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BIANCHI TYPE-III STRING COSMOLOGICAL MODELS FOR STIFF AND ANTI-STIFF FLUID IN GENERAL RELATIVITY

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Bianchi type-III string cosmological models in the presence of stiff and anti-stiff fluids are studied. The scalar expansion is assumed to be proportional to the shear. In the present study, we consider two cases (i) $p + \rho = 0$ and (ii) $p - \rho = 0$, where ρ and p are the rest energy density and the pressure of the fluid, respectively. The physical behavior of these models is also discussed.

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1. *Introduction*

Today a precise understanding of structure formation in the universe is one of the outstanding problems in cosmology. It is generally believed that during the phase transition, the symmetry of the early universe was broken. This lead to the formation of topological defects such as domain walls, cosmic strings, monopoles and other 'hybrid' creatures [1]. Cosmic strings play an important role in the study of the early universe. They are topologically stable objects and may have been found during a phase transition in the early universe. They are considered to have arisen during the phase transition after the big-bang explosion as the temperature cooled down below some critical temperature as predicted by the grand unified theories [1 − 5]. Zel'dovich [6] believed that cosmic strings have given rise to density perturbations which lead to the formation of galaxies. It is interesting to study the gravitational effects of cosmic strings because they have stress-energy coupling to the gravitational field.

The study of Bianchi type cosmological models creates more interest because some of these models contain special isotropic cases and permit arbitrarily small anisotropy levels at some instant of cosmic time. Such property makes them to be known as a suitable model of our universe. One of the simplest anisotropic universe models which plays an important role in the understanding of essential features of the universe is Bianchi type-III.

In recent years, there has been considerable interest in string cosmology. Bali et al. [7 − 13] obtained Bianchi type-I, -III and -IX string cosmological models in general relativity. Yadav et al. [14] studied some Bianchi type-I viscous fluid string cosmological models with magnetic field. Wang [15 − 18] also discussed LRS Bianchi type-I and Bianchi type-III cosmological models for a cloud string with bulk viscosity. Yadav, Pradhan and Rai [19] obtained the integrability of cosmic string in Bianchi type-III space-time in the presence of bulk viscous fluid by applying a new technique.

Also, Pradhan and Amirhashchi [20] studied LRS Bianchi type-II string cosmological models for perfect fluid. Roy and Banerjee [21] investigated some LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Upadhaya and Dave [22] studied some magnetized Bianchi type-III massive string cosmological models in general relativity. Recently, Adhav et al. [23] studied Bianchi type-III string cosmological models. In their study, they did not consider the effect of cosmic pressure.

In this article we investigate Bianchi type-III string cosmological models for stiff and anti-stiff fluid distribution under two conditions (i) $p+\rho = 0$ and (ii) $p-\rho = 0$.

2. The metric and field equations

We consider the Bianchi type-III metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2\alpha x} dy^2 - C^2 dz^2, \tag{1}$$

where A, B and C are functions of t only. The energy momentum tensor for a cloud of strings with perfect fluid distribution is taken as

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu - p g_{\mu\nu} - \lambda x_\mu x_\nu \,, \tag{2}$$

where u_μ and x_μ satisfy the conditions

$$u_\mu u^\mu = -x^\mu x_\mu = 1, \quad u^\mu x_\mu = 0 \,. \tag{3}$$

p is the isotropic pressure, ρ is the proper energy density for a cloud string with particles attached to them, λ is the string tension density, v^μ the four-velocity of the particles, and x^μ is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u^\mu = (0,0,0,1), \quad x^\mu = (0,0,\frac{1}{C},0). \tag{4}$$

The particle density of the configuration is

$$\rho = \rho_p + \lambda\,. \tag{5}$$

The Einstein's field equations (in gravitational units $c = 1$, $8\pi G = 1$) are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}\,, \tag{6}$$

For the metric (1), these lead to

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = p\,, \tag{7}$$

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} = p\,, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\alpha^2}{A^2} = p + \lambda\,, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \rho\,, \tag{10}$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0\,. \tag{11}$$

The particle density ρ_p, expansion scalar θ and the shear scalar σ are given by

$$\rho_p = \frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} + 3\frac{\dot{A}\dot{C}}{AC}\,, \tag{12}$$

$$\theta = u^\mu_{;\mu} = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\,, \tag{13}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right]^2, \tag{14}$$

where an overdot stands for the first and double overdot for the second derivative with respect to t.

