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BIANCHI TYPE-I MAGNETIZED STIFF FLUID MODELS IN LYRA GEOMETRY

RAJ BALI[a] and NARESH K. CHANDNANI[b]

[a]*Department of Mathematics, University of Rajasthan, Jaipur-302004, India*
E-mail address: balir5@yahoo.co.in

[b]*Department of Mathematics, Jaipur Engineering College, Jaipur India*
E-mail address: chandnaninaresh1980@yahoo.co.in

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Bianchi type-I magnetized cosmological models with time-dependent gauge function β for stiff fluid distribution within the framework of Lyra geometry are investigated. To get the deterministic model of the universe, we have assumed that eigenvalue (σ_1^1) of shear tensor (σ_i^j) is proportional to the expansion (θ). This leads to $A = (BC)^n$ where A, B and C are metric potentials. The physical and geometrical aspects of the models and singularities in the models are discussed.

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1. *Introduction*

The relevance of the stiff equation of state $(\rho = p)$ to the matter content of the universe in its early stage is discussed by Barrow [1]. Wesson [2] investigated an exact solution of Einstein's field equation for stiff fluid distribution. Götz [3] obtained a plane-symmetric solution of Einstein's field equation for stiff perfect fluid distribution. Asseo and Sol [4] speculated the large-scale inter-galactic magnetic field that is of primordial origin and at present measures 10^{-8} G and gives rise to a density of the order of 10^{-35} g cm^{-3}. The present-day magnitude of magnetic energy is very small in comparison with the estimated matter density. It might not have been negligible during the early stage of evolution of the universe. FRW models are approximately valid as the present day magnetic field is very weak. The existence of a primordial magnetic field is limited to Bianchi types I, II, III, VI_0 and VII_0 as shown by Hughston and Jacobs [5]. The detailed discussion of the primordial magnetic field in the case of Bianchi type-I models has been given by Thorne [6]. Jacobs [7, 8] investigated Bianchi type-I cosmological model for barotropic perfect fluid distribution with magnetic field. Collins [9] gave a qualitative analysis of Bianchi type-I models with magnetic field. Roy and Prakash [10] investigated a plane-symmetric cosmological model with an incident magnetic field for perfect fluid distribution in which free gravitational field is of Petrov type-I degenerate. The cosmological models in the presence of magnetic field have also been investigated by Roy and Singh [11] and Bali et al. [12 − 14] in different contexts.

Einstein succeeded in geometrizing gravitation by expressing gravitational potential in terms of metric tensor. The idea of geometrizing gravitation inspired Weyl [15] to develop a theory to geometrize both gravitation and electromagnetism. But Weyl theory was criticized due to the non-integrability of length of vector under parallel displacement. Lyra [16] proposed a modification to Riemann geometry by introducing a gauge function into the structureless manifold which is in a close resemblance to Weyl's geometry. In continuation of these investigations, Sen [17], and Sen and Dunn [18] developed a new scalar-tensor theory of gravitation and constructed a field equation analog of the Einstein's field equation based on Lyra's geometry. Halford [19] has shown that the constant displacement field ϕ_μ in Lyra's geometry plays the role of cosmological constant Λ in general relativity. Soleng [20] investigated cosmological models based on Lyra's geometry and indicated that ϕ_μ includes either a creation field and is equal to Hoyle-Narlikar creation field cosmology [21, 22] or contains a special vacuum field which with gauge vector can be considered as the cosmological term. The cosmological models based on Lyra's geometry in different contexts have been investigated by Singh and Singh [23, 24], Reddy and Venkateshwarlu [25], Chakraborty and Ghosh [26], Rahaman et al. [27 − 29], Pradhan et al. [30 − 32], Mohanty et al. [33] and Bali and Chandnani [34, 35].

In this paper, we investigate Bianchi type-I cosmological models in the presence and absence of magnetic field, based on Lyra's geometry. The magnetic field is due to an electric current produced along the x-axis. The physical and geometrical aspects of the models and singularities in these models are also discussed.

2. The metric and field equations

We consider Bianchi type-I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where A, B, C are functions of t alone.

