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INHOMOGENEOUS PERFECT FLUID UNIVERSE WITH ELECTROMAGNETIC FIELD IN LYRA GEOMETRY

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A new class of cylindrically symmetric inhomogeneous cosmological models for perfect fluid distribution with electromagnetic field is obtained in the context of Lyra's geometry. We obtained two types of solutions by considering the uniform as well as time dependent displacement field. The source of the magnetic field is due to an electric current produced along the z-axis. Only F_{12} is a non-vanishing component of the electromagnetic field tensor. To get the deterministic solution, it has been assumed that the expansion θ in the model is proportional to the shear σ. It has been found that the solutions are consistent with the recent observations of type Ia supernovae and that the displacement vector $\beta(t)$ affects entropy. Physical and geometric aspects of the models are also discussed in the presence and absence of magnetic field.

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1. *Introduction and motivations*

The inhomogeneous cosmological models play a significant role in the understanding of some essential features of the universe such as the formation of galaxies during the early stages of evolution and the process of homogenization. The early attempts at the construction of such models have been done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6],

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Collins and Szafron [7], Szafron and Collins [8]. Senovilla [9] obtained a new class of exact solutions of Einstein's equations without the big-bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere, satisfying energy and causality conditions. Later, Ruiz and Senovilla [10] examined a fairly large class of singularity-free models through a comprehensive study of general cylindrically symmetric metric with separable function of r and t as metric coefficients. Dadhich et al. [11] established a link between the FRW model and the singularity-free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former. Recently, Patel et al. [12] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid, assuming that the background space-time admits two space-like commuting Killing vectors and has separable metric coefficients. Singh, Mehta and Gupta [13] obtained inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. Recently, Pradhan et al. [14] investigated cylindrically-symmetric inhomogeneous cosmological models in various contexts.

The occurrence of magnetic field on galactic scale is a well-established fact today, and its importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [15]. Also Harrison [16] suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein's system of field equations leads to the cosmological models more general than the Robertson-Walker model [17]. The presence of primordial magnetic field in the early stages of the evolution of the universe is discussed by many [18]−[27]. Strong magnetic field can be created due to the adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and decays slowly as compared to the case when the pressure is held isotropic [28, 29]. Such fields can be generated at the end of an inflationary epoch [30]−[34]. Anisotropic magnetic field models have a significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [35] obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Pradhan et al. [36] investigated magnetized cosmological models in various contexts.

In 1917 Einstein introduced the cosmological constant into his field equations of general relativity in order to obtain a static cosmological model since, as is well known, without the cosmological term his field equations admit only non-static solutions. After the discovery of the red-shift of galaxies and explanation thereof Einstein regretted for the introduction of the cosmological constant. Recently, there has been much interest in the cosmological term in the context of quantum field theories, quantum gravity, super-gravity theories, Kaluza-Klein theories and the inflationary-universe scenario. Shortly after Einstein's general theory of relativity, Weyl [37] suggested the first so-called unified field theory based on a generalization of Riemannian geometry. With its backdrop, it would seem more appropriate to call Weyl's theory a geometrized theory of gravitation and electromagnetism (just as the general theory was a geometrized theory of gravitation only), instead a unified field theory. It is not clear as to what extent the two fields have been unified, even though they acquire (different) geometrical significance in the same geometry. The theory was never taken seriously in as much as it was based on the concept of non-integrability of length transfer; and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl's geometry provides an interesting example of non-Riemannian connections, and recently Folland [38] has given a global formulation of Weyl manifolds clarifying thereby considerably many of Weyl's basic ideas.

In 1951 Lyra [39] proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. But in Lyra's geometry, unlike that of Weyl, the connection is metric preserving as in Remannian; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the "normal" gauge the curvature scalar identical to that of Weyl. In consecutive investigations, Sen [40], Sen and Dunn [41] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein's field equations based on Lyra's geometry. It is, thus, possible [40] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory, however, without the inconvenience of non-integrability of length transfer.

Halford [42] pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of the cosmological constant Λ in the normal general relativistic treatment. It was shown by Halford [43] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits of the Einstein's theory. Several authors, Sen and Vanstone [44], Bhamra [45], Karade and Borikar [46], Kalyanshetti and Wagmode [47], Reddy and Innaiah [48], Beesham [49], Reddy and Venkateswarlu [50], Soleng [51], studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one for convenience and there is no *a priori* reason for it. Beesham [52] considered FRW models with time dependent displacement field. He has shown that by assuming the energy density of the universe to be equal to its critical value, the models have the $k = -1$ geometry. Singh and Singh [53] – [56], Singh and Desikan [57] studied Bianchi-type I, III, Kantowaski-Sachs and a new class of cosmological models with the time-dependent displacement field and made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [51] pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [56] – [60] or contain a special vacuum field, which together with the gauge vector term, may be considered as a cosmological term. In the latter case, the solutions are equal to the general relativistic cosmologies with a cosmological term.

