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A CLASS OF INHOMOGENEOUS MESONIC PERFECT FLUID MODELS WITH TIME-DEPENDENT Λ-TERM

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A class of non-static inhomogeneous cosmological models are investigated with cosmological term $\Lambda(t)$ when source of the gravitational field is generated by a mixture of meson field and perfect fluid. Using gamma law equation of state, Einstein's field equations are solved for two particular cases which are physically important. The cosmological term $\Lambda(t)$ is found to be a decreasing function of time, which is supported by results found from recent type Ia supernovae observations. Some physical and geometrical aspects of the models are also discussed.

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1. *Introduction*

The cosmological term Λ has been introduced in 1917 by Einstein to modify his own equations of general relativity. Now this Λ-term remains a focal point of interest in modern theories. In 1930's, distinguished cosmologists A. S. Eddington and Abbé Georges Lemaître felt that introduction of Λ-term has attractive features in cosmology and models based on it should be discussed deeply. Moreover, models with Λ-term are becoming popular as they help to solve the cosmological constant problem in a natural way. Further, cosmological observations by High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [1], Perlmutter et al. [2], Riess et al. [3], Schmidt et al. [4]) strongly favor to a significant positive Λ with the magnitude $\Lambda(Gh/c^3) \approx 10^{-123}$ (Pradhan et al. [5]). These observations on magnitudes and red-shift of type Ia supernova suggest that our universe may be an accelerating one with a large fraction of the cosmological density in the form of Λ-term. The recent studies on cosmological constant problems by authors like Tsagas and Maartens [6], Sahni and Starobinsky [7], Peeble [8], Padmanabhan [9], Vishwakarma [10], Pradhan et al. [11], Sahu and Panigrahi [12] and Sahu and Mohapatra [13] motivates us for more and deep studies on the cosmological models involved with Λ-term.

Nowadays, cosmologists have taken keen interest to study the nature of scalar fields with or without a mass parameter interacting with a perfect fluid distribution in order to draw a resemblance of the physics of the cosmos with experimental results. A perfect fluid satisfactorily describes the distribution of matter due to the large-scale distribution of galaxies in our universe (Bali and Jain [14]). The physical phenomena observed, such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest an analysis of dissipative effects in cosmology (Pradhan et al. [15]). Plane-symmetric space-times with perfect fluid as the source have been studied in general relativity owing to their possible applications in astrophysics, cosmology and special relativistic hydrodynamics (Sahu and Mohanty [16]). The study of interacting fields, one of the fields being massless scalar field, is basically an attempt to look into yet another unsolved problem of the unification of gravitational and quantum theories (Reddy [17]). To our knowledge, no author has studied this theory for the plane-symmetric space-time with cosmological constant Λ when source of the gravitational field is generated by a mixture of massless scalar field and perfect fluid.

Therefore, in the present paper, we have considered this problem to study and construct a class of plane-symmetric inhomogeneous models in general relativity. Since the field equations are highly non-liner in nature, we derived solutions in two particular forms which are physically important. The kinematical and dynamical properties of all solutions found in the models are also studied. The work reported here may be considered as the extension work of Sahu and Mohapatra [13], Sahu and Mohanty [16] and generalization of work done by Panigrahi et al. [18].

2. *Einstein's field equations*

We consider the metric for a plane-symmetric space-time in the general form

$$\mathrm{d}s^2 = D^2\mathrm{d}t^2 - A^2\mathrm{d}x^2 - B^2(\mathrm{d}y^2 + \mathrm{d}z^2)\,, \tag{1}$$

where A, B and D are functions of x and t.

