Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

LETTER TO THE EDITOR

ACCELERATED EXPANSION OF THE HIGHER-DIMENSIONAL SPATIALLY FLAT UNIVERSE

R. A. EL-NABULSI

College of Mathematics and Information Science, Qujing Normal University, Qujing, Yunnan 655011, PR China
E-mail address: nabulsiahmadrami@yahoo.fr

Received 31 March 2009; Revised manuscript received 28 July 2010
Accepted 15 December 2010 Online 6 April 2011

We discuss in this work two independent scenarios of a higher-dimensional spatially flat universe. In the 1st scenario, we conjecture that the cosmological constant varies according to the ansatzs $\Lambda = \rho^{\alpha}$, $\alpha \in \mathbb{R}$, where ρ is the energy density assumed to vary like $\rho = DH^{m}$, $(D, m) \in \mathbb{R}$. In the 2nd scenario, we conjecture that the cosmological constant varies like $\Lambda = G^{\beta}$, $\beta \in \mathbb{R}$, where G is the gravitational coupling constant. Both scenarios reveal many interesting and appealing consequences which are discussed in some details.

PACS numbers: 04.50.+h, 11.25.Yb, 98.80.Es, 95.36.+x UDC 524.83
Keywords: higher-dimensional cosmology, dark energy, accelerated expansion

Various cosmological and astronomical observations like the Type Ia Supernova, the first acoustic peak of the CMB background radiation, the large scale structure favour a spatially flat universe undergoing a late phase of accelerated expansion with the equation of state (EoS) parameter $w = p/\rho < -1/3$ (ρ and p are, respectively, the density and pressure of the perfect fluid) [1 − 7]. Dozens of phenomenological models have been presented for the explanation of this mysterious phenomenon such as quintessence [8], viscous fluid [9], Chaplygin gas [10, 11], generalized Chaplygin gas model [12 − 15], holographic dark energy [16, 17] and so on. Other models are based on certain modifications of Einstein's general relativity which can have very similar consequences for special choices of their corresponding parameters [18 − 28]. On the other hand, it is widely believed that extra-dimensional spacetimes play a crucial role at the cosmological and elementary particle levels [12, 29 − 38]. In this context, we want to investigate the dynamical properties of vacuum energy or cosmological constant in a kind of non-trivial higher-dimensional cosmological model. A phenomenological approach is proposed here to build a model which is able to fit the available data and leads to accelerated expansion of the universe. To this aim, we will discuss two independent scenarios.

1) In the 1st scenario, we conjecture that the cosmological constant varies according to the ansatz $\Lambda = \rho^\alpha$, $\alpha \in \mathbb{R}$, where ρ is the energy density which in its turn is assumed to vary like $\rho = DH^m$, $(D, m) \in \mathbb{R}$.

2) In the 2nd scenario, we conjecture that the cosmological constant varies like $\Lambda = G^\beta$, $\beta \in \mathbb{R}$, where G is the gravitational coupling constant.

Here H is the Hubble parameter of the higher-dimensional homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime metric described by the combination of the standard $(n + 2)$ FRW metric as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)\left[\mathrm{d}r^2 + r^2(\mathrm{d}\theta_1^2 + \sin^2\theta_1 \mathrm{d}\theta_2^2 + \ldots + \sin^2\theta_{n-1}\mathrm{d}\theta_n^2)\right]. \tag{1}$$

The motivations for introducing the ansatzs $\Lambda = \rho^\alpha$ and $\Lambda = G^\beta$ are in reality based on most recent studies on the evolution of dark energy and phantom energy [39]. The ansatz $\rho = DH^m$ is motivated by the observations that many of the cosmological models predict that, during its evolution, the universe undergoes a series of regimes where the energy density scales like $\rho \propto (H^3, H^2, H, \ldots)$, e.g. pressureless regime, radiation regime, brane cosmology and so on [18, 19]. Besides all these, we will implement in our theory a dynamical gravitational coupling constant G. Particularly significant are found those cosmological models in which the cosmological constant and G-variation laws were inferred from some fundamental physical theories, such as perturbative particle physics theory with the Hilbert-Einstein action treated semiclassically [40 – 44], or the quantum gravity approach where nonperturbative solutions were obtained within the "Hilbert-Einstein truncation" [45], gravitational holography [46, 47] or those theories sorting from string, superstring and M-theories (Refs. [18], [19], [48] and references therein).