Recently, Pradhan et al. [61], Casama et al. [62], Rahaman et al. [63], Bali and

Chandnani [64], Kumar and Singh [65], Singh [66] and Rao, Vinutha and Santhi [67] studied cosmological models based on Lyra's geometry in various contexts. With these motivations, in this paper, we obtained exact solutions of Einstein's modified field equations in cylindrically-symmetric inhomogeneous space-time within the frame work of Lyra's geometry in the presence and absence of magnetic field for uniform and time varying displacement vector. This paper is organized as follows. In Section 1 the motivation for the present work is discussed. The metric and the field equations are presented in Section 2, in Section 3 the solution of field equations, the Section 4 contains the solution of uniform displacement field ($\beta = \beta_0$, constant). The Section 5 deals with the solution with time varying displacement field ($\beta = \beta(t)$). Subsections 5.1, 5.2 and 5.3 describe the solutions of empty universe, Zeldovich universe and radiating universe with the physical and geometric aspects of the models respectively. The solutions in absence of magnetic field are given in Section 6. Sections 7 and 8 deal with the solutions for uniform and time dependent displacement field. Finally, in Section 9, discussion and concluding remarks are given.

2. The metric and field equations

We consider the cylindrically symmetric metric in the form

$$\mathrm{d}s^2 = A^2(\mathrm{d}x^2 - \mathrm{d}t^2) + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where A is the function of t alone and B and C are functions of x and t. The energy momentum tensor is taken as

$$T_i^j = (\rho + p)u_i u^j + p g_i^j + E_i^j, \tag{2}$$

where ρ and p are, respectively, the energy density and pressure of the cosmic fluid, and u_i is the fluid four-velocity vector satisfying the conditions

$$u^i u_i = -1, \quad u^i x_i = 0. \tag{3}$$

In Eq. (2), E_i^j is the electromagnetic field given by Lichnerowicz [68]

$$E_i^j = \bar{\mu}\left[h_l h^l \left(u_i u^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}\, {}^*F_{ji} u^j, \tag{5}$$

where the dual electromagnetic field tensor ${}^*F_{ij}$ is defined by Synge [69]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{6}$$

Here F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density.

The co-ordinates are considered to be co-moving so that $u^1 = 0 = u^2 = u^3$ and $u^4 = 1/A$. If we consider that the current flows along the z-axis, then F_{12} is the only non-vanishing component of F_{ij}. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \tag{7}$$

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = 0\,, \tag{8}$$

require that F_{12} is the function of x-alone. We assume that the magnetic permeability is the functions of x and t both. Here the semicolon represents a covariant differentiation.

The field equations (in gravitational units $c = 1$, $G = 1$), in normal gauge for Lyra's manifold, were obtained by Sen [40] as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -8\pi T_{ij}, \tag{9}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta), \tag{10}$$

where β is either a constant or a function of t. The other symbols have their usual meaning as in Riemannian geometry.

For the line-element (1), the field Eq. (9) with Eqs. (2) and (10) lead to the following system of equations

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC}\right] - \frac{3}{4}\beta^2$$

$$= 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right), \tag{11}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{C_{11}}{C}\right) - \frac{3}{4}\beta^2 = 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right), \tag{12}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{B_{11}}{B}\right) - \frac{3}{4}\beta^2 = 8\pi\left(p - \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right), \tag{13}$$

$$\frac{1}{A^2}\left[-\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC}\right] + \frac{3}{4}\beta^2$$

$$= 8\pi \left(\rho + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right), \tag{14}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) = 0, \tag{15}$$

where the subscript indices 1 and 4 at A, B, C and elsewhere denote ordinary differentiation with respect to x and t, respectively.

3. Solution of field equations

Equations (11) − (15) are five independent equations in seven unknowns A, B, C, ρ, p, β and F_{12}. For the complete determinacy of the system, we need two extra conditions which are narrated hereinafter. The research on exact solutions is based on some physically reasonable restrictions used to simplify the field equations.

To get a determinate solution, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$A = \left(\frac{B}{C} \right)^n, \tag{16}$$

where n is a constant. The motive behind this condition is explained with reference to Thorne [70]. The observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent [71, 72]. To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the ratio of shear, σ, to Hubble constant, H, in the neighbourhood of our Galaxy today. Collins et al. [73] pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition σ/θ is constant.