The Einstein's field equations with the cosmological term Λg_{ij} are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi(T^p_{ij} + T^m_{ij})\,, \tag{2}$$

where

$$T^p_{ij} = (\rho + p)u_i u_j - pg_{ij}; \quad g_{ij}u^i u^j = 1\,, \tag{3}$$

and

$$T^m_{ij} = \nu_i \nu_j - \frac{1}{2} g_{ij} \nu_k \nu^k \tag{4}$$

are respectively the energy-momentum tensors corresponding to a perfect fluid and massless scalar field. The massless scalar field ν satisfies the Klein-Gordon equation

$$g_{ij} \nu_{;ij} = 0\,. \tag{5}$$

Here ρ, p, u_i, ν and Λ are, respectively, the energy density, isotropic pressure, four velocity vector of the fluid, massless scalar field and the cosmological constant. Hereafter, the semicolon (;) denotes covariant differentiation. Now we use the co-moving coordinates as $u_i = (0,0,0,D)$ and hence $u^i = (0,0,0,1/D)$. Thus, using co-moving coordinate system, the set of field Eqs. (2) for the metric (1) reduces to the following forms,

$$\frac{2}{BD^2}\left[B_{44} - \frac{DB_1D_1}{A^2} - \frac{B_4D_4}{D}\right] - \frac{1}{B^2}\left[\frac{B_1^2}{A^2} - \frac{B_4^2}{D^2}\right] - \Lambda = -8\pi\left[p + \frac{1}{2}\left(\frac{\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right] \tag{6}$$

$$\frac{2}{B}\left[B_{14} - \frac{B_1A_4}{A} - \frac{D_1B_4}{D}\right] = -8\pi \nu_1 \nu_4\,, \tag{7}$$

$$\frac{1}{BD^2}\left[B_{44} - \frac{DB_1D_1}{A^2} - \frac{B_4D_4}{D}\right] - \frac{1}{A^2B}\left[B_{11} - \frac{A_1B_1}{A} - \frac{AA_4B_4}{D^2}\right] \tag{8}$$

$$+\frac{1}{A^2D^2}\left(AA_{44} - \frac{AA_4D_4}{D} - DD_{11} + \frac{DA_1D_1}{A}\right) - \Lambda = -8\pi\left[p + \frac{1}{2}\left(\frac{-\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right]$$

$$\frac{2}{A^2B}\left[B_{11} - \frac{A_1B_1}{A} - \frac{AA_4B_4}{D^2}\right] + \frac{1}{B^2}\left[\frac{B_1^2}{A^2} - \frac{B_4^2}{D^2}\right] + \Lambda = -8\pi\left[\rho + \frac{1}{2}\left(\frac{\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right] \tag{9}$$

The Klein-Gordon Eq. (5) for the metric (1) yields

$$\frac{\nu_{44}}{D^2} + \left(\frac{A_4}{A} + \frac{2B_4}{B} - \frac{D_4}{D}\right)\frac{\nu_4}{D^2} + \left(\frac{A_1}{A} - \frac{2B_1}{B} - \frac{D_1}{D}\right)\frac{\nu_1}{A^2} - \frac{\nu_{11}}{A^2} = 0\,. \tag{10}$$

Here and further on, the suffixes 1 and 4 of a field variable indicate partial differentiation with respect to x and t, respectively. Now we find an underdetermined system with five equations in seven unknowns viz. ρ, p, Λ, ν, A, B and D. Thus, to overcome the under-determinacy, we need two extra conditions. Let us assume the equation of state

$$p = \gamma\rho, \qquad 0 \le \gamma \le 1 \tag{11}$$

as an additional condition, where γ is called the adiabatic parameter. Assigning different values for γ, we can solve the field equations for different epochs.

3. Solutions of the field equations

Due to the inhomogeneity of the space-time and high non-linearity of the field equations, the second additional condition we consider are the explicit solutions of the field equations, which are physically important. Thus the metric functions A, B and D are taken in the following form (Davidson [19]):

$$A = t^{\alpha}(1+x^2)^a, \quad B = t^{\beta}(1+x^2)^b, \quad \text{and} \quad D = (1+x^2)^d, \tag{12}$$

where α, β, a, b and d are real constants ($\alpha \neq 0,\ \beta \neq 0$).

Further, to avoid mathematical complexities, we consider the scalar field ν to be the function of t only. Using the values of A, B and D from Eq. (12) in Eq. (10), we get

$$\nu_{44} + \frac{k\nu_4}{t} = 0\,, \tag{13}$$

where $k = \alpha + 2\beta$.