The Einstein field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -8\pi G T_{ij} = -8\pi G\big[(p+\rho)u_i u_j - p g_{ij}\big], \tag{2}$$

where g_{ij} is the metric tensor. The Friedmann equation takes the special form

$$\frac{n(n+1)}{2}\,\frac{\dot{a}^2}{a^2} = 8\pi G\rho + \Lambda\,, \tag{3}$$

whereas the Bianchi identity

$$\left(R^{ij} - \frac{1}{2}g^{ij}R\right)_{;i} = -\big(8\pi G T^{ij} + \Lambda g^{ij}\big)_{;i} = 0\,, \tag{4}$$

leads directly with the equation of state $p = (\gamma - 1)\rho$, $\gamma = constant$, to

$$\dot{\rho} + (n+1)\gamma\rho\,\frac{\dot{a}}{a} = 0\,, \tag{5}$$

and

$$\dot{\Lambda} + 8\pi\dot{G}\rho = 0\,. \tag{6}$$

First scenario: With $\Lambda = \rho^\alpha$, Eq. (6) gives

$$\frac{\alpha}{\alpha-1}\,\rho^{\alpha-1} = -8\pi G\,. \tag{7}$$

With $\rho = DH^m$, Eq. (5) gives easily

$$(n+1)\gamma H^{m+1} + mH^{m-1}\dot{H} = 0\,. \tag{8}$$

It is easy at this stage to check that the solution of Eq. (8) is

$$a(t) = \left(\frac{(n+1)\gamma}{m}t\right)^{m/\{(n+1)\gamma\}}\,. \tag{9}$$

Therefore, the energy density, the cosmological constant and the gravitational coupling constant vary, respectively, like

$$\rho = DH^m = D\left(\frac{m}{(n+1)\gamma}\right)^m t^{-m}\,, \tag{10}$$

$$\Lambda = \rho^\alpha = D^\alpha\left(\frac{m}{(n+1)\gamma}\right)^{m\alpha} t^{-m\alpha}\,, \tag{11}$$

$$G = \frac{1}{8\pi}\,\frac{\alpha}{1-\alpha}\rho^{\alpha-1} = \frac{1}{8\pi}\,\frac{\alpha}{1-\alpha}D^{\alpha-1}\left(\frac{m}{(n+1)\gamma}\right)^{m(\alpha-1)} t^{-m(\alpha-1)}\,. \tag{12}$$

Besides, Eq. (3) gives the following constraints between the parameters,

$$\frac{n(n+1)}{2}H^2 = \frac{1}{1-\alpha}\,D^\alpha H^{m\alpha}\,, \tag{13}$$

and accordingly $m\alpha = 2$. However, a positive gravitational coupling constant requires $0 < \alpha < 1$ and then $m > 2$. Besides, accelerated expansion occurs if, for instance, $m > (n+1)\gamma$, i.e. $(n+1)\gamma = 2$. Then for $n = 2$, which corresponds to a four-dimensional spacetime, $\gamma = 2/3$ (dark energy dominated stage with equation of state parameter $w = \gamma - 1 = -1/3$) and for $n = 3$, which corresponds to a five-dimensional spacetime, we find $\gamma = 1/2$ (dark energy dominated stage with equation of state parameter $w = \gamma - 1 = -1/2$). For a very large number of extra-dimensions, $\gamma \to 0$ and hence the universe is dominated by a vacuum energy. In our framework, $\Lambda \propto t^{-2}$, $\rho \propto t^{-m}$ and $G \propto t^{m-2}$ with $m > 2$ (increasing gravitational coupling constant). The present day variation (PDV) of the gravitational constant is $(\dot{G}/G)_0 = (m-2)(n+1)\gamma H_0/m = 2(m-2)H_0/m$ per year which for $H_0 = 10^{-10}$ y^{-1} is allowable value if compared with the results of observations for $m = 2$ [49, 50].

Second scenario: With $\Lambda = G^{\beta}$, Eq. (6) gives

$$\rho = -\frac{\beta}{8\pi} G^{\beta-1}\,, \tag{14}$$

and accordingly, Eq. (5) gives

$$G = a^{(n+1)\gamma/(\beta-1)}\,. \tag{15}$$

It is easy to check from Eq. (3), making use of Eqs. (14) and (15), that the scale factor behaves like

$$a(t) = \left(\frac{2(\beta-1)}{\gamma\beta(n+1)} \sqrt{\frac{2(3-\beta)}{3n(n+1)}}\; t \right)^{2(\beta-1)/\{\gamma\beta(n+1)\}} \tag{16}$$

with $\beta < 0$ in order to satisfy a positive energy density. The cosmological constant varies like

$$\Lambda(t) = \left(\frac{2(\beta-1)}{\gamma\beta(n+1)} \sqrt{\frac{2(3-\beta)}{3n(n+1)}}\; t \right)^{-2}\,. \tag{17}$$