Energy momentum tensor T_i^j for perfect fluid distribution in the presence of magnetic field is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + E_i^j. \tag{2}$$

Einstein's modified field equation in normal gauge for Lyra's manifold obtained by

Sen [17] is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}\phi_k\phi^k g_i^j = -T_i^j\,, \tag{3}$$

(in geometrized units where $8\pi G = 1$ and $c = 1$). where $v_i = (0, 0, 0, -1)$; $v^i v_i = -1$; $\phi_i = (0, 0, 0, \beta(t))$; $v_4 = -1$; $v^4 = 1$, p is the isotropic pressure, ρ the matter density, v^i the fluid flow vector and β the gauge function.

E_i^j is the electro-magnetic field tensor given by Lichnerowicz [36] as

$$E_i^j = \bar{\mu}\left[|h|^2(v_i v^j + \frac{1}{2}g_i^j) - h_i h^j\right], \tag{4}$$

$\bar{\mu}$ being magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}}\epsilon_{ijkl}F^{kl}v^j\,, \tag{5}$$

where F^{kl} is the electro-magnetic field tensor and ϵ_{ijkl} the Levi-Civita tensor density. We assume that current is flowing along the x-axis, so magnetic field is in the yz plane. Thus $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$ and F_{23} is the only non vanishing component of F_{ij}. This leads to $F_{12} = 0 = F_{13}$ by virtue of (5).We also find $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conductivity of the fluid (Maartens [37]). A cosmological model, which contains a global magnetic field, is necessarily anisotropic since the magnetic vector specifies a preferred spatial direction (Bronnikov et al. [38]). The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0\,,$$

and

$$F^{ij}_{;j} = 0\,,$$

are satisfied by $F_{23} =$ constant$= H$(say).

Equation (5) leads to

$$h_1 = \frac{AH}{\bar{\mu}BC}\,. \tag{6}$$

From Eq. (4), we have

$$E_1^1 = -\frac{H^2}{2\bar{\mu}B^2C^2} = -E_2^2 = -E_3^3 = E_4^4. \tag{7}$$

The modified Einstein's field Eq. (3) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{3}{4}\beta^2 = -\left(p - \frac{H^2}{2\bar{\mu}B^2C^2}\right), \tag{8}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{3}{4}\beta^2 = -\left(p + \frac{H^2}{2\bar{\mu}B^2C^2}\right), \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{3}{4}\beta^2 = -\left(p + \frac{H^2}{2\bar{\mu}B^2C^2}\right), \tag{10}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{3}{4}\beta^2 = \rho + \frac{H^2}{2\bar{\mu}B^2C^2}. \tag{11}$$

The energy conservation equation $T^j_{i;j} = 0$ leads to

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) - \left[\frac{\partial}{\partial t}\left(\frac{H^2}{2\bar{\mu}B^2C^2}\right) + \frac{H^2}{\bar{\mu}B^2C^2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] = 0. \tag{12}$$

and conservation of L.H.S. of (3) leads to

$$\left(R^j_i - \frac{1}{2}Rg^j_i\right)_{;j} + \frac{3}{2}\left(\phi_i\phi^j\right)_{;j} - \frac{3}{4}\left(\phi_k\phi^k g^j_i\right)_{;j} = 0. \tag{13}$$

This leads to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j} + \phi^l\Gamma^j_{lj}\right] + \frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j} - \phi_l\Gamma^l_{ij}\right]$$
$$-\frac{3}{4}g^j_i\phi_k\left[\frac{\partial\phi^k}{\partial x^j} + \phi^l\Gamma^k_{lj}\right] - \frac{3}{4}g^j_i\phi^k\left[\frac{\partial\phi_k}{\partial x^j} - \phi_l\Gamma^l_{kj}\right] = 0. \tag{14}$$

Equation (14) is automatically satisfied for $i = 1, 2, 3$.

