_i = 1$ characterizes the chemical equilibrium. When we allow γ_i to take values other than unity, it turns out to be possible to relate γ_i to the collision energy in order to reproduce almost all features observed for the particle ratios, especially on K^+/π^+, Eq. (2). This means that γ_i indirectly plays here the role of a fitting parameter. The new values of γ_q^- and γ_s^- are given in Table 1. Their values are greater than those indicating a phase-space oversaturation.

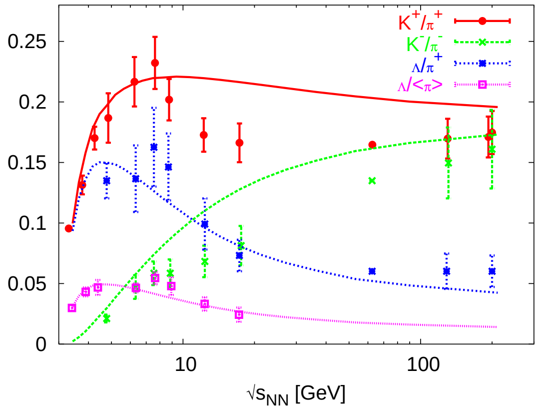

Fig. 1. K^+/π^+, K^-/π^-, Λ/π^+ and $\Lambda/\langle\pi\rangle$ ratios at AGS ($\sqrt{s} \le 4.84$ GeV), SPS ($6.26 \le \sqrt{s} \le 17.27$ GeV) and RHIC energies ($62.4 \le \sqrt{s} \le 200$ GeV) (symbols). In calculating the curves, both quark occupancy parameters γ_q and γ_s are explicitly assigned the value one.

TABLE 1. The values of γ_q and γ_s at certain collision energies. The second and third columns are related to the results depicted in Fig. 2. The last two columns are related to the results given in Fig. 3. According to the different quarks constituents of different hadrons, $\gamma = \gamma_q\gamma_s$ is accordingly different.

Collision energy	γ_q	γ_s	γ_q^-	γ_s^-
3.5	1.0	0.8	0.7	0.9
7.5	1.0	1.0	1.75	1.32
17	1.0	0.7	1.39	1.43
130	1.0	0.8	1.57	1.48

When we have a look at γ_q/γ_s ratios, we find that they first increase up to the collision energy at which the sharp peak in K^+/π^+ ratio has been predicted. After that they sharply decrease. At higher energies, they smoothly increase again, indicating that the mechanism responsible for particle production remains unchanged at energies up to the RHIC energy.

In calculating the other particle ratios, we use the same parameters γ_q and γ_s, which we obtained from the K^+/π^+ ratio. For K^-/π^-, the ratio γ_s/γ_q multiplied by λ_s/λ_q, as in Eq. (3), is apparently the leading term determining these particle ratios. For the Λ/π^+ ratio, the quarks phase-space occupancy parameters $\gamma_q^2\gamma_s$, as given in Eq. (4), are responsible for the good agreement we obtained so far. From this discussion we realize that Eqs. (2), (3) and (4) can help us to understand the dependence of particle ratios on the energy.

3. *Results*

In Fig. 1, we plot results from hadron resonance gas model (lines) at $\gamma_q = \gamma_s = 1$, on top of the experimental data at AGS ($\sqrt{s} \leq 4.84$ GeV), SPS ($6.26 \leq \sqrt{s} \leq 17.27$ GeV) and RHIC $62.4 \leq \sqrt{s} \leq 200$ GeV) energies (symbols). For the K^+/π^+ ratio, there is a good agreement at AGS energy. At the SPS energy, $\sqrt{s} \simeq 10$ GeV, there is a very mild maximum. At higher energies, our results obviously overestimate the K^+/π^+ ratio. The same feature, i.e. overestimates, has been observed in other work, for instance, an overestimation of 20% has been found at $\sqrt{s_{NN}} = 130$ GeV in Ref. [26]. At this energy, we find that $K^+/\pi^+ \simeq 0.163$. It is necessary to mention here that the particle numbers used to estimate the ratios are originally calculated through comparison between statistical models and experimental models in order to determine the intensive freeze-out parameters, $T_{\rm ch}$ and μ_B [14, 27].