From Eqs. (11)-(13), we have

$$\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} - \frac{B_{44}}{B} - \frac{B_4 C_4}{BC} = \frac{C_{11}}{C} - \frac{B_1 C_1}{BC} = K(constant) \tag{17}$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu} B^2} = -\frac{C_{44}}{C} + \frac{C_{11}}{C} + \frac{B_{44}}{B} - \frac{B_{11}}{B}. \tag{18}$$

We also assume that

$$B = f(x) g(t),$$

$$C = f(x)k(t). \tag{19}$$

Using Eqs. (16) and (19) in (15) and (17) one obtains

$$\frac{k_4}{k} = \frac{(2n-1)}{(2n+1)}\frac{g_4}{g}, \tag{20}$$

$$(n-1)\frac{g_{44}}{g} - n\frac{k_{44}}{k} - \frac{g_4}{g}\frac{k_4}{k} = K, \tag{21}$$

$$ff_{11} - f_1^2 = Kf^2. \tag{22}$$

Equation (20) leads to

$$k = cg^{\alpha}, \tag{23}$$

where $\alpha = (2n-1)/(2n+1)$ and c is the constant of integration. From Eqs. (21) and (23), we have

$$\frac{g_{44}}{g} + \ell\frac{g_4^2}{g^2} = N, \tag{24}$$

where

$$\ell = \frac{n\alpha(\alpha-1)+\alpha}{n(\alpha-1)+1},\ N = \frac{K}{n(1-\alpha)-1}.$$

Equation (22) leads to

$$f = \exp\left(\frac{1}{2}K(x+x_0)^2\right), \tag{25}$$

where x_0 is an integrating constant. Equation (24) leads to

$$g = \left(c_1 e^{bt} + c_2 e^{-bt}\right)^{1/(\ell+1)}, \tag{26}$$

where $b = \sqrt{(\ell+1)N}$ and c_1, c_2 are integrating constants. Hence from (23) and (26), we have

$$k = c\left(c_1 e^{bt} + c_2 e^{-bt}\right)^{\alpha/(\ell+1)}. \tag{27}$$

Therefore we obtain

$$B = \exp\left(\frac{1}{2}K(x+x_0)^2\right)\left(c_1 e^{bt} + c_2 e^{-bt}\right)^{1/(\ell+1)}, \tag{28}$$

$$C = \exp\left(\frac{1}{2}K(x+x_0)^2\right)c\left(c_1 e^{bt} + c_2 e^{-bt}\right)^{\alpha/(\ell+1)}, \tag{29}$$

$$A = a\left(c_1 e^{bt} + c_2 e^{-bt}\right)^{n(1-\alpha)/(\ell+1)}, \tag{30}$$

where $a = c_3/c$, c_3 being a constant of integration.

After using suitable transformation of the coordinates, the model (1) reduces to the form

$$ds^2 = a^2(c_1 e^{bT} + c_2 e^{-bT})^{2n(1-\alpha)/(\ell+1)}(dX^2 - dT^2) + e^{KX^2}(c_1 e^{bT} + c_2 e^{-bT})^{2/(\ell+1)} dY^2$$
$$+ e^{KX^2}(c_1 e^{bT} + c_2 e^{-bT})^{2\alpha/(\ell+1)} dZ^2, \tag{31}$$

where $x + x_0 = X$, $t = T$, $y = Y$, $cz = Z$.

For the specification of the displacement vector β within the framework of Lyra geometry and for realistic models of physical importance, we consider the following two cases described in Sections 4 and 5.

4. *When β is a constant i.e. $\beta = \beta_0$ (constant)*

Using Eqs. (28), (29) and (30) in Eqs. (11) and (14), the expressions for pressure p and density ρ for the model (31) are given by

$$8\pi p = \frac{1}{a^2 \psi_2^{2n(1-\alpha)/(\ell+1)}} \left[K^2 X^2 - \frac{2(3+\alpha) b^2 c_1 c_2}{(\ell+1)\psi_2^2} \right.$$
$$\left. - \frac{(2n\alpha^2 + \alpha^2 + 2\alpha - 2n + 3) b^2}{2(\ell+1)^2} \frac{\psi_1^2}{\psi_2^2} \right] - \frac{3}{4}\beta_0^2, \tag{32}$$

$$8\pi \rho = \frac{1}{a^2 \psi_2^{2n(1-\alpha)/(\ell+1)}} \left[-3K^2 X^2 - 2K + \frac{2b^2(\alpha-1) c_1 c_2}{(\ell+1)\psi_2^2} \right.$$
$$\left. - \frac{(2n\alpha^2 - \alpha^2 - 2\alpha - 2n + 1) b^2}{2(\ell+1)^2} \frac{\psi_1^2}{\psi_2^2} \right] + \frac{3}{4}\beta_0^2, \tag{33}$$

where

$$\psi_1 = c_1 e^{bT} - c_2 e^{-bT},$$

$$\psi_2 = c_1 e^{bT} + c_2 e^{-bT}.$$

From Eq. (18) the non-vanishing component F_{12} of the electromagnetic field tensor is obtained as

$$F_{12}^2 = \frac{\bar{\mu}}{8\pi} \frac{b^2(1-\alpha)}{(\ell+1)^2} e^{KX^2} \psi_2^{2/(\ell+1)} \left[\frac{4(\ell+1)c_1 c_2 + (1+\alpha)\psi_1^2}{\psi_2^2} \right]. \tag{34}$$

From the above equation, it is observed that the electromagnetic field tensor increases with time.