On integration, Eq. (13) reduces to

$$\nu_4 = \frac{k_1}{t^k}\,, \tag{14}$$

where k_1 is the non-zero constant of integration. Further, integration of Eq. (14) yields

$$\nu = \frac{k_1 t^{-k+1}}{-k+1} + k_2\,, \tag{15}$$

where k_i, $i = 1, 2$ are constants of integration. In view of Eqs. (12) and (15), Eqs. (6) – (9) yield

$$\frac{3\beta^2 - 2\beta}{t^2(1+x^2)^{2d}} - \frac{4b(2d+b)x^2}{t^{2\alpha}(1+x^2)^{2a+2}} + \frac{4\pi k_1^2}{t^{2k}(1+x^2)^{2d}} = \Lambda - 8\pi p\,, \tag{16}$$

$$\beta(d-b) + \alpha b = 0\,, \tag{17}$$

$$\frac{\alpha^2+\beta^2-\alpha-\beta+\alpha\beta}{t^2(1+x^2)^{2d}} + \frac{(4ad-4bd+2d-4d^2-4b^2+2b+4ab)x^2 + 2(d+b)}{t^{2\alpha}(1+x^2)^{2a+2}}$$

$$+\frac{4\pi k_1^2}{t^{2k}(1+x^2)^{2d}} = \Lambda - 8\pi p\,, \tag{18}$$

and

$$\frac{4b\{(3b-2a-1)x^2+1\}}{t^{2\alpha}(1+x^2)^{2a+2}} - \frac{\beta(2\alpha+\beta)}{t^2(1+x^2)^{2d}} + \frac{4\pi k_1^2}{t^{2k}(1+x^2)^{2d}} = -\Lambda - 8\pi\rho\,. \tag{19}$$

Comparing Eqs. (16) and (18), we find

$$b + d = 0\,, \tag{20a}$$

$$3\beta^2 - 2\beta = \alpha^2 + \beta^2 - \alpha - \beta + \alpha\beta\,, \tag{20b}$$

and

$$4ad - 4bd + 2d - 4d^2 - 4b^2 + 2b + 4ab = -4b(2d + b)\,. \tag{20c}$$

Now, corresponding to the Eqs. (20 a,b,c), we have two sets of solutions, i.e.

$$(i) \quad b = d = 0 \quad \text{and} \quad \alpha = \beta, \qquad (ii) \quad b = d = 0 \quad \text{and} \quad \alpha = -2\beta + 1\,.$$

$$\textbf{Case 1}: \quad \text{When} \quad b = 0,\ d = 0,\ \alpha = \beta = r(\text{say})\ (r \neq \frac{1}{3})\,. \tag{21}$$

Using the values from Eq. (21), Eqs. (15), (16) and (19) yield

$$\nu = \frac{k_1}{1 - 3r}\, t^{1-3r} + k_2\,, \tag{22}$$

$$\frac{2r - 3r^2}{t^2} - \frac{4\pi k_1^2}{t^{6r}} = 8\pi p - \Lambda\,, \tag{23}$$

$$\frac{3r^2}{t^2} - \frac{4\pi k_1^2}{t^{6r}} = 8\pi\rho + \Lambda\,. \tag{24}$$

Again, adding Eqs. (23) and (24), we obtain

$$p + \rho = \frac{1}{8\pi}\left[\frac{2r}{t^2} - \frac{8\pi k_1^2}{t^{6r}}\right]. \tag{25}$$

Sub-case I: If $\gamma = 1$, then Eq. (11) reduces to

$$p = \rho \tag{26}$$

Using Eq. (26), Eq. (25) yields

$$p = \rho = \frac{1}{8\pi}\left[\frac{r}{t^2} - \frac{4\pi k_1^2}{t^{6r}}\right]. \tag{27}$$

Using the value of p from Eq. (27) in Eq. (23), we get

$$\Lambda = \frac{r(3r-1)}{t^2}\,. \tag{28}$$

Thus the geometry of our universe, in this case, is described by the metric (1)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{2r}(1+x^2)^{2a}\mathrm{d}x^2 - t^{2r}(\mathrm{d}y^2 + \mathrm{d}z^2)\,, \tag{29}$$

or

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{2r}[\mathrm{d}\bar{x}^2 + \mathrm{d}y^2 + \mathrm{d}z^2]\,, \tag{30}$$

where $\mathrm{d}\bar{x} = (1+x^2)^a \mathrm{d}x$.