For $i = 4$, Eq. (14) leads to

$$\frac{3}{2}\beta\left[\frac{\partial}{\partial x^4}(g^{44}\phi_4) + \phi^4\Gamma^4_{44}\right] + \frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial t} - \phi_4\Gamma^4_{44}\right]$$
$$-\frac{3}{4}g^4_4\phi_4\left[\frac{\partial\phi_4}{\partial x^4} + \phi^4\Gamma^4_{44}\right] - \frac{3}{4}g^4_4 g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial t} - \phi^4\Gamma^4_{44}\right] = 0, \tag{15}$$

which leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{16}$$

3. Solution of field equations

For the complete determination of the model of the universe, we assume that eigenvalue (σ^1_1) of shear tensor (σ^j_i) is proportional to the expansion (θ). This leads to,

$$A = (BC)^n, \tag{17}$$

where n is a constant and we have assumed the proportionality constant as unity.

The motive behind assuming this condition is explained as follows: Referring to Thorne [6], the observations of the velocity-redshift relation for extragalactic sources suggest that the Hubble expansion of the universe is isotropic today to within 30 percent [39, 40]. More precisely, the red-shift studies place the limit $\sigma/H \leq 0.30$, where σ is the shear and H is the Hubble constant. Collins et al. [41] have pointed out that for spatially homogeneous metric, the normal congurence to the homogeneous hyper-surface satisfies the condition σ/θ is constant where σ is shear and θ the expansion in the model.

Now, Eqs. (8) and (9) lead to

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} = \frac{C_4}{C}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) + \frac{H^2}{\bar{\mu}B^2C^2}\,. \tag{18}$$

From Eqs. (9) and (10), we have

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right). \tag{19}$$

Using the condition (17), Eq. (19) leads to

$$\frac{B_{44}C - C_{44}B}{B_4C - C_4B} = -n\left(\frac{B_4}{B} + \frac{C_4}{C}\right). \tag{20}$$

Equation (20) leads to

$$C^2\left(\frac{B}{C}\right)_4 = L(BC)^{-n}, \tag{21}$$

where L is constant of integration.

Let us assume

$$BC = \mu\,, \tag{22}$$

$$\frac{B}{C} = \nu\,. \tag{23}$$

Using Eqs. (22) and (23) in (21), we have

$$\frac{\nu_4}{\nu} = L\mu^{-(n+1)}, \tag{24}$$

which leads to

$$\frac{\nu_{44}}{\nu} = -(n+1)L\mu^{-(n+2)}\mu_4 + L^2\mu^{-2(n+1)}. \tag{25}$$

Using above equation, Eq. (18) leads to

$$(1-2n)\frac{\mu_{44}}{\mu} + n(1-2n)\frac{\mu_4^2}{\mu^2} + \frac{\nu_{44}}{\nu} - \frac{\nu_4^2}{\nu^2} + (1+n)\frac{\mu_4\nu_4}{\mu\nu} = \frac{2H^2}{\bar{\mu}\mu^2}\,. \tag{26}$$

Using Eqs. (24) and (25) in (26), we get

$$(1-2n)\frac{\mu_{44}}{\mu}+n(1-2n)\frac{\mu_4^2}{\mu^2}=\frac{2H^2}{\bar{\mu}\mu^2}\,, \tag{27}$$

which leads to

$$2(1-2n)\mu_{44}+2n(1-2n)\frac{\mu^2}{\mu}=\frac{4K}{\mu}\,, \tag{28}$$

where $K=H^2/\bar{\mu}$.

We assume that

$$\mu_4=f(\mu)\,.$$

Thus

$$\mu_{44}=ff'\,,$$

where $f'=\mathrm{d}f/\mathrm{d}\mu$.

Therefore, Eq. (28) leads to

$$\frac{\mathrm{d}f^2}{\mathrm{d}\mu}+\frac{2n}{\mu}f^2=\frac{4K}{(1-2n)\mu}\,, \tag{29}$$

which again leads to

$$f^2=\frac{2K}{n(1-2n)}+l\mu^{-2n}, \tag{30}$$

where l is a constant of integration.

Equation (30) leads to

$$\frac{\mathrm{d}\mu}{\sqrt{2K/\{n(1-2n)\}+l\mu^{-2n}}}=\mathrm{d}t\,. \tag{31}$$

Also, Eq. (24) leads to

$$\frac{\mathrm{d}\nu}{\nu}=\frac{L}{\mu^{n+1}}\mathrm{d}t\,, \tag{32}$$

which leads to

$$\log\nu=\int\frac{L}{\mu^{n+1}}\frac{\mathrm{d}t}{\mathrm{d}\mu}\mathrm{d}\mu\,. \tag{33}$$

Using Eq. (31) in (33), we get

$$\nu=N\left[\mu^{-n}+\sqrt{\mu^{-2n}+\gamma K}\right]^{-L/n\sqrt{l}}\,, \tag{34}$$

where $\gamma = 2/\{ln(1-2n)\}$ and N is a constant of integration.