The reality conditions (Ellis [74])

$$(i)\rho + p > 0, \quad (ii)\rho + 3p > 0,$$

lead to

$$\frac{b^2(n - n\alpha^2 - 1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2} - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} > K(KX^2+1), \tag{35}$$

and

$$\frac{b^2(4n - 4n\alpha^2 - \alpha^2 - 2\alpha - 5)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2} - \frac{4b^2(\alpha+5)c_1c_2}{(\ell+1)\psi_2^2}$$

$$> 2K + \frac{3}{2}\beta_0^2 a^2 \psi_2^{2n(1-\alpha)/(\ell+1)}, \tag{36}$$

respectively.

The dominant energy conditions (Hawking and Ellis [75])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0,$$

lead to

$$\frac{b^2(\alpha+1)^2}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2} + \frac{4b^2(\alpha+1)c_1c_2}{(\ell+1)\psi_2^2} + \frac{3}{2}\beta_0^2 a^2 \psi_2^{2n(1-\alpha)/(\ell+1)} \geq 2K(2KX^2+1), \tag{37}$$

and

$$\frac{b^2(n - n\alpha^2 - 1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2} - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} \geq K(KX^2+1), \tag{38}$$

respectively. The conditions (36) and (37) impose a restriction on the constant displacement vector β_0.

5. *When β is a function of t i.e. $\beta = \beta(t)$*

In this case, to find the explicit value of displacement field $\beta(t)$, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{39}$$

where γ ($0 \leq \gamma \leq 1$) is a constant. Using Eqs. (28) – (30) and (39) in equations (11) and (14), we obtain

$$4\pi(1+\gamma)\rho = \frac{1}{a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[-K^2X^2 - K - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} - \frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right], \tag{40}$$

and

$$(1+\gamma)\beta^2(t)=\frac{4}{3a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[K^2X^2(1+\gamma)+2K\gamma+\frac{2b^2c_1c_2\{(1-\alpha)(1-\gamma)-4\}}{(\ell+1)\psi_2^2}\right.$$

$$\left.+\frac{b^2}{(\ell+1)^2}\{(2n\alpha^2-\alpha^2-2\alpha-2n+1)(1+\gamma)-2(n\alpha^2-n+1)\}\frac{\psi_1^2}{\psi_2^2}\right]. \tag{41}$$

Here we consider the three cases of physical interest in following Subsections 5.1, 5.2 and 5.3.

5.1. *Empty universe*

Putting $\gamma = 0$ in (39) reduces to $p = 0$. Thus, from Eqs. (40) and (41), we obtain the expressions for physical parameters ρ and $\beta^2(t)$

$$4\pi\rho=\frac{1}{a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[-K^2X^2-K-\frac{4b^2c_1c_2}{(\ell+1)\psi_2^2}+\frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right], \tag{42}$$

$$\beta^2(t)=\frac{4}{3a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[K^2X^2-\frac{2b^2(\alpha+4)c_1c_2}{(\ell+1)\psi_2^2}-\frac{b^2(\alpha+1)^2}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right]. \tag{43}$$

From Eqs. (42) and (43), we observe that $\rho > 0$ and $\beta^2(t) > 0$ according to

$$\frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\psi_1^2-K(KX^2+1)\psi_2^2>\frac{4b^2c_1c_2}{(\ell+1)}, \tag{44}$$

and

$$K^2X^2\psi_2^2-\frac{b^2(\alpha+1)^2}{(\ell+1)^2}\psi_1^2>\frac{2b^2(\alpha+4)c_1c_2}{(\ell+1)}, \tag{45}$$

respectively.

Halford [6] has pointed out that the displacement field ϕ_i in Lyra's geometry plays the role of the cosmological constant Λ in the normal general relativistic treatment. From Eq. (43), it is observed that the displacement vector $\beta(t)$ is a decreasing function of time which is corroborated with Halford as well as with the recent observations [76, 77] leading to the conclusion that $\Lambda(t)$ is a decreasing function of t.