3. Solution of the field equations

In this, paper following Bali and Jain [24], and Pradhan et al. [25, 26], we assume that the expansion (θ) is proportional to the shear (σ). This assumption is in accord with the Thorne study [27] which quotes that the observations of the velocity redshift relation for extragalactic sources suggests that Hubble expansion of the universe is isotropic today to approximately within 30 percent [28, 29]. More precisely, redshift studies place the limit $\dfrac{\sigma}{H} \leq 0.3$, where σ and H are the shear and Hubble constants, respectively.

The field equations (7) − (11) are a system of five equations with six unknown parameters A, B, C, p, ρ and λ. One additional constraint relating these parameters is required to obtain explicit solutions of the system. If we assume that the expansion (θ) is proportional to the shear (σ), we have

$$C = A^n, \tag{15}$$

where n is a constant.

From Eq. (11) we get

$$A = mB\,. \tag{16}$$

For simplicity, we assume $m = 1$. With this assumption, we have

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} = p\,, \tag{17}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} - \frac{\alpha^2}{A^2} = p + \lambda\,, \tag{18}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = \rho\,. \tag{19}$$

In order to overcome the under-determinacy occurring here, we consider the following two cases.

3.1. Case I (Anti-stiff fluid)

In this case

$$p + \rho = 0\,. \tag{20}$$

From (15), (17) and (19), we obtain

$$(n+1)\frac{\ddot{A}}{A} + (n^2 + 2n + 1)\frac{\dot{A}^2}{A^2} = \frac{\alpha^2}{(n+1)}\frac{1}{A^2}\,. \tag{21}$$

Equation (21) can be written as

$$2\ddot{A} + 2(n+1)\frac{\dot{A}^2}{A} = \frac{2\alpha^2}{(n+1)}\frac{1}{A}\,. \tag{22}$$

Let $\dot{A} = f(A)$, which implies that $\ddot{A} = ff'$, where $f' = \dfrac{\mathrm{d}f}{\mathrm{d}A}$, and

$$\frac{\mathrm{d}}{\mathrm{dA}}(f^2) + 2(n+1)\frac{f^2}{A} = \frac{2\alpha^2}{(n+1)}\frac{1}{A}\,. \tag{23}$$

Equation (23), after integration, reduces to

$$f^2 = \left(\frac{\mathrm{d}A}{\mathrm{d}t}\right)^2 = \frac{\alpha^2}{(n+1)^2} + c_1 A^{-2(n+1)}, \tag{24}$$

where c_1 is an integrating constant. Thus the metric (1) reduces to

$$\mathrm{d}s^2 = \left[\frac{\mathrm{d}A^2}{\frac{\alpha^2}{(n+1)^2} + c_1 A^{-2(n+1)}}\right] - A^2\mathrm{d}x^2 - A^2 e^{-2\alpha x}\mathrm{d}y^2 - A^{2n}\mathrm{d}z^2, \tag{25}$$