The same behavior can also be seen in K^-/π^- ratio (20% overestimation at $\sqrt{s_{NN}} = 130$ GeV). The overestimation here starts up earlier than that of the K^+/π^+ ratio. For the strangeness baryons, as expected, we find a decreasing population of strange quarks with increasing energy. For Λ/π^+, there are underpredictions in two energy regions. The first one is at the SPS energy. This in turn indicates to a peak corresponding to that of the K^+/π^+ ratio. The second region is at the RHIC energy (25% underestimation at $\sqrt{s_{NN}} = 130$ GeV, for instance). Here the resulting values lie beneath the error bars. The same behavior is observed in the $\Lambda/\langle\pi\rangle$ ratio.

We summarize the above results. The potential maxima *do not all occur in a very narrow range of center-of-mass energy* and, therefore, it has been concluded that *the case for a phase transition is not very strong* [14]. Furthermore, we conclude here that the particle ratios calculated by statistical models at $\gamma_q = \gamma_s = 1$, disagree with the experimental results, especially at the RHIC energies.

In Fig. 2, we draw the results at constant $\gamma_q = 1$ and varying $\gamma_s \neq 1$, i.e., we assume that only strangeness is out-of-equilibrium at the stage of chemical freeze-out, and the light quark numbers in this case are in equilibrium. Using varying γ_s and $\gamma_q = 1$ has a tradition [13, 28, 29]. It has been based on assuming that the strangeness saturation is a signature for QGP [30]. We apply here the same process. We use γ_s as a kind of *fit-parameter* for the K^+/π^+ ratio. We find that γ_s is always smaller than one. The strangeness is then always undersaturated. We will leave open the discussion on the physical interpretation, why the strangeness quantum number has to be out-of-equilibrium, while the light quark quantum numbers are explicitly in equilibrium. The results are convincing that the varying γ_s and $\gamma_q = 1$ result in an obvious underestimation for the particle ratios Λ/π^+ and $\Lambda/\langle\pi\rangle$.

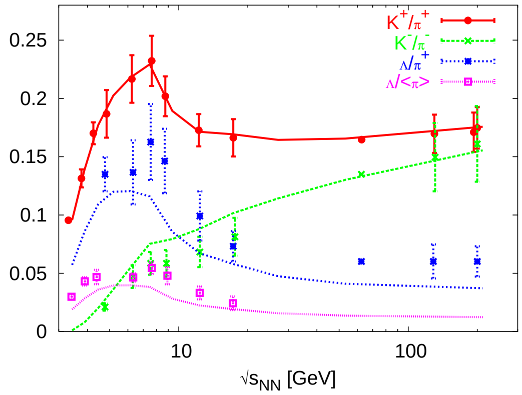

Fig. 2. K^+/π^+, K^-/π^-, Λ/π^+ and $\Lambda/\langle\pi\rangle$ ratios. γ_q is assigned the value one, while γ_s is a free parameter. Although the K^+/π^+ ratio has been successfully fitted, the calculation of the other particle ratios underestimates the experimental results.

In Fig. 3, we depict the results at varying γ_i. In estimating the γ_i-values, we *fit* our results from the hadron resonance gas model to the experimental results of the K^+/π^+ ratio. γ_i are the only parameters of the fit that we apply in the present work. In Refs. [13] and [31], γ_q is given as an input depending on T and correspondingly on $\sqrt{s_{NN}}$. In our calculations, we assumed that the particle production occurs along the freeze-out line, at which we calculate T, which depends on the chemical potential $T_{\rm ch}(\mu_B)$, according to the condition that $s/T^3 = 7$ [6–8]. Here s stands for the entropy density. In other words, we fix all thermodynamic intensive parameters at each energy value but γ_i.

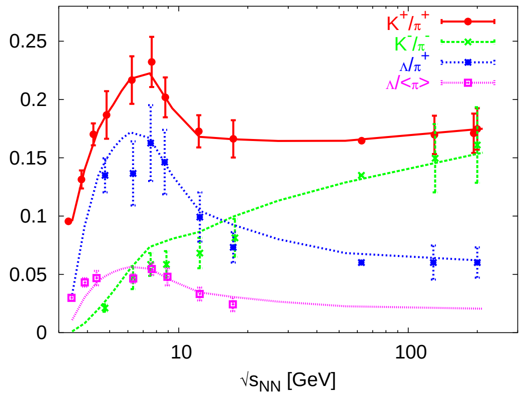

Fig. 3. The K^+/π^+, K^-/π^-, Λ/π^+ and $\Lambda/\langle\pi\rangle$ ratios at energies ranging from 3.5 to 200 GeV. *The calculated ratios for K^+/π^+ have been fitted to the experimental data by varying γ_q and γ_s. The other particle ratios have been calculated by applying the same parameter set.*