Now, using Eqs. (31) and (34), the metric (1) leads to

$$\begin{aligned} \mathrm{d}s^2 = \quad & -\frac{\mathrm{d}T^2}{l(T^{-2n}+\gamma K)} + T^{2n}\mathrm{d}X^2 + T\left[T^{-n}+\sqrt{T^{-2n}+\gamma K}\right]^{-L/n\sqrt{l}}\mathrm{d}Y^2 \\ & +T\left[T^{-n}+\sqrt{T^{-2n}+\gamma K}\right]^{L/n\sqrt{l}}\mathrm{d}Z^2, \end{aligned} \tag{35}$$

where $T=\mu$, $x=X$, $\sqrt{N}y=Y$, $z/\sqrt{N}=Z$ and cosmic time t is given by

$$t = \int \frac{\mathrm{d}T}{\sqrt{2K/\{n(1-2n)\}+lT^{-2n}}}\,. \tag{36}$$

4. Some physical and geometrical properties

Taking the stiff fluid condition, i.e. $p=\rho$, in Eq. (12) and using (17) and (22), we have

$$\rho_4 + 2(n+1)\rho\left(\frac{\mu_4}{\mu}\right) - \left[\frac{\partial}{\partial t}\left(\frac{K}{2\mu^2}\right) + \frac{K}{\mu^2}\left(\frac{\mu_4}{\mu}\right)\right] = 0\,, \tag{37}$$

which leads to

$$\rho_4 + 2\rho\frac{(n+1)\sqrt{l}}{\mu^{n+1}}\sqrt{1+\gamma\,,K\mu^{2n}} = 0 \tag{38}$$

and

$$\rho = MT^{-2(n+1)\sqrt{l}} = p\,, \tag{39}$$

where M is a constant of integration and $\mu = T$.

The expansion (θ) is given by

$$\theta = \left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right), \tag{40}$$

which leads to

$$\theta = \frac{(n+1)\sqrt{l}}{T^{n+1}}\sqrt{1+\gamma K T^{2n}}\,. \tag{41}$$

Components of the shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3}\left(\frac{2A_4}{A}-\frac{B_4}{B}-\frac{C_4}{C}\right), \tag{42}$$

$$\sigma_2^2 = \frac{1}{3}\left(\frac{2B_4}{B}-\frac{A_4}{A}-\frac{C_4}{C}\right), \tag{43}$$

$$\sigma_3^3 = \frac{1}{3}\left(\frac{2C_4}{C} - \frac{B_4}{B} - \frac{A_4}{A}\right), \tag{44}$$

$$\sigma_4^4 = 0\,, \tag{45}$$

so we obtain

$$\sigma_1^1 = \frac{(2n-1)\sqrt{l}}{3T^{(n+1)}}\sqrt{1+\gamma K T^{2n}}\,, \tag{46}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{(1-2n)\sqrt{l}}{2T^{(n+1)}}\sqrt{1+\gamma K T^{2n}} + \frac{3L}{2T^{(n+1)}}\right], \tag{47}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{(1-2n)\sqrt{l}}{2T^{(n+1)}}\sqrt{1+\gamma K T^{2n}} - \frac{3L}{2T^{(n+1)}}\right], \tag{48}$$

$$\sigma_4^4 = 0\,. \tag{49}$$

Now, the shear σ is given by

$$\sigma^2 = \tfrac{1}{2}\left[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2\right],$$

which leads to

$$\sigma^2 = \frac{(2n-1)^2 l[1+\gamma K T^{2n}] + 3L^2}{12T^{2(n+1)}}\,. \tag{50}$$

Thus we find

$$\frac{\sigma_1^1}{\theta} = \frac{(2n-1)}{3(n+1)}\,,$$

which is a constant. Hence, the anisotropy is maintained throughout.