After the transformation $A = T$, the metric (25) reduces to

$$\mathrm{d}s^2 = \left[\frac{\mathrm{d}T^2}{\frac{\alpha^2}{(n+1)^2} + c_1 T^{-2(n+1)}}\right] - T^2\mathrm{d}x^2 - T^2 e^{-2\alpha x}\mathrm{d}y^2 - T^{2n}\mathrm{d}z^2, \tag{26}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (26) are given by

$$p = \frac{n^2\alpha^2}{(n+1)^2}\frac{1}{T^2} - (2n+1)c_1 T^{-2(n+2)}, \tag{27}$$

$$\rho = (2n+1)c_1 T^{-2(n+2)} - \frac{n^2\alpha^2}{(n+1)^2}\frac{1}{T^2}\,, \tag{28}$$

$$\lambda = -\frac{2n\alpha^2}{(n+1)}\frac{1}{T^2}\,, \tag{29}$$

$$\rho_p = \frac{n(n+2)\alpha^2}{(n+1)^2}\frac{1}{T^2} + (2n+1)c_1 T^{-2(n+2)}, \tag{30}$$

$$\theta^2 = (n+2)^2 \left[\frac{\alpha^2}{(n+1)^2} + c_1 T^{-2(n+2)} \right], \tag{31}$$

$$\sigma^2 = \frac{(n-1)^2}{3} \left[\frac{\alpha^2}{(n+1)^2} + c_1 T^{-2(n+2)} \right], \tag{32}$$

$$V^3 = \sqrt{-g} = e^{-\alpha x} T^{n+2}. \tag{33}$$

From Eqs. (31) and (32), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = constant\,. \tag{34}$$

The model (26) starts with a big-bang at $T = 0$. The expansion in the model decreases as time increases. The expansion stops at $T = \infty$. When $T \to 0$ then $\rho \to \infty$, $\lambda \to \infty$. When $T \to \infty$ then $\rho \to 0$, $\lambda \to 0$. Also $p \to \infty$ when $T \to 0$ and $p \to 0$ when $T \to \infty$. Since $\dfrac{\sigma}{\theta} = constant$, the model does not approach isotropy.

According to Refs. [1] and [30], when $\dfrac{\rho_p}{|\lambda|} > 1$, in the process of evolution, the universe is dominated by massive strings, and when $\dfrac{\rho_p}{|\lambda|} < 1$, the universe is dominated by the strings. In this case, from Eqs. (29) and (30), we obtain, $c_1 > 0$,

$$\lim_{T \to 0} \frac{\rho_p}{|\lambda|} > 1\,. \tag{35}$$

Thus, in our model, the universe is dominated by massive strings in the early era after the big-bang. Also, for $T \to \infty$, we have

$$\lim_{T \to \infty} \frac{\rho_p}{|\lambda|} < 1\,. \tag{36}$$

So in this case the universe is dominated by the strings when $T \to \infty$.

3.2. *Case II (Stiff fluid)*

In this case

$$p - \rho = 0\,. \tag{37}$$

Equations (15), (17)and (19) lead to

$$(-n^2 + 2n + 1)\frac{\dot{A}^2}{A^2} - (n+1)\frac{\ddot{A}}{A} - \frac{\alpha^2}{A^2}\,. \tag{38}$$

Equation (38) can be written as

$$2\ddot{A} + \frac{2(n^2-2n-1)}{(n+1)}\frac{\dot{A}^2}{A} = -\frac{2\alpha^2}{(n+1)}\frac{1}{A}\,. \tag{39}$$

Let $\dot{A} = f(A)$ which implies that $\ddot{A} = ff'$, where $f' = \dfrac{\mathrm{d}f}{\mathrm{d}A}$. Hence (39) reduces to

$$\frac{\mathrm{d}}{\mathrm{dA}}f^2 + \frac{2(n^2-2n-1)}{(n+1)}\frac{f^2}{A} = -\frac{2\alpha^2}{(n+1)}\frac{1}{A}\,. \tag{40}$$