It is interesting to look at the overall agreement between our results from the resonance gas model and experimental data. Here we used the parameters which we obtained from fitting the K^+/π^+ particle ratio, as discussed above. Although the K^-/π^- ratio grows with the energy, we notice a weak slope within the energy region $7 < \sqrt{s_{NN}} < 12$ GeV. This might be an indication about the strangeness asymmetry in this energy region. The same behavior can be seen in the experimental data as well. While our calculations lie above the experimental results at SPS energies, the agreement with the data at AGS and RHIC energies is excellent. We also notice that the K^-/π^- ratio is not so much sensitive to the change in the light-quark occupancy parameter from $\gamma_q = 1$ as in Fig. 2 to the out-of equilibrium values as in Fig. 3.

The most important finding here is that, although the heights of the peaks are different (according to strangeness asymmetry), all peaks are located at almost the same value of energy, $\sqrt{s_{NN}} \simeq 7.5$ GeV. This energy value corresponds to the baryo-chemical potential $\mu_B \simeq 0.43$ GeV [14]. This energy value can be achieved from the lead beam accelerated to 40 AGeV. Another conclusion we can make is that the non-equilibrium process in both light and strange quark numbers is responsible for the particle yields [32]. We know that there are calculations in which it was suggested that the particle production is an equilibrium process.

We like to go one step further to clarify the physical reason behind the location of the peaks. The behavior of the single-particle entropy in dependence on energy, which is related to the averaged phase space density, gives a tool probing the critical phenomenon. This is exactly what we study in this work. We study the response of non-equilibrium quark occupancy (phase space) on producing maxima in strangeness/non-strangeness ratios. In the Boltzmann limit, we get

$$\frac{S(T,\mu)}{N(T,\mu)} \equiv \frac{s}{n} \quad = \quad \frac{m}{T}\left[\frac{K_3(m/T)}{K_2(m/T)} - \frac{\mu}{m}\right] \tag{5}$$

where m is the effective mass. T and μ are estimated at chemical freeze-out, as discussed above. These two intensive parameters depend on energy $\sqrt{s_{NN}}$. The maximum values of S/N are to be determined by T and μ values, at which the first derivative of Eq. (5) vanishes.

The results on the ratio of the single-particle entropy and particle number, s/n, as a function of the energy $\sqrt{s_{NN}}$ are depicted in Fig. 4. For $\gamma_q = \gamma_s = 1$, we find that for AGS and low SPS energies ($\sqrt{s_{NN}} < 10$ GeV) the value of s/n is an increasing function. As in Fig. 1, the mild maximum in K^+/π^+ ratio is located at the same value of $\sqrt{s_{NN}}$ as in the s/n ratio. For higher energies, s/n is saturated (remains constant with energy). This is a clear indication for a strong compensation of energy. Although we increase the energy, the phase space remains constant.

For the out-of-equilibrium case, i.e., when varying γ_q and γ_s, we find almost the same behavior up to $\sqrt{s_{NN}} \simeq 6.5$ GeV. For the rest of SPS energies, s/n rapidly decreases. This is a clear indication that the phase space of the interacting system rapidly increases at the SPS energy. Furthermore, there is an interesting behavior at RHIC energies. The entropy per particle s/n here decreases slowly with increasing energiy. On the one hand, the different slopes of s/n evidently indicate to different kinds of phase transitions. The singularity at $\sqrt{s_{NN}} \simeq 6.5$ GeV might refer to the QCD tricritical endpoint, which connects the first-order phase transition line at low $\sqrt{s_{NN}}$ with the cross-over region at high $\sqrt{s_{NN}}$. As we shown above, the peaks of all particle ratios are also located at almost the same energy value, $\sqrt{s_{NN}}$. This would mean that the peaks are due to fluctuations at the tricritical endpoint [33 – 37]. Studying the particle ratios with different constituents of light and strange quarks apparently magnifies this behavior.

