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EVIDENCE OF ENERGY DEPENDENCE OF SELF-SIMILAR CASCADING RATE OF PIONS IN NUCLEAR COLLISION

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This paper presents a study on energy dependence of self-similar cascading rate of produced pions in nuclear collision in the entire accelerator energy 2.1A GeV – 200A GeV $\left[{}^{16}\mathrm{O}\right.$-AgBr interactions at 2.1A GeV and 60A GeV, Pb-Pb interactions at 158A GeV and ^{32}S-AgBr interactions at 200A GeV$\left.\right]$ using the results of F-moments methodology and Levy stable index in both pseudorapidity (η) space and azimuthal angle (ϕ) space. This analysis reveals a positive dependence of self-similar cascading rate on beam energy.

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1. *Introduction*

In high energy nuclear physics, a large number of theoretical and experimental investigations have given a progressive outlook for the multiparticle production processes at relativistic and ultra-relativistic energies. The observations from these experiments reveal the existence of non-statistical fluctuations during the multiparticle production. So the analyses of non-statistical fluctuations are believed to throw light on the inner dynamics of particle production process. Thus the study on non-statistical fluctuations during multiparticle production is of major interest today. A large number of methodologies have been developed for studying the large non-statistical fluctuations, such as correlation, intermittency, etc. These efforts have revealed the power-law dependence of the scaled factorial moments of the multiplicity distribution of produced particles on the bin size. The phenomenon is known as intermittency and is an evidence for the dynamical fluctuations [1]. Intermittent type of fluctuations has been observed in e^+e^- annihilation [2], muon-hadron [3], hadron-hadron [4], hadron-nucleus [5] and nucleus-nucleus interactions [6]. However, a single mechanism that could explain intermittency has not been found.

Non-statistical fluctuations may be a manifestation of quark-gluon plasma (QGP) phase transition which might occur at ultra-relativistic nucleus-nucleus interactions. One of the signatures of QGP formation is larger non-statistical fluctuations among produced pions. The concept of self-similarity is closely related to the fractal theory and is a consequence of cascade mechanism prevailing in multiparticle production. Multifractality has been studied previously and at present time in relativistic heavy-ion collisions. It would be interesting to study whether the self-similar cascading rate is dependent on beam energy or not. No studies in this regards have been reported so far. In view of this, we used the data of fluctuation study (F-moments) of shower particles at lower and higher energy (^{16}O-AgBr interactions at 2.1A GeV and 60A GeV, Pb-Pb interactions at 158A GeV and ^{32}S-AgBr interactions at 200A GeV) to study the energy dependence of the Levy index (μ) in both pseudorapidity (η) and azimuthal angle (ϕ) space. For the analysis of intermittent behaviour the variable rapidity and the azimuthal angle (ϕ) have usually been used. But for emulsion studies, it is very difficult to measure rapidity of each particle. Pseudorapidity (η) is used as the basic variable instead of rapidity for this data analysis because rapidity reduces to pseudorapidity for the relativistic particles. The degree of multifractality in multiparticle production can be measured by the parameter μ ($0 < \mu < 2$) [7], which also estimates the cascading rate in self-similar branching processes. The values of μ also give information about the nature of phase transition during the cascading process, as they suggest a thermal phase transition if $0 < \mu < 1$ and a non-thermal one if $\mu > 1$.

2. Experimental details

The data set used in the present analysis was obtained by exposing Ilford G5 emulsion stacks to ^{16}O beam of energy 2.1A GeV at BERKELEY BEVALAC, Ilford G5 emulsion stacks to ^{16}O beam of energy 60A GeV at CERN SPS and Ilford G5 emulsion stacks exposed to ^{32}S beam of energy 200A GeV at CERN SPS. The scanning of the plates is carried out with the help of a high-resolution Leitz Metalloplan microscope provided with semi-automatic scanning and measuring system. The scanning is done using objective 10× in conjunction with a 25× ocular lens. To increase the scanning efficiency, two independent observers scanned the plates independently. For measurement 100× oil-immersion objective was used in conjunction with 25× ocular lens. The measuring system fitted with it has 1 μm resolution along the X- and Y-axes and 0.5 μm resolution along the Z-axis.

The events were chosen according to the following criteria:

(i) The incident beam track should lie within a very narrow forward cone to the mean beam direction of the pellicle. The opening angle depends on the energy. It is done to ensure that we have taken the real projectile beam.