Equation (40), after integration, leads to

$$f^2 = \left(\frac{\mathrm{d}A}{\mathrm{d}t}\right)^2 = -\frac{\alpha^2}{(n^2-2n-1)} + c_2 A^{-2\frac{(n^2-2n-1)}{(n+1)}}, \tag{41}$$

where c_2 is a integrating constant. Thus the metric (1) reduces to

$$\mathrm{d}s^2 = \left[\frac{\mathrm{d}A^2}{-\frac{\alpha^2}{(n^2-2n-1)} + c_2 A^{-2\frac{(n^2-2n-1)}{(n+1)}}}\right] - A^2\mathrm{d}x^2$$

$$-A^2 e^{-2\alpha x}\mathrm{d}y^2 - A^{2n}\mathrm{d}z^2. \tag{42}$$

After the transformation $A = T$, the metric (42) reduces to

$$\mathrm{d}s^2 = \left[\frac{\mathrm{d}T^2}{-\frac{\alpha^2}{(n^2-2n-1)} + c_2 T^{-2\frac{(n^2-2n-1)}{(n+1)}}}\right] - T^2\mathrm{d}x^2$$

$$-T^2 e^{-2\alpha x}\mathrm{d}y^2 - T^{2n}\mathrm{d}z^2. \tag{43}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (43) are given by

$$p = \rho = -\frac{n^2\alpha^2}{(n^2-2n-1)}\frac{1}{T^2} + (2n+1)c_2 T^{-2n(\frac{n-1}{n+1})}, \tag{44}$$

$$\lambda = \frac{2n\alpha^2}{(n^2-2n-1)}\frac{1}{T^2} - \frac{(4n^2-2n-2)}{(n+1)}c_2 T^{-2n(\frac{n-1}{n+1})}, \tag{45}$$

$$\rho_p = -\frac{(n^2+2n)\alpha^2}{(n^2-2n-1)}\frac{1}{T^2} + \frac{(6n^2+n-1)}{(n+1)} c_2 T^{-2n(\frac{n-1}{n+1})}, \tag{46}$$

$$\theta^2 = (n+2)^2 \left[-\frac{\alpha^2}{(n^2-2n-1)}\frac{1}{T^2} c_2 T^{-2n(\frac{n-1}{n+1})} \right], \tag{47}$$

$$\sigma^2 = \frac{(n-1)^2}{3} \left[-\frac{\alpha^2}{(n^2-2n-1)}\frac{1}{T^2} c_2 T^{-2n(\frac{n-1}{n+1})} \right], \tag{48}$$

$$V^3 = \sqrt{-g} = e^{-\alpha x} T^{n+2}. \tag{49}$$

From Eqs. (47) and (48), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = constant\,. \tag{50}$$

The model (43) starts with a big-bang at $T = 0$. The expansion in the model decreases as time increases. The expansion in the model stops at $T = \infty$. When $T \to 0$, then $\rho \to \infty$, $\lambda \to \infty$. When $T \to \infty$, then $\rho \to 0$, $\lambda \to 0$. Also $p \to \infty$, when $T \to 0$, and $p \to 0$, when $T \to \infty$. Since $\frac{\sigma}{\theta} = constant$, the model does not approach isotropy.

From Eqs. (45) and (46), we obtain

$$\lim_{T\to 0} \frac{\rho_p}{|\lambda|} < 1 \quad (\text{for} \quad n \le 2)\,, \tag{51}$$

$$\lim_{T\to \infty} \frac{\rho_p}{|\lambda|} < 1 \quad (\text{for} \quad n \ge 3)\,, \tag{52}$$

$$\lim_{T\to \infty} \frac{\rho_p}{|\lambda|} > 1 \quad (\text{for} \quad n \le 2)\,, \tag{53}$$

$$\lim_{T\to 0} \frac{\rho_p}{|\lambda|} > 1 \quad (\text{for} \quad n \ge 3)\,. \tag{54}$$

Hence, in this case, when $T \to 0$ for $n \le 2$ and when $T \to \infty$ for $n \ge 3$, the universe is dominated by the strings. When $T \to \infty$ for $n \le 2$ and when $T \to 0$ for $n \ge 3$, the universe is dominated by massive strings.

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References

[1] T. W. B. Kibble, J. Phys. A: Math. Gen. **9** (1976) 1387.