The model obtained in Eq. (30) is a stiff-fluid filled universe but not a de-Sitter universe nor an Einstein space. The model (30) is an important model in relativistic cosmology for the description of early stages of the universe.

Sub-case II: When $\gamma = 0$ and $\rho > 0$, Eq. (11) yields

$$p = 0 \qquad \text{(dust distribution)}. \tag{31}$$

Putting the value of p in Eqs. (23) and (25), we obtain

$$\Lambda = \frac{r(3r-2)}{t^2} + \frac{4\pi k_1^2}{t^{6r}}\,, \tag{32}$$

and

$$\rho = \frac{r}{4\pi t^2} - \frac{k_1^2}{t^{6r}}\,. \tag{33}$$

In this case, the geometry of our universe can also be represented by Eq. (30), which is a dust-filled universe.

Sub-case III: Let $\gamma = 1/3$. In this case Eq. (11) yields

$$\rho = 3p \qquad \text{(disordered radiation)}. \tag{34}$$

Now using $\rho = 3p$ in Eq. (25), we get

$$p = \frac{r}{16\pi t^2} - \frac{k_1^2}{4t^{6r}} \tag{35}$$

and

$$\rho = 3p = \frac{3r}{16\pi t^2} - \frac{3k_1^2}{4t^{6r}} \tag{36}$$

Applying Eq. (25) in Eq. (23), we get

$$\Lambda = \frac{3r(2r-1)}{2t^2} + \frac{2\pi k_1^2}{t^{6r}} \tag{37}$$

Thus the geometry of the space time (1), corresponding to solutions (35), (36) and (37), can be written by the same Eq. (30), which represents a radiation dominated universe.

Sub-case IV: When $\Lambda = 0$ and $\nu = 0$, the results reduce to the results obtained by Pradhan et al. in Ref. [20].

Sub-case V: When $\nu = 0$, the results reduce to those of Sahu and Mahapatra in Ref. [13].

$$\textbf{Case 2}: \quad \text{When} \quad b = 0,\ d = 0,\ \alpha = -2\beta + 1\,. \tag{38}$$

After substitution of the value of α from Eq. (38) in Eq. (13), we get

$$\nu_{44} + \frac{\nu_4}{t} = 0\,. \tag{39}$$

On integration, Eq. (39) yields

$$\nu = k_3 \ln t + k_4 \tag{40}$$

where k_3 and k_4 are constants of integration and $k_3 \neq 0$. Using Eq. (38) in Eqs. (16) and (19), we obtain

$$\frac{3\beta^2 - 2\beta}{t^2} + \frac{4\pi k_1^2}{t^2} = \Lambda - 8\pi p\,, \tag{41}$$

and

$$\frac{3\beta^2 - 2\beta}{t^2} + \frac{4\pi k_1^2}{t^2} = -\Lambda - 8\pi \rho\,. \tag{42}$$

Comparing Eqs. (41) and (42), we get

$$\Lambda = 4\pi(p - \rho)\,. \tag{43}$$

Sub-case I: When $\gamma = 1$, Eq. (11) reduces to Eq. (26). Applying Eq. (26) in Eq. (43), we find

$$\Lambda = 0\,. \tag{44}$$

Also, applying Eq. (44) in Eq. (41), we get

$$p = \rho = \frac{\beta(2-3\beta)}{8\pi t^2} - \frac{k_1^2}{2t^2}\,. \tag{45}$$

Hence, in this case, the geometry of our universe is described for the metric (1) as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{2(-2\beta+1)}(1+x^2)^{2a}\mathrm{d}x^2 - t^{2\beta}(\mathrm{d}y^2 + \mathrm{d}z^2)\,, \tag{46}$$

The model represented by Eq. (46) is a stiff-fluid filled universe. But if $a = 0$ and $\beta = 1/3$, then the model (46) reduces to the Einstein-de-Sitter universe.