5.2. *Zeldovich universe*

Putting $\gamma = 1$ in Eq. (39) reduces to $\rho = p$. In this case the expressions for physical quantities are given by

$$\beta^2(t) = \frac{4}{3a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[K^2X + K - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} - \frac{b^2(\alpha+1)^2}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right], \tag{46}$$

$$8\pi p = 8\pi\rho = \frac{1}{a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[-K^2X^2 - K - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} + \frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right]. \tag{47}$$

The reality condition (Ellis [74])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0,$$

lead to

$$\frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\psi_1^2 - K(KX^2+1)\psi_2^2 > \frac{4b^2c_1c_2}{(\ell+1)}. \tag{48}$$

5.3. *Radiating universe*

Putting $\gamma = \frac{1}{3}$ in Eq. (39) reduces to $p = \frac{1}{3}\rho$. In this case the expressions for $\beta(t)$, p and ρ are obtained as

$$\beta^2(t) = \frac{2}{3a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[2K^2X^2 + K + \frac{2b^2(\alpha+5)c_1c_2}{(\ell+1)\psi_2^2}\right.$$
$$\left. + \frac{b^2(n\alpha^2-2\alpha^2-4\alpha-n-1)}{3(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right], \tag{49}$$

$$8\pi p = \frac{1}{2a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[-K^2X^2 - K - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} + \frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right], \tag{50}$$

$$8\pi\rho = \frac{3}{2a^2\psi_2^{2n(1-\alpha)/(\ell+1)}}\left[-K^2X^2 - K - \frac{4b^2c_1c_2}{(\ell+1)\psi_2^2} + \frac{b^2(n-n\alpha^2-1)}{(\ell+1)^2}\frac{\psi_1^2}{\psi_2^2}\right]. \tag{51}$$

From Eq. (49), it is observed that the displacement vector β is a decreasing function of time and therefore it behaves as the cosmological term Λ.

The reality conditions (Ellis [74])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0,$$

are satisfied under the condition (48).

The dominant energy conditions (Hawking and Ellis[75])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0,$$

lead to

$$\frac{b^2(n - n\alpha^2 - 1)}{(\ell+1)^2}\psi_1^2 - K(KX^2+1)\psi_2^2 \geq \frac{4b^2 c_1 c_2}{(\ell+1)}. \tag{52}$$

Some geometric properties of the model

The expressions for the expansion θ, shear scalar σ^2, deceleration parameter q and proper volume V^3 for the model (31) are given by

$$\theta = \frac{b\{n(1-\alpha)+(1+\alpha)\}}{(\ell+1)a\psi_2^{n(1-\alpha)/(\ell+1)}}\frac{\psi_1}{\psi_2}, \tag{53}$$

$$\sigma^2 = \frac{b^2\left[\{n(1-\alpha)+(1+\alpha)\}^2 - 3n(1-\alpha)(1+\alpha) - 3\alpha\right]}{3(\ell+1)^2 a^2 \psi_2^{2n(1-\alpha)/(\ell+1)}}\frac{\psi_1^2}{\psi_2^2}, \tag{54}$$

$$q = -1 - \frac{6c_1c_2(\ell+1)}{n(1-\alpha^2)\left(c_1e^{bT} - c_2e^{-bT}\right)^2}, \tag{55}$$

$$V^3 = \sqrt{-g} = a^2\psi_2^{2n(1+\alpha)(1-\alpha)/(\ell+1)}e^{KX^2}. \tag{56}$$

From Eqs. (53) and (54) we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{\{n(1-\alpha)+(1+\alpha)\}^2 - 3n(1-\alpha^2) - 3\alpha}{3\{n(1-\alpha)+(1+\alpha)\}^2} = constant. \tag{57}$$

The rotation ω is identically zero.

The rate of expansion H_i in the directions of x, y and z are given by

$$H_x = \frac{A_4}{A} = \frac{nb(1-\alpha)}{(\ell+1)}\frac{\psi_1}{\psi_2},$$

$$H_y = \frac{B_4}{B} = \frac{b}{(\ell+1)}\frac{\psi_1}{\psi_2},$$

$$H_z = \frac{C_4}{C} = \frac{b\alpha}{(\ell+1)}\frac{\psi_1}{\psi_2}. \tag{58}$$

Generally, the model (31) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. The model (31) starts expanding at $T > 0$

and goes on expanding indefinitely when $n(1-\alpha)/(\ell+1) < 0$. Since $\sigma/\theta = constant$, the model does not approach isotropy. As T increases, the proper volume also increases. The physical quantities p and ρ decrease as F_{12} increases. However, if $n(1-\alpha)/(\beta+1) > 0$, the process of contraction starts at $T > 0$ and at $T = \infty$ the expansion stops. The electromagnetic field tensor does not vanish when $b \neq 0$, and $\alpha \neq 1$. It is observed from Eq. (55) that $q < 0$ when $c_1 > 0$ and $c_2 > 0$, which implies an accelerating model of the universe. Recent observations of type Ia supernovae [76, 77] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our models of the universe are consistent with recent observations. Either when $c_1 = 0$ or $c_2 = 0$, the deceleration parameter q approaches the value (-1) as in the case of de-Sitter universe.