(ii) The events, which are within 20 μm thickness from the top or bottom surface of the plate, are rejected. It is done to reduce the loss of tracks as well as to reduce the error in angle measurement.

(iii) The events, of which primary beam tracks are observed to be a secondary track of other interaction, are not be analyzed and are rejected.

According to the emulsion terminology [8], the particles emitted after interactions are classified as:

a. Black particles: Black particles consist of both single and multiply charged fragments. They are target fragments of element like carbon, with ionization greater than or equal to $10I_0$, I_0 being the minimum ionization of a singly-charged relativistic particle. These black particles having maximum ionizing power are less energetic and consequently they are short-ranged. Their range is less than 3 mm in the emulsion medium. They have velocities less than $0.3c$ and energy less than 30 MeV, where c is the velocity of light in vacuum. In the emulsion experiments, it is very difficult to measure the charge of these fragments. So identification of the exact nucleus is not possible.

b. Grey particles: They are mainly fast target recoil protons with energy up to 400 MeV. They have ionization $1.4 \leq I \leq 10I_0$. These particles have range greater than 3 mm in the emulsion medium and having velocities $0.3 \leq V \leq 0.7c$.

c. Shower particles: The relativistic shower tracks with ionization I less than or equal to $1.4I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle. These shower particles have energy in the GeV range.

d. Projectile fragments: Along with those tracks, there are a few projectile fragments. In high-energy nuclear collisions, the projectile beam which collides with the target nucleus also undergoes fragmentation. These particles have constant ionization, long range and small emission angle. They generally lie within a very narrow forward cone with respect to the main beam direction and the opening angle depends on the energy. Great care should be taken to identify these projectile fragments.

Event by event analysis demands separation of events into ensembles of collisions of different projectiles with hydrogen (H), light nuclei (C, N, O) and heavy nuclei (Ag, Br). Usually events with the number of heavy ionizing tracks $N_h \leq 1$ are classified as collisions with hydrogen. Events with $2 \leq N_h \leq 7$ are classified as collisions with light nuclei and $N_h \geq 8$ as collisions with heavy nuclei. By this method, the separation of events for the AgBr target is quite accurate in the sample with $N_h > 8$ but for $N_h \leq 8$, there is an admixture of CNO events and peripheral collisions with the AgBr target. In our experiment, only events with a number of heavy tracks, i.e. the number of black tracks plus the number of grey tracks $N_h > 8$, have been selected to exclude CNO interactions. In this investigation, we performed our analysis on central and quasi-central events only. Here we add one point. In emulsion experiment, it is very difficult to assess the impact parameter quantitatively, and hence to comment on the centrality of a collision. However, it may be stated that for a perfect central collision, there should not be any projectile fragments with charge $Z \geq 2$ [9]. In our analysis for all data sets, most of the events have no projectile fragments with charge $Z \geq 2$. The average number of projectile fragments with charge $Z \geq 2$ is less than 0.3 [10−13]. This excludes the possibility of existence of peripheral collisions in the considered data sets.

According to the above selection procedure, we have chosen 730 events of ^{16}O-AgBr interactions at 2.1A GeV [14], 250 events of ^{16}O-AgBr interactions at 60A GeV [12] and 140 events of ^{32}S-AgBr interactions at 200A GeV [12]. The emission angle (θ) and azimuthal angles (ϕ) are measured for each track with respect to the beam direction by taking the readings of the coordinates of the interaction point (X_0, Y_0, Z_0), the coordinates (X_1, Y_1, Z_1) at any point on each secondary track and the coordinates (X_i, Y_i, Z_i) of a point on the incident beam. The variables used for this analysis are pseudorapidity (η) and azimuthal angle (ϕ). The pseudorapidity (η) values of the shower particles are calculated from the emission angles (θ) of the tracks with the help of the relation

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right). \tag{1}$$

It is important to mention that the emulsion is a very sensitive detector which enrolls all charged particles in 4π geometry and provides very high spatial resolution. The shower tracks have considerably long range. So the probability of missing a shower track by the observers is almost zero. Hence the information loss due to detector biasing and observers inability is negligible.