Sub-case II: When $\gamma = 0$ and $\rho > 0$,

In this case Eq. (11) reduces Eq. (31). Applying Eq. (31) in Eq. (41), we find

$$\Lambda = \frac{3\beta^2 - 2\beta}{t^2} + \frac{4\pi k_1^2}{t^2}\,. \tag{47}$$

Further, using of Eq. (47) in Eq. (42), we get

$$\rho = \frac{\beta(2-3\beta)}{4\pi t^2} - \frac{k_1^2}{t^2}\,. \tag{48}$$

Therefore, the model of the universe described by the space-time (1) is the same as the model given in Eq. (46).

As in the sub-case I, for $a = 0$ and $\beta = 1/3$, the model (46) reduces to the Einstein-de-Sitter and dust-filled universe.

Sub-case III: For $\gamma = 1/3$, Eq. (11) reduces to Eq. (34). Using Eq. (34) in Eq. (43), we get

$$\Lambda = -8\pi p\,. \tag{49}$$

Using Eq. (49) in Eq. (41), we find

$$p = \frac{\beta(2-3\beta)}{16\pi t^2} - \frac{k_1^2}{4t^2}\,. \tag{50}$$

Putting the value of p in Eqs. (34) and (49), we get

$$\rho = 3p = \frac{3\beta(2-3\beta)}{16\pi t^2} - \frac{3k_1^2}{4t^2}\,. \tag{51}$$

and

$$\Lambda = -8\pi\left[\frac{\beta(2-3\beta)}{16\pi t^2} - \frac{k_1^2}{4t^2}\right] = \frac{\beta(3\beta-2)}{2t^2} + \frac{2\pi k_1^2}{t^2}\,. \tag{52}$$

In this case also, the model of the universe of space-time (1) is represented by Eq. (46), which is a radiation-dominated universe.

Sub-case IV: When $\Lambda = 0$ and D is a non-zero constant, then the results are the same as already found by Panigrahi et al. [18].

4. *Some physical and geometrical properties*

Case 1:

The reality conditions given by Ellis [21], (a) $\rho + p > 0$, (b) $\rho + 3p > 0$, (c) $\rho > 0$, are satisfied in all three sub-cases provided $r > 0$ for the sub-case I and $r > 1/3$ for the sub-cases II and III.

As $t \to 0$, $\nu \to \infty$ (provided $r \leq 1/3$) and ν tends to a constant (provided $r > 1/3$). Also, as $t \to \infty$, ν tends to a constant (provided $r \leq 1/3$) and $\nu \to -\infty$ (provided $r > 1/3$).

Again, as $t \to 0$, $\Lambda \to \infty$, ρ and p are undefined in the sub-cases I and III, while only ρ is undefined in the sub-case II. But as $t \to \infty$, $\Lambda \to 0$, $\rho \to 0$ and $p \to 0$ in all the three sub-cases.

From these results it is evident that the space-time admits a singularity, which may be a Big-Bang singularity. Moreover, it is found that Λ, the pressure p and the density ρ are decreasing functions of time t, and the parameters (physical and kinematical) remain finite and well behaved for $t > 0$.

The physical quantities, like the expansion scalar θ and the shear scalar σ^2 are given by the following expressions,

$$\theta = u^{\mu}_{;\mu} = \frac{A_4}{AD} + \frac{2B_4}{BD} + \frac{D_1}{A^2} = \frac{3r}{t} \,,$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right] = 0\,.$$

Since $\theta \to \infty$ as $t \to 0$ and $\theta \to 0$ as $t \to \infty$, we infer that in the models in the case 1 universe is expanding with time. However, the expansion in the models decreases as the time increases and expansion in the models stops when $t \to \infty$. Further, $\sigma = 0$ says that the models are sheer-free and approach isotropy as confirmed by $\sigma/\theta = 0$.

Also, as the rotation w given by the vorticity tensor $w_{\mu r}$, i.e. $w^2 = \frac{1}{2} w_{\mu r} w^{\mu r} = 0$ and acceleration $\dot{u}_{\mu} = 0$, hence the models found in the case 1 are non-rotating in nature and the mesonic fluid flow is geodesic.

Case 2:

The reality conditions in all the three sub-cases of case 2 are satisfied provided

$$\frac{1}{3} - \frac{1}{3}\sqrt{1 + 12\pi k_1^2} < \beta < \frac{1}{3} + \frac{1}{3}\sqrt{1 + 12\pi k_1^2}\,.$$

Here it is found that Λ, ρ and p are decreasing functions of t, whereas $\Lambda = 0$ in the sub-case I.