6. *Solution in the absence of magnetic field*

In the absence of magnetic field, the field Eq. (9) with Eqs. (2) and (10) for metric (1) read as

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC}\right] = 8\pi p + \frac{3}{4}\beta^2, \tag{59}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{C_{11}}{C}\right) = 8\pi p + \frac{3}{4}\beta^2, \tag{60}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{B_{11}}{B}\right) = 8\pi p + \frac{3}{4}\beta^2, \tag{61}$$

$$\frac{1}{A^2}\left[-\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC}\right] = 8\pi \rho - \frac{3}{4}\beta^2, \tag{62}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) = 0. \tag{63}$$

Eqs. (60) and (61) lead to

$$\frac{B_{44}}{B} - \frac{B_{11}}{B} - \frac{C_{44}}{C} + \frac{C_{11}}{C} = 0. \tag{64}$$

Eqs. (19) and (64) lead to

$$\frac{g_{44}}{g} - \frac{k_{44}}{k} = 0. \tag{65}$$

Eqs. (23) and (65) lead to

$$\frac{g_{44}}{g} + \alpha \frac{g_4^2}{g^2} = 0, \tag{66}$$

which on integration gives

$$g = (c_4 t + c_5)^{1/(\alpha+1)}, \tag{67}$$

where c_4 and c_5 are constants of integration. Hence from (23) and (67), we have

$$k = c(c_4 t + c_5)^{\alpha/(\alpha+1)}. \tag{68}$$

In this case Eq. (22) again leads to

$$f = \exp\left(\frac{1}{2} K (x + x_0)^2\right). \tag{69}$$

Therefore, we have

$$B = \exp\left(\frac{1}{2} K (x + x_0)^2\right)(c_4 t + c_5)^{1/(\alpha+1)}, \tag{70}$$

$$C = \exp\left(\frac{1}{2} K (x + x_0)^2\right) c(c_4 t + c_5)^{\alpha/(\alpha+1)}, \tag{71}$$

$$A = a(c_4 t + c_5)^{n(1-\alpha)/(1+\alpha)}, \tag{72}$$

where a is already defined in previous section.
After using suitable transformation of the co-ordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = a^2 (c_4 T)^{2n(1-\alpha)/(1+\alpha)} (\mathrm{d}X^2 - \mathrm{d}T^2) + e^{KX^2} (c_4 T)^{2/(\alpha+1)} \mathrm{d}Y^2$$

$$+ e^{KX^2} (c_4 T)^{2\alpha/(\alpha+1)} \mathrm{d}Z^2, \tag{73}$$

where $x + x_0 = X$, $y = Y$, $cz = Z$, $t + c_5/c_4 = T$.

For the specification of the displacement field $\beta(t)$ within the framework of Lyra geometry and for realistic models of physical importance, we consider the following two cases given in Sections 7 and 8.

7. *When β is a constant i.e. $\beta = \beta_0$ (constant)*

Using Eqs. (70) – (72) in Eqs. (59) and (62) the expressions for pressure p and density ρ for the model (73) are given by

$$8\pi p = \frac{1}{a^2 (c_4 T)^{2n(1-\alpha)/(1+\alpha)}} \left[\left\{ \frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2} \right\} \frac{1}{T^2} + K^2 X^2 \right] - \frac{3}{4}\beta_0^2, \tag{74}$$

$$8\pi \rho = \frac{1}{a^2 (c_4 T)^{2n(1-\alpha)/(1+\alpha)}} \left[\left\{ \frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2} \right\} \frac{1}{T^2} - K(2+3KX^2) \right] + \frac{3}{4}\beta_0^2, \tag{75}$$

The dominant energy conditions (Hawking and Ellis [75])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0,$$

lead to

$$\frac{3}{4}\beta_0^2 a^2 (c_4 T)^{2n(1-\alpha)/(1+\alpha)} \geq K(1+2KX^2), \tag{76}$$

and

$$\left\{ \frac{n(1-\alpha^2)+\alpha}{(1+\alpha)^2} \right\} \frac{1}{T^2} \geq K(1+KX^2), \tag{77}$$

respectively.

The reality conditions (Ellis [74])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0,$$

lead to

$$\left\{ \frac{n(1-\alpha^2)+\alpha}{(1+\alpha)^2} \right\} \frac{1}{T^2} > K(1+KX^2), \tag{78}$$

and

$$\frac{2[n(1-\alpha^2)+\alpha]}{(1+\alpha)^2} \frac{1}{T^2} > K + \frac{3}{4}\beta_0^2 (c_4 T)^{2n(1-\alpha)/(1+\alpha)}. \tag{79}$$

The conditions (76) and (79) impose a restriction on β_0.