However, like other detectors, nuclear emulsion plates are not free from systematic errors. Systematic errors may be introduced in emulsion plates due to the presence of background events which may result from the cosmic rays during the exposure time. These background events can be eliminated by properly choosing the primary events due to incident beam. Systematic errors may also arise in the measurement of polar angles due to fading of tracks and variation of shrinkage factor with temperature. However, these errors are small and hence can not influence the final results.

3. *Method of study*

We have adapted the scaled factorial moment method to investigate the fluctuation pattern of produced pions. The number of particles considered is finite at the available projectile energy. Hence, the statistical fluctuation is very much obvious and the standard moments fail to reveal the dynamical fluctuation of particle density distribution. The scaled factorial moment method is free from the hazards of statistical noise pollution and so it is used for this analysis.

Consider a certain interval in some x space, defined as $\Delta x = x_{\max} - x_{\min}$. The considered region is divided into M bins of equal size having the width $\delta x = \Delta x/M$.

The normalized scaled factorial moment is defined as [1]

$$\langle F_q \rangle = \left\langle M^{q-1} \sum_{m=1}^{M} \frac{k_m(k_m-1)\cdots(k_m-q+1)}{\langle k_m \rangle^q} \right\rangle \tag{2}$$

where k_m is the number of particles in the mth bin ($m = 1, 2, \cdots, M$), angular bracket denotes an average over the whole sample of events and q is the order of moment which can assume any positive integral value starting from 2.

If the non-statistical fluctuations are self-similar in nature, in the limit of small bin size, factorial moment obeys the scaling relation

$$\langle F_q \rangle \propto M^{\alpha_q}. \tag{3}$$

From Eq. (3), we get

$$\ln\langle F_q \rangle = \alpha_q \ln M + C\,. \tag{4}$$

In analogy with turbulent fluid dynamics, this property is called “intermittency”. In connection with high energy interactions, α_q measures the strength of intermittency and it is called intermittency exponent. From this intermittency exponent α_q, one can find out the signals of non-thermal phase transition with the help of the relation

$$\lambda_q = \frac{1+\alpha_q}{q}\,. \tag{5}$$

The generalized fractal dimension (D_q) is calculated from α_q using the relation

$$D_q = 1 - \frac{\alpha_q}{q-1}\,. \tag{6}$$

Thus the ‘anomalous fractal dimension’ d_q can be defined as

$$d_q = 1 - D_q\,. \tag{7}$$

The ratios of higher-order anomalous fractal dimensions with respect to the second order anomalous fractal dimension can be written as,

$$\frac{B_q}{B_2} = \frac{d_q}{d_2}(q-1)\,, \tag{8}$$

$$\beta_q = \frac{d_q}{d_2}(q-1)\,, \tag{9}$$

where, $\beta_q = B_q/B_2$ represents the degree of multifractality. β_q is related to the Levy index (μ) [7] by the equation

$$\beta_q = \frac{q^\mu - q}{2^\mu - 2}\,. \tag{10}$$

4. *Results and discussion*

In this paper, analysis is done on produced pions in the light of scaled factorial moment method. For the present analysis, we have divided the η and ϕ space into $M = 3, 4, \cdots, 30$ bins for ^{16}O-AgBr interactions at 2.1A GeV and 60A GeV and ϕ space into the same number of bins for ^{32}S-AgBr interactions at 200A GeV. For each event, F-moments are calculated for different order of moments q ($q = 2, 3, 4$ and 5) using Eq. (2) for ^{16}O-AgBr interactions at 2.1A GeV and 60A GeV for both phase spaces, and that of ^{32}S-AgBr interactions at 200A GeV for the ϕ space. The exponent α_q of the power-law behaviour is obtained by least-squares fitting of the data points. We have collected the values of α_q for the ^{32}S-AgBr interaction at 200A GeV in the η space from our previous publication [15].

The generalized fractal dimensions D_qs (q = 2, 3, 4 and 5) have been calculated from the slopes of the best linear fits for the produced pions following the relation (6). We have also calculated the anomalous fractal dimensions (d_q) for different orders of moments (q = 3, 4 and 5) for all interactions in both the η and ϕ space following the relation (7). The values of the degree of multifractality β_q have also been calculated for all interactions for both phase spaces using the relation (9). For all these interactions, the values of β_q have been tabulated in Table-1 and Table-2 for η and ϕ spaces, respectively. We also include the results of the similar analysis for Pb-Pb interactions at 158A GeV [16]. We have then calculated the Levy index