[2] Ya. B. Zel'dovich, I. Yu. Kobzarev and L. B. Okun, Zh. Eksp. Teor. Fiz. **67** (1975) 3; Sov. Phys.-JETP **40** (1975) 1.

[3] T. W. B. Kibble, Phys. Rep. **67** (1980) 183.

[4] A. E. Everett, Phys. Rev. **24** (1981) 858.

[5] A. Vilenkin, Phys. Rev. D **24** (1981) 2082.

[6] Ya. B. Zel'dovich, Mon. Not. Roy. Astron. Soc. **192** (1980) 663.

[7] R. Bali and S. Dave, Pramana-J. Phys. **56** (2001) 513.

[8] R. Bali and R. D. Upadhaya, Astrophys. Space Sci. **283** (2003) 97.

[9] R. Bali and S. Dave, Astrophys. Space Sci. **288** (2003) 503.

[10] R. Bali and D. K. Singh, Astrophys. Space Sci. **300** (2005) 387.

[11] R. Bali and Anjali, Astrophys. Space Sci. **302** (2006) 201.

[12] R. Bali, U. K. Pareek and A. Pradhan, Chin. Phys. Lett. **24** (2007) 2455.

[13] R. Bali and A. Pradhan, Chin. Phys. Lett. **24** (2007) 585.

[14] M. K. Yadav, A. Pradhan and S. K. Singh, Astrophys. Space Sci. **311** (2007) 423.

[15] X. X. Wang, Astrophys. Space Sci. **293** (2004) 933.

[16] X. X. Wang, Chin. Phys. Lett. **21** (2004) 1205.

[17] X. X. Wang, Chin. Phys. Lett. **22** (2005) 29.

[18] X. X. Wang, Chin. Phys. Lett. **23** (2006) 1702.

[19] M. K. Yadav, A. Pradhan and A. Rai, Int. J. Theor. Phys. **46** (2007) 2677.

[20] A. Pradhan, H. Amirhashchi and M. K. Yadav, Fizika B (Zagreb) **18** (2009) 35.

[21] S. R. Roy and S. K. Banerjee, Class. Quant. Grav. **11** (1995) 1943 .

[22] R. D. Upadhaya and S. Dave, Braz. J. Phys. **38** (2008) 615.

[23] K. S. Adhav, M. V. Dawande and V. B. Raut, Int. J. Theor. Phys. **48** (2009) 700.

[24] R. Bali and V. C. Jain, Astrophys. and Space Sci. **262** (1999) 145.

[25] A. Pradhan, L. Yadav and A. K. Yadav, Czech. J. Phys **54** (2003) 487 .

[26] A. Pradhan and S. K. Singh, Int. J. Mod. Phys. D **13** (2004) 503 .

[27] K. S. Thorne, Astrophys. J. **148** (1967) 51.

[28] R. Kantowski and R. K. Sachs, J. Math. Phys. **7** (1966) 433.

[29] J. Kristian and R. K. Sachs, Astrophys. J. **143** (1966) 379; G. Mohanty, R. R. Sahoo and K. L. Mahanta, Astrophys. Space Sci. **312** (2007) 321.

[30] K. D. Krori, T. Chaudhuri, C. R. Mahanta and A. Mazumdar, Gen. Rel. Grav. **22** (1990) 123.

BIANCHIJEVI KOZMOLOŠKI MODELI III VRSTE SA STRUNAMA ZA UKOČENU I NEUKOČENU TEKUĆINU U OPĆOJ RELATIVNOSTI

Proučavamo Bianchijeve kozmološke modele III vrste sa strunama za ukočenu i neukočenu tekućinu. Pretpostavlja se razmjernost širenja i posmika. U ovom se radu razmatraju dva slučaja: (i) $p + \rho = 0$ i (ii) $p - \rho = 0$, gdje su ρ i p gustoća energije mirovanja odnosno tlak tekućine. Raspravljaju se fizička svojstva tih modela.