8. *When β is a function of t*

In this case, to find the explicit value of displacement field $\beta(t)$, we assume that the fluid obeys an equation of state given by (39). Using Eqs. (70) – (72) and (39) in Eqs. (59) and (62), we obtain expressions for $\rho(t)$ and $\beta(t)$ given by

$$8\pi(1+\gamma)\rho = \frac{1}{a^2(c_4T)^{2n(1-\alpha)/(1+\alpha)}}\left[\left\{\frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2}\right\}\frac{2}{T^2} - 2K(1+KX^2)\right], \quad (80)$$

$$(1+\gamma)\beta^2(t) = \frac{4}{3a^2(c_4T)^{2n(1-\alpha)/(1+\alpha)}}\left[\left\{\frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2}\right\}\frac{(1-\gamma)}{T^2}\right.$$

$$\left. +2K\gamma + KX^2(1+3\gamma)\right]. \quad (81)$$

It is observed that $\rho > 0$ and $\beta^2(t) > 0$ according to

$$\left\{\frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2}\right\}\frac{1}{T^2} > K(1+KX^2), \quad (82)$$

and

$$\left\{\frac{n(1-\alpha^2)+\alpha}{(\alpha+1)^2}\right\}\frac{1}{T^2} > K^2X^2, \quad (83)$$

respectively.

It is worth mentioning here that by putting $\gamma = 0, 1, \frac{1}{3}$ in Eqs. (80) and (81), one can derive the expressions for energy density $\rho(t)$ and displacement vector $\beta(t)$ for empty universe, Zeldovich universe and radiating universe, respectively. It is also observed that these three types of models have similar properties as we have already discussed above. Therefore, we have not mentioned the expressions for physical quantities of these models.

Some geometric properties of the model

The expressions for the expansion θ, Hubble parameter H, shear scalar σ^2, deceleration parameter q and proper volume V^3 for the model (73) in absence of magnetic field are given by

$$\theta = 3H = \frac{n(1-\alpha)+(1+\alpha)}{a(1+\alpha)c_4^{n(1-\alpha)/(1+\alpha)}}\frac{1}{T^{\{n(1-\alpha)+(1+\alpha)\}/(1+\alpha)}}, \quad (84)$$

$$\sigma^2 = \frac{\{n(1-\alpha)+(1+\alpha)\}^2 - 3n(1-\alpha^2) - 3\alpha}{3a^2(1+\alpha)^2c_4^{n(1-\alpha)/(1+\alpha)}}\frac{1}{T^{\{2n(1-\alpha)+2(1+\alpha)\}/(1+\alpha)}}, \quad (85)$$

$$q = -1 + \frac{3(\alpha+1)}{2n(1-\alpha)+2(1+\alpha)}, \quad (86)$$

$$V^3 = \sqrt{-g} = a^2 e^{KX^2} (c_4 T)^{\{2n(1-\alpha)+(1+\alpha)\}/(1+\alpha)}. \tag{87}$$

From Eqs. (84) and (85) we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{\{n(1-\alpha)+(1+\alpha)\}^2 - 3n(1-\alpha^2) - 3\alpha}{3\{n(1-\alpha)+(1+\alpha)\}^2} = constant. \tag{88}$$

The rotation ω is identically zero.

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{A_4}{A} = \frac{n(1-\alpha)}{(1+\alpha)}\frac{1}{T},$$

$$H_y = \frac{B_4}{B} = \frac{1}{(1+\alpha)}\frac{1}{T},$$

$$H_z = \frac{C_4}{C} = \frac{\alpha}{(1+\alpha)}\frac{1}{T}. \tag{89}$$

The model (73) starts expanding with a big bang at $T = 0$ and it stops expanding at $T = \infty$. It should be noted that the universe exhibits initial singularity of the point-type at $T = 0$. The space-time is well behaved in the range $0 < T < T_0$. In the absence of magnetic field, the model represents a shearing and non-rotating universe in which the flow vector is geodetic. At the initial moment $T = 0$, the parameters ρ, p, β, θ, σ^2 and H tend to infinity. So the universe starts from an initial singularity with infinite energy density, infinite internal pressure, infinitely large gauge function, infinite rate of shear and expansion. Moreover, ρ, p, β, θ, σ^2 and H are monotonically decreasing toward a non-zero finite quantity for T in the range $0 < T < T_0$ in the absence of magnetic field. Since $\sigma/\theta = constant$, the model does not approach isotropy. As T increases, the proper volume also increases. It is observed that for all three models, i.e., for empty universe, Zeldovice universe and radiating universe, the displacement vector $\beta(t)$ is a decreasing function of time and, therefore, it behaves like the cosmological term Λ. It is observed from Eq. (86) that $q < 0$ when $\alpha < (2n-1)/(2n+1)$ which implies an accelerating model of the universe. When $\alpha = -1$, the deceleration parameter q approaches the value (-1) as in the case of the de-Sitter universe. Thus, also in the absence of magnetic field, our models of the universe are consistent with recent observations.