For the displacement vector (β), from Eq. (11), we have

$$\frac{3}{4}\beta^2 = \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \left(\rho + \frac{H^2}{2\bar{\mu}B^2C^2}\right), \tag{51}$$

which leads to

$$\beta^2 = \frac{(4n+1)l - L^2}{3T^{2(n+1)}} + \frac{4(n+1)(2n+1)K}{3n(1-2n)T^2} - \frac{4M}{3T^{2(n+1)}\sqrt{l}}\,. \tag{52}$$

5. *Solution in the absence of magnetic field*

To find the solution in the absence of magnetic field, we put $K = 0$ in Eq. (31) and get

$$\mu^n \mathrm{d}\mu = \sqrt{l}\mathrm{d}t\,, \tag{53}$$

which leads to

$$\mu = \left[(n+1)\sqrt{l}t + d\right]^{1/(n+1)}, \tag{54}$$

where d is a constant of integration.

Now, Eq. (32) leads to

$$\frac{\mathrm{d}\nu}{\nu} = \frac{L}{\left[(n+1)\sqrt{l}t + d\right]}\mathrm{d}t, \tag{55}$$

which leads to

$$\nu = Q\left[(n+1)\sqrt{l}t + d\right]^{L/\{(n+1)\sqrt{l}\}}, \tag{56}$$

where Q is a constant of integration.

Thus the metric (1) in the absence of magnetic field is given by

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{(n+1)^2 l} + T^{2n/(n+1)}\mathrm{d}X^2 + T^{(\sqrt{l}+L)/\{(n+1)\sqrt{l}\}}\mathrm{d}Y^2 + T^{(\sqrt{l}-L)/\{(n+1)\sqrt{l}\}}\mathrm{d}Z^2, \tag{57}$$

where $(n+1)\sqrt{l}\,t + d = T$, $x = X$, $\sqrt{Q}y = Y$ and $z/\sqrt{Q} = Z$.

In this case, the matter density (ρ) and isotropic pressure (p) are given by

$$\rho = \frac{M}{T^{2\sqrt{l}}} = p. \tag{58}$$

The expansion (θ) is given by

$$\theta = \frac{(n+1)\sqrt{l}}{T}. \tag{59}$$

Components of the shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{(2n-1)\sqrt{l}}{3T}, \tag{60}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{(1-2n)\sqrt{l} + 3L}{2T}\right], \tag{61}$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{(1-2n)\sqrt{l} - 3L}{2T}\right], \tag{62}$$

$$\sigma_4^4 = 0. \tag{63}$$

Hence, σ is given by

$$\sigma^2 = \frac{(2n-1)^2 l + 3L^2}{12T^2} \,. \tag{64}$$

Therefore

$$\frac{\sigma_1^1}{\theta} = \frac{(2n-1)}{3(n+1)} \,,$$

which is a constant.

Hence in the absence of magnetic field also the anisotropy is maintained throughout.

For the displacement vector (β), from Eq. (11), we have

$$\beta^2 = \frac{(4n+1)l - L^2}{3T^2} - \frac{4M}{3T^2\sqrt{l}} \,. \tag{65}$$

6. *Special case*

To get the deterministic model in terms of the cosmic time t, we put $n = 1$ in Eq. (31) and get

$$\frac{\mu \mathrm{d}\mu}{\sqrt{l - 2K\mu^2}} = \mathrm{d}t \,, \tag{66}$$

which after integration leads to

$$\mu^2 = \frac{l}{2K} - 2K(t+b)^2, \tag{67}$$

where b is a constant of integration.

For $n = 1$, Eq. (24) leads to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^2} \,, \tag{68}$$

Using (64) in (65), we have

$$\frac{\nu_4}{\nu} = \frac{L}{l/(2K) - 2K(t+b)^2} \,, \tag{69}$$

which after integration leads to

$$\nu = \eta \left[\frac{\sqrt{l} + 2K(t+b)}{\sqrt{l} - 2K(t+b)} \right]^{L/2\sqrt{l}} \,, \tag{70}$$

where η is a constant of integration.