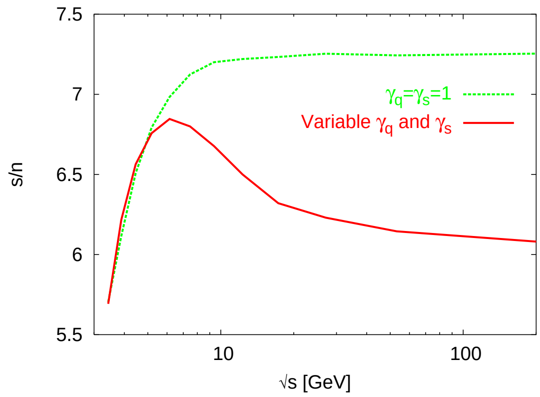

Fig. 4. The entropy per particle s/n as a function of $\sqrt{s_{NN}}$. Only for varying quark occupancy parameters γ_q and γ_s there is a singularity in s/n ratio. It is important to notice that the singularity is located at almost the same collision energy as that of the peaks of particle ratios. This might be an indication that the sharp peaks are due to the QCD critical endpoint.

4. *Summary and discussion*

We have studied the dependence on energy of the ratios of strangeness and non-strangeness of hadrons in the range from 3 to 200 GeV. We assumed that the particle production occurs along the freeze-out line, which is characterized by the constant s/T^3. Fixing all thermodynamical parameters but the quark phase-space occupancy parameters, we fitted K^+/π^+ particle ratio at all energies. Using the resulting parameters without any further fitting, we calculated the other particle ratios. We found that the overall agreement is excellent. The Ξ/π and Ω/π ratios are given in Ref. [13]. Within the statistical acceptance, we can conclude that almost all peaks are located at one value of energy.

Different models have been suggested in order to interpret the maxima observed in the particle ratios. Besides the statistical model at early stage [9, 10], there are two additional ones. The first one relates the peak to a transition from baryon-rich to meson-rich hadronic matter [14]. According to the second model, the peaks separate a high entropy phase from a low entropy phase [13].

The γ_q and γ_s play the key role in our calculations, which perfectly reproduce the particle ratios at all energies. This implies that the particle production is likely due to out-of-equilibrium processes in both light and strange quarks. Assuming equilibrium with only one of these quantum numbers, light q or strange quarks s, one can not reproduce the particle ratios shown in Fig. 2.

From Fig. 4, we find that the sharp peaks of particle ratios are associated with a maximum entropy per particle s/n. Thus we conclude that the tricritical endpoint, at which the line of the first-order phase transition ends up and the region of rapid cross-over sets on, might be responsible for the sharp peaks. For lattice calculations on the tricritical endpoint we refer to Refs. [38] and [39]. The tricritical endpoint given in Ref. [38] has the coordinates $T_c = 162 \pm 2$ MeV and $\mu_B = 0.36 \pm 0.04$ GeV, intensive thermodynamic parameters. According to Ref. [39], the corresponding $\mu_B \simeq 0.42$ GeV. We have to mention here that the lattice calculations exclusively assume equilibrium, i.e., $\gamma_q = \gamma_s = 1$.

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OMJERI ČESTICA U SUDARIMA TEŠKIH IONA

Izračunali smo niz čestičnih omjera u širokom području energije $3.5 < \sqrt{s_{NN}} \leq 200$ GeV rabeći hadronski rezonantni plinski model koji je bio ranije uspješno primijenjen za opis QCD termodinamike na rešetki i za davanje značajki faznom prijelazu i krivuljama u faznom dijagramu. U ovom radu pretpostavljamo tvorbu čestica duž krivulje smrzavanja, koja se određuje stalnom temperaturom normalizirane gustoće entropije, s/T^3, na svim energijama sudara. Pretpostavlja se da kvarkovski fazno-prostorni parametri zauzeća, γ_i, mogu biti različiti od jedan. Na taj smo način uspjeli opisati omjer K^+/π^+ u cijelom području energije, uključivo i oštar vrh koji je opažen na višim energijama SPS. Parametre izvedene u ovoj analizi smo primijenili za računanje omjera čestica K^-/π^-, Λ/π^+ i $\Lambda/\langle\pi\rangle$ i našli smo, unutar statističke prihvatljivosti, da se vrhovi u svim tim čestičnim omjerima nalaze u vrlo uskom području energije u sustavu centra mase, $\sqrt{s_{NN}} \simeq 7.5$ GeV, što bi moglo biti u svezi s bario-kemijskim potencijalom $\mu_B \simeq 0.43$ GeV. Također smo našli da entropija po čestici postaje najveća u tom području energije što ukazuje na fluktuacije na trokritičnoj krajnjoj točki.