9. *Discussion and concluding remarks*

In this paper, we have obtained a new class of exact solutions of Einstein's modified field equations for cylindrically symmetric space-time with perfect fluid distribution within the framework of Lyra's geometry, both in presence and absence of magnetic field. The solutions are obtained using the functional separability of the metric coefficients. The source of the magnetic field is due to an electric current produced along the z-axis. F_{12} is the only non-vanishing component of electromagnetic field tensor. The electromagnetic field tensor is given by Eq. (34), $\bar{\mu}$ remains undetermined as function of both x and t. The electromagnetic field tensor does not vanish if $b \neq 0$ and $\alpha \neq 1$. It is observed that in the presence of magnetic field, the rate of expansion of the universe is faster than that in the absence of magnetic field. The idea of primordial magnetism is appealing because it can potentially explain all large-scale fields seen in the universe today, specially those found in remote proto-galaxies. As a result, the literature contains many studies examining the role and the implications of magnetic fields for cosmology. In presence of magnetic field, the model (31) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. But in the absence of magnetic field, the model (73) is found that in the universe all matter and radiation are concentrated at the big-bang epoch and the cosmic expansion is driven by the big-bang impulse. The universe has singular origin and it exhibits a power-law expansion after the big-bang impulse. The rate of expansion slows down and finally stops at $T \to \infty$. In the absence of magnetic field, the pressure, energy density and displacement field become zero, whereas the spatial volume becomes infinitely large as $T \to \infty$.

It is possible to discuss entropy in our universe. In thermodynamics, the expression for entropy is given by

$$\mathrm{d}S = \mathrm{d}(\rho V^3) + p(\mathrm{d}V^3), \tag{90}$$

where $V^3 = A^2BC$ is the proper volume in our case. To solve the entropy problem of the standard model, it is necessary to treat $\mathrm{d}S > 0$ for at least a part of evolution of the universe. Hence Eq. (90) reduces to

$$T\mathrm{d}S = \rho_4 + (\rho + p)\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0. \tag{91}$$

The conservation equation $T^j_{i;j} = 0$ for (1) leads to

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{92}$$

Therefore, Eqs. (91) and (92) lead to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0, \tag{93}$$

which gives to $\beta < 0$. Thus, the displacement vector $\beta(t)$ affects entropy because for entropy $\mathrm{d}S > 0$ leads to $\beta(t) < 0$.

In spite of the homogeneity at large scale, our universe is inhomogeneous at small scale, so physical quantities being position-dependent are more natural in our observable universe if we do not go to super high scale. This result shows this kind of physical importance. It is observed that the displacement vector $\beta(t)$ coincides with the nature of the cosmological constant Λ which has been supported by the work of several authors as discussed in the physical behaviour of the model in Sections 5 and 8. In recent time Λ-term has attracted theoreticians and observers for many a reason. The nontrivial role of the vacuum in the early universe generates a Λ-term that leads to inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see Refs. [78] and [79]). Assuming that Λ owes its origin to vacuum interaction, as suggested particularly by Sakharov [80], it follows that it would, in general, be a function of space and time coordinates, rather than a strict constant. In a homogeneous universe, Λ will be at most time dependent [81]. In the case of inhomogeneous universe, this approach can generate Λ that varies both in space and time. In considering the nature of local massive objects, however, the space dependence of Λ cannot be ignored. For details, reference may be made to Refs. [82], [83], [84]. In recent past, there is an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology [85, 86, 87]. Therefore, the study of cosmological models in Lyra's geometry may be relevant for inflationary models. Also, the space dependence of the displacement field β is important for inhomogeneous models for the early stages of evolution of the universe. In the present study, we also find $\beta(t)$ as both space and time dependent which may be useful for a better understanding of the evolution of universe in cylindrically symmetric space-time within the framework of Lyra's geometry. There seems a good possibility of Lyra's geometry to provide a theoretical foundation for relativistic gravitation, astrophysics and cosmology. However, the importance of Lyra's geometry for astrophysical bodies is still an open question. In fact, it needs a fair trial by experiment.

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NEHOMOGEN SVEMIR S PERFEKTNOM TEKUĆINOM I ELEKTROMAGNETSKIM POLJEM U LYRINOJ GEOMETRIJI

Našli smo novu klasu cilindrično-simetričnih nehomogenih kozmoloških modela za perfektnu raspodjelu tekućine i s elektromagnetskim poljem u okviru Lyrine geometrije. Postigli smo dvije vrste rješenja razmatrajući jednoliko i vremenski ovisno posmično polje. Magnetsko se polje proizvodi električnom strujom duž z-osi. Tako je prisutna jedino sastavnica F_{12} elektromagnetskog tenzora polja. Radi postizanja određenosti rješenja, u modelu se pretpostavlja razmjernost širenja θ s posmikom σ. Našli smo da su rješenja u skladu s nedavnim opažanjima Ia supernova te da posmični vektor $\beta(t)$ utječe na entropiju. Raspravljaju se fizička i geometrijska svojstva modela uz i bez prisutnosti magnetskog polja.