Using the above equations, the metric takes the form

$$\begin{aligned} \mathrm{d}s^2 = \quad & -\mathrm{d}T^2 + \left[\frac{l}{2K} - 2KT^2\right]\mathrm{d}X^2 + \left[\frac{l}{2K} - 2KT^2\right]^{1/2}\left[\frac{\sqrt{l}+2KT}{\sqrt{l}-2KT}\right]^{L/2\sqrt{l}}\mathrm{d}Y^2 \\ & + \left[\frac{l}{2K} - 2KT^2\right]^{1/2}\left[\frac{\sqrt{l}+2KT}{\sqrt{l}-2KT}\right]^{-L/2\sqrt{l}}\mathrm{d}Z^2, \end{aligned} \tag{71}$$

where $t+b=T$, $x=X$, $\sqrt{\eta}\,y=Y$ and $z/\sqrt{\eta}=Z$.

Now, from Eq. (39), the matter density (ρ) and isotropic pressure (p) are given by

$$\rho = \frac{M}{[l/(2K) - 2KT^2]^{2\sqrt{l}}} = p\,, \tag{72}$$

The expansion (θ) is given by

$$\theta = \frac{4KT}{2KT^2 - l/(2K)}\,. \tag{73}$$

The components of shear (σ_i^j) are given by

$$\sigma_1^1 = \frac{2KT}{3[2KT^2 - l/(2K)]} \tag{74}$$

$$\sigma_2^2 = \frac{-1}{6}\left[\frac{3L+2KT}{[2KT^2 - l/(2K)]}\right] \tag{75}$$

$$\sigma_3^3 = \frac{1}{6}\left[\frac{3L-2KT}{[2KT^2 - l/(2K)]}\right] \tag{76}$$

$$\sigma_4^4 = 0 \tag{77}$$

Therefore,

$$\sigma^2 = \frac{4K^2T^2 + 3L^2}{12[2KT^2 - l/(2K)]^2} \tag{78}$$

and

$$\sigma_1^1/\theta = \frac{1}{6}\,,$$

which is a constant. Hence, anisotropy is maintained throughout.

For $n = 1$, Eq. (11) leads to

$$\beta^2 = \frac{(5l - L^2)}{3\left[l/(2K) - 2KT^2\right]^2} - \frac{8K}{\left[l/(2K) - 2KT^2\right]} - \frac{4M}{3\left[l/(2K) - 2KT^2\right]^{2\sqrt{l}}} \,. \tag{79}$$

Also the spatial volume (R^3) is given by

$$R^3 = ABC = A^2 = \left[\frac{l}{2K} - 2KT^2\right]. \tag{80}$$

The deceleration parameter (q) is given by

$$q = -\frac{R_{44}/R}{R_4^2/R^2} \,, \tag{81}$$

which leads to

$$q = \left[\frac{6l + 8K^2T^2}{16K^2T^2}\right] \,. \tag{82}$$

To discuss this case in the absence of magnetic field, we put $K = 0$ in Eq. (66) and get

$$\mu \, \mathrm{d}\mu = \sqrt{l} \, \mathrm{d}t \,, \tag{83}$$

which leads to

$$\mu^2 = (at + d) \,, \tag{84}$$

where $a = 2\sqrt{l}$ and d is a constant of integration.

Equation (68) leads to

$$\frac{\nu_4}{\nu} = \frac{L}{(at + d)} \,, \tag{85}$$

which again leads to

$$\nu = W(at + d)^{L/a}, \tag{86}$$

where W is a constant of integration.

Now, using these values of μ and ν, the metric takes the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{a^2} + T\mathrm{d}X^2 + T^{(1/2+L/a)}\mathrm{d}Y^2 + T^{(1/2-L/a)}\mathrm{d}Z^2 \tag{87}$$

where $(at + d) = T$, $x = X$, $\sqrt{W}y = Y$, $z/\sqrt{W} = Z$.