It is observed that $|\nu| \to \infty$, $p \to \infty$ and $\rho \to \infty$ as $t \to 0$. But in the sub-case II, we see $p = 0$. Similarly, as $t \to \infty$, $\nu \to \infty$, $p \to 0$ and $\rho \to 0$, while in the sub-case II, we have $p = 0$.

The above results admit a singularity, which may be a Big-Bang singularity. As in the case 1, here the scalar expansion θ and sheer scalar σ^2 are found, respectively, as

$$\theta = \frac{1}{t}, \qquad \text{and} \qquad \sigma = \frac{\sqrt{2}}{\sqrt{3}}\,\frac{1-3\beta}{t}\,.$$

Again, we get $\dfrac{\sigma}{\theta} = \dfrac{\sqrt{2}}{\sqrt{3}}(1-3\beta)$, which is a non-zero constant ($\beta \neq \frac{1}{3}$).

Now, $\theta \to \infty$ as $t \to 0$ and $\theta \to 0$ as $t \to \infty$. Thus the models in the case 2 are also of expanding in nature, which possess the same properties as the models in the case 1.

But as $t \to 0$, $\sigma \to \infty$, and as $t \to \infty$, $\sigma \to 0$. Thus the shape of the universes changes in x and y direction only, and the rate of change of the shape of the universes becomes slow with the increase of time. Also, $\lim\limits_{t\to\infty} \dfrac{\sigma}{\theta} \neq 0$ indicates that the models in the case 2 are anisotropic in nature.

As in the case 1, here also the vorticity tensor w_{ij} and the acceleration $\dot{u}_\mu$ are found to be zero each. Hence the models are of non-rotating nature and the fluid flow is geodesic.

5. *Conclusion*

In the present paper, we have made an attempt to study a class of inhomogeneous mesonic perfect-fluid models with cosmological term $\Lambda(t)$ in general relativity. It is observed that at $t = 0$, the parameters, both physical and kinematical, involved in the models diverge, while the parameters remain finite and well behaved for $t > 0$. It is also observed that $\dfrac{\sigma}{\theta} = \dfrac{\sqrt{2}}{\sqrt{3}}\,\dfrac{(\alpha-\beta)}{(\alpha-2\beta)}$ is an unique expression for $\dfrac{\sigma}{\theta} = 0$ and $\dfrac{\sigma}{\theta} = \dfrac{\sqrt{2}}{\sqrt{3}}(1-3\beta)$ as discussed in the case 1 and the case 2 of Sec. 4. It is further observed that the models found in the case 1 and the case 2 are expanding in nature and possess Big-Bang singularities. The models in the case 1 are shear-free and non-rotating, while in the case 2 the models are anisotropic and non-rotating. In all models, the physical parameters pressure and density are found to be decreasing functions of time. However, in the sub-case II of the case 2, we get $p = 0$. Similarly the cosmological term $\Lambda(t)$ is found to be a decreasing function of time t, whereas $\Lambda(t) = 0$ in the model of the sub-case I of the case 2.

Moreover, the cosmological constant found in the models (as discussed in Sec. 3) approaches small positive values at finite large times. These results are supported by the results from the supernova observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [1], Perlmutter et al. [2], Riess et al. [3], Schmidt et al. [4]).

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VRSTA NEHOMOGENIH MEZONSKIH MODELA S PERFEKTNOM TEKUĆINOM I VREMENSKI-OVISNIM ČLANOM Λ

Proučavamo vrstu nestatičkih i nehomogenih kozmoloških modela s kozmološkim članom $\Lambda(t)$ kada je izvor gravitacijskog polja mješavina mezonskog polja i perfektne tekućine. Primjenom zakona za γ za jednadžbu stanja rješavamo za Einsteinove jednadžbe polja za dva posebna slučaja koji su važni za fiziku. Nalazimo kozmološki član $\Lambda(t)$ kao opadajuću funkciju vremena, što je u skladu s opažanjima supernova Ia. Raspravljaju se neka fizička i geometrijska svojstva modela.