TABLE 1. Values of the degree of multifractality (β_q) for different values of $q-1$, (q = 3, 4 and 5) and Levy index (μ), for ^{16}O-AgBr *interactions at 2.1A* GeV, ^{16}O-AgBr *interactions at 60A* GeV, Pb-Pb *interactions at 158* GeV *and* ^{32}S-AgBr *interactions at 200A* GeV *in η space for shower particles.*

Interactions	$q-1$	β_q	μ
^{16}O-AgBr (2.1A GeV)	2	2.374 ± 0.046	1.149 ± 0.046
	3	3.993 ± 0.049	
	4	5.640 ± 0.052	
^{16}O-AgBr (60A GeV)	2	2.531 ± 0.033	1.379 ± 0.034
	3	4.603 ± 0.034	
	4	7.075 ± 0.037	
Pb-Pb (158A GeV)	2	1.336 ± 0.040	1.949 ± 0.040
	3	2.034 ± 0.041	
	4	2.692 ± 0.041	
^{32}S-AgBr (200A GeV)	2	2.050 ± 0.004	1.709 ± 0.006
	3	3.600 ± 0.006	
	4	5.500 ± 0.005	

TABLE 2. Values of the degree of multifractality (β_q) for different values of $q-1$, ($q = 3, 4$ and 5) and Levy index (μ), for ^{16}O-AgBr interactions at 2.1A GeV, ^{16}O-AgBr interactions at 60A GeV, Pb-Pb interactions at 158 GeV and ^{32}S-AgBr interactions at 200A GeV in ϕ space for shower particles.

Interactions	$q-1$	β_q	μ
^{16}O-AgBr (2.1A GeV)	2	3.130 ± 0.033	1.149 ± 0.031
	3	6.535 ± 0.031	
	4	7.840 ± 0.027	
^{16}O-AgBr (60A GeV)	2	3.004 ± 0.019	1.279 ± 0.027
	3	5.721 ± 0.023	
	4	8.704 ± 0.027	
Pb-Pb (158A GeV)	2	1.332 ± 0.042	1.929 ± 0.040
	3	2.028 ± 0.039	
	4	2.680 ± 0.039	
^{32}S-AgBr (200A GeV)	2	2.962 ± 0.019	1.609 ± 0.022
	3	5.565 ± 0.022	
	4	8.464 ± 0.024	

(μ) by fitting β_q vs. q curve in the form of Eq. (10). The values of (μ) have been tabulated in Table 1 and Table 2 for the four interactions in η and ϕ spaces, respectively. The energy dependence of (μ) values are also shown in Fig. 1 and Fig. 2 for η and ϕ spaces, respectively, for all interactions. It is observed that (μ) value increases with energy in both phase spaces. However, the Levy index for Pb-Pb case does not properly fit the systematics of Levy indices for other interactions. This may be attributed to the following facts: The Pb-Pb system is a totally symmetric system, whereas the other systems are highly asymmetric, and the impact parameter selection criterion for Pb-Pb interactions is different from that of the other interactions.

The present study yields some notable features which can be summarized below.

- In both η space and ϕ space, Levy index increases with energy which indicates that the self-similar cascading rate increases with beam energy.
- The μ-value corresponding to Pb-Pb interaction at the energy 158A GeV is the highest.
- The Levy indices for pseudo-rapidity space are very close to those for azimuthal angle space for all cases. Though at this stage it is difficult to point out the reason behind such a behaviour, the feature is no doubt very interesting.

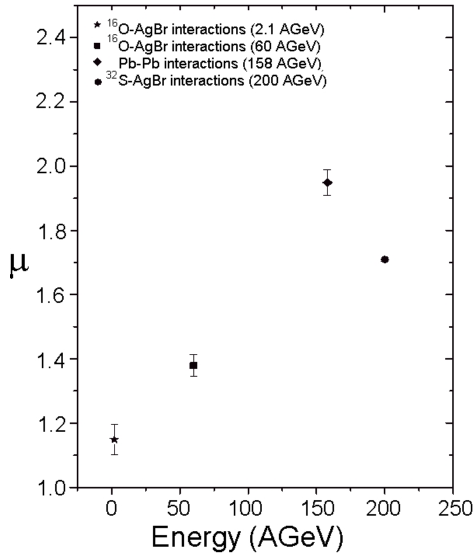

Fig. 1. Plot of Levy index (μ) vs. energy per nucleon for shower particles of ^{16}O-AgBr *interactions at 2.1A* GeV, ^{16}O-AgBr *interactions at 60A* GeV, *Pb-Pb interactions at 158* GeV *and* ^{32}S-AgBr *interactions at 200A* GeV *in pseudorapidity (η) space.*