The matter density (ρ), pressure (p) and expansion (θ) for the model (87) are given by

$$\rho = \frac{M}{T^a} = p\,, \tag{88}$$

$$\theta = \frac{a}{T}\,. \tag{89}$$

Components of shear (σ_i^j) are given by

$$\sigma_1^1 = \frac{a}{6T}\,, \tag{90}$$

$$\sigma_2^2 = \frac{(6L - a)}{12T}\,, \tag{91}$$

$$\sigma_3^3 = -\frac{6L + a}{12T}\,, \tag{92}$$

$$\sigma_4^4 = 0\,. \tag{93}$$

Therefore,

$$\sigma^2 = \frac{(a^2 + 12L^2)}{48T^2}\,. \tag{94}$$

and

$$\frac{\sigma_1^1}{\theta} = \frac{1}{6}$$

which is a constant. Hence, anisotropy is maintained throughout.

The displacement vector (β) is given by

$$\beta^2 = \frac{(5l - L^2)}{3T^2} - \frac{4M}{3T^a}\,. \tag{95}$$

The spatial volume (R^3) is given by

$$R^3 = T\,. \tag{96}$$

The deceleration parameter (q) is given by

$$q = -\frac{R_{44}/R}{R_4^2/R^2}\,,$$

which leads to

$$q = 2\,.$$

It is possible to discuss the entropy. To solve the entropy problem of the standard model, it is necessary to have $\mathrm{d}S > 0$ for at least a part of evolution of the universe. In Riemannian geometry without a cosmological constant we have

$$T\mathrm{d}S = \mathrm{d}(\rho R^3) + p\,\mathrm{d}R^3 = 0\,, \tag{97}$$

where R is the scale factor. The conservation equation $T^j_{i;j} = 0$ for the metric (1) in the presence of magnetic field is given by Eq. (12) which leads to

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0\,. \tag{98}$$

In our case $R^3 = ABC$. Since

$$T\mathrm{d}S = \rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0 \tag{99}$$

because entropy increases.

Therefore, Eqs. (98) and (99) lead to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0\,,$$

which requires $\beta < 0$. Thus, the displacement vector β is related to entropy because for entropy $\mathrm{d}S > 0$ which leads to $\beta < 0$.

7. *Discussion and conclusions*

The model (35) starts with a big-bang at $T = 0$ in the presence of magnetic field when $n > 0$ and the expansion in the model decreases as time increases. However, if $n < 0$ then the expansion in the model increases as time increases. Since $\sigma_1^1/\theta = constant$, the anisotropy is maintained throughout. For $n = 1/2$, the model (35) gives isotropy. The model (35) has point-type singularity at $T = 0$ when $n > 0$ and it has cigar-type singularity when $n < 0$ [Mac Callum[42]].

In the absence of magnetic field, the model (57) starts expanding with a big-bang at $T = 0$ and the expansion in the model decreases as time increases. When $T \to \infty$ then $\theta \to 0$. When $l > L^2$ the model (57) has point-type singularity at $T = 0$ and cigar-type singularity at $T = 0$ when $l < L^2$. The model (71) starts with a big-bang at $T = 0$ in the presence of magnetic field and the expansion in the model decreases as time increases. Since $\sigma_1^1/\theta = constant$, the anisotropy is maintained throughout. The model (71) has point-type singularity at $T = \sqrt{l}/(2K)$. Since the deceleration factor $q > 0$, the model (71) gives decelerating universe.

In the special case $n = 1$, the model (87) starts expanding with a big bang at $T = 0$ and the expansion in the model decreases as time increases. The spatial volume increases as time increases. Since $\sigma_1^1/\theta = constant$, the anisotropy is maintained throughout. Since the deceleration factor $q > 0$, the model (87) represents a decelerating universe. For $n = 1$, we obtain the same model as obtained by Bali and Chandnani [34] in the absence of magnetic field. The model (87) has point-type singularity at $T = 0$ when $L/a > 1/2$.

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BIANCHIJEVI MODELI TIPA I U LYRINOJ GEOMETRIJI S MAGNETIZIRANOM KRUTOM TEKUĆINOM

Istražujemo Bianchijeve kozmološke modele s vremenski-ovisnom baždarnom funkcijom β za krutu tekućinsku raspodjelu u okviru Lyrine geometrije. Radi postizanja određenja u modelima, pretpostavljamo da je svojstvena vrijednost (σ_1^1) tenzora smicanja (σ_i^j) razmjerna širenju (θ). To vodi na $A = (BC)^n$ gdje su A, B and C metrički potencijali. Raspravljamo također fizičke i geometrijske značajke modela i singularnosti.