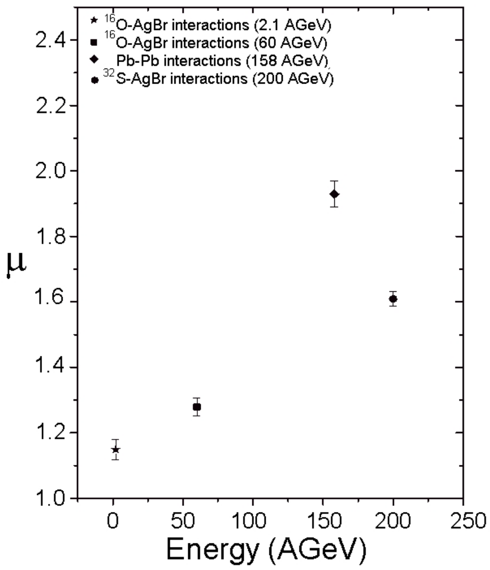

Fig. 2. Plot of Levy index (μ) vs. energy per nucleon for shower particles of ^{16}O-AgBr *interactions at 2.1A* GeV, ^{16}O-AgBr *interactions at 60A* GeV, *Pb-Pb interactions at 158* GeV *and* ^{32}S-AgBr *interactions at 200A* GeV *in azimuthal angle (ϕ) space.*

5. *Conclusion*

Self-similar cascading rate increases with the energy of the projectile beam with an exception for the Pb-Pb interactions. Moreover the cascading rates are almost equal in pseudorapidity and azimuthal angle spaces.

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References

[1] A. Bialas and R. Peschanski, Nucl. Phys. B **273** (1986) 703.

[2] DELPHI Collab, Phys. Lett. B **247** (1990) 137.

[3] W. Shaoshun, Z. Jie, Y. Yunxiu, X. Chinguo and Z. Yu, Phys. Rev. D **49** (1990) 5785.

[4] UA1 Collab., Nucl. Phys. B **345** (1990) 1.

[5] KLM Collab., Phys. Rev. C **40** (1989) R2449.

[6] EMU-01 Collab., Phys. Rev. Lett. **65** (1990) 412.

[7] W. Ochs, Phys. Lett. B 247 (1990) 101; W. Ochs, Z. Phys. C **50** (1991) 339.

[8] C. F. Powell, P. H. Fowler and D. H. Perkins, *The Study of Elementary Particles by Photographic Method*, Oxford, Pergamon (1959) p. 450.

[9] H. Heckman et al. Phys. Rev. C **17** (1978) 1651.

[10] D. Ghosh et al., Phys. Rev. C **47** (1993) 1120.

[11] D. Ghosh et al., Phys. Lett. B **218** (1989) 431.

[12] D. Ghosh et al., Europhys. Lett. **65** (2004) 311.

[13] D. Ghosh et al., J. Phys. G **16** (1990) 1505.

[14] D. Ghosh et al., Phys. Rev. C **49** (1994) R1747.

[15] D. Ghosh et al., Z. Phys. C **71** (1996) 243.

[16] A. M. Tawfik, hep-ph/0104004v1, 1 Apr 2001.

DOKAZI O ENERGIJSKOJ OVISNOSTI SLIČNIH SLJEDOVA EMITIRANIH PIONA U NUKLEARNIM SUDARIMA

Predstavljamo proučavanje energijske ovisnosti sličnih sljedova piona proizvedenih u nuklearnim sudarima u širokom području energije od 2.1A GeV do 200A GeV $\left[{}^{16}\text{O-AgBr}\right.$ sudari na 2.1A GeV i 60A GeV, Pb-Pb sudari na 158A GeV, te ^{32}S-AgBr sudari na 200A GeV$\left.\right]$. Primjenjujemo metodologiju F momenata i Levyjeve stabilne indekse za prostor pseudorapiditeta (η) i prostor azimutalnog kuta (ϕ). Analize pokazuju pozitivnu ovisnost sličnih sljedova piona o energiji sudara.