A decaying and positive cosmological constant is satisfied if, for instance, also the following constraint is satisfied: $\gamma > 0$. Besides, accelerated expansion will occur if $\gamma(n+1) > 2$. For $n = 2$ (4D), $\gamma > 2/3$ (dark energy dominated stage with equation of state parameter $w = \gamma - 1 > -1/3$), while for $n = 3$ (5D) $\gamma > 1/2$ (dark energy dominated stage with equation of state parameter $w = \gamma - 1 > -1/2$). In our framework, the gravitational coupling constant varies like

$$G(t) = \left(\frac{2(\beta-1)}{\gamma\beta(n+1)} \sqrt{\frac{2(3-\beta)}{3n(n+1)}}\; t \right)^{-2/\beta}\,. \tag{18}$$

The present day variation (PDV) of G is now $(\dot{G}/G)_0 = (n+1)\gamma H_0/(1-\beta)$ per year which for $H_0 = 10^{-10}$ y^{-1} is as well an allowable value if compared with the results of observations for $\beta \ll 0$.

In summary, we have discussed two independent cosmological scenarios which in addition to their simplicity hold several interesting consequences and new features which depart from the standard FRW cosmology. In both scenarios, the cosmological constant and the energy density decays in time, besides the universe is accelerating in time and dominated by dark energy. However, in the 1st scenario, the 4D universe is dominated by dark energy with an equation of state parameter

$\gamma - 1 = -1/3$, whereas in the second scenario, the equation of state parameter is $\gamma - 1 > -1/3$. In addition, in both models, the PDV is close to the results of observations for a certain range of the parameters. One more interesting feature is the following: in the 1$^{\text{st}}$ scenario, we have $(n+1)\gamma = 2$ and hence for a very large number of extra-dimensions, the equation of state parameter tends to -1 which corresponds for a vacuum dominated energy, whereas in the 2$^{\text{nd}}$ scenario, we have $(n+1)\gamma > 2$ and therefore for $n \to \infty$, $\gamma - 1 > -1$, which corresponds to a dark energy dominance. We have simply presented our results as possible departures from the current standard model. Further details are under current investigation.

References

[1] A. G. Riess et al., Astron. J. **116** (1998) 1009.

[2] S. Perlmutter et al., Astrophys. J. **517** (1999) 565.

[3] B. R. Schmidt et al., Astrophys. J. **507** (1998) 46.

[4] P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D **59** (1999) 123504.

[5] M. Persic, P. Salucci and F. Stel, Mon. Not. Roy. Astron. Soc. **281** (1996) 27.

[6] A. G. Riess et al., Astrophys. J. **607** (2004) 665.

[7] J. S. Alcaniz, Phys. Rev. D **69** (2004) 083521.

[8] Ph. Brax and J. Martin, J. Phys. Lett. B **468** (1999) 45.

[9] J. C. Fabris, S. V. B. Concalves and R. de Sa Ribeiro, Gen. Rel. Grav. **38** (2006) 495.

[10] J. C. Fabris, S. V. B. Concalves and P. E. de Souz, Gen. Rel. Grav. **34** (2002) 53.

[11] N. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D **66** (2002) 043507.

[12] N. Bilić, G. P. Tupper and R. D. Viollier, Phys. Lett. B **535** (2002) 17.

[13] J. C. Fabris et al., Phys. Lett. A **367** (2007) 423.

[14] S. Mukherjee, Class. Quantum Grav. **23** (2006) 6927.

[15] R. A. El-Nabulsi, Braz. J. Phys. **40**, 2 (2010) 150.

[16] M. R. Setare and M. Saridakis, JCAP**0809** (2008) 026.

[17] M. R. Setare and M. Saridakis, Phys. Lett. B **668** (2008) 177.

[18] R. A. El-Nabulsi, Int. J. Mod. Phys. D **18**, 15 (2009) 691.

[19] R. A. El-Nabulsi, Int. J. Mod. Phys. D **18**, 2 (2009) 289.

[20] R. A. El-Nabulsi, Comm. Theor. Phys. **53** (2010) 869.

[21] R. A. El-Nabulsi, Mod. Phys. Lett. A **23**, 6 (2008) 40.

[22] R. A. El-Nabulsi, Gen. Rel. Grav. **42**, 6 (2010) 1381.

[23] R. A. El-Nabulsi, Astrophys. Space Scie. **327**, 1 (2010) 111.

[24] S. Nojiri and S. D. Odintsov, Phys. Rev. D **74** (2006) 086005.

[25] S. K. Srivastava, Int. J. Mod. Phys. D **17**, 5 (2008) 755.

[26] R. A. El-Nabulsi, Fizika B **16**, 2 (2007) 79.

[27] T. Clifton and J. D. Barrow, Phys. Rev. D **72** (2005) 103005.

[28] J. D. Barrow and J. Middleton, Phys. Rev. D **75** (2007) 123515.

[29] C. Csaki, M. Graesser and J. Terning, Phys. Lett. B **456** (1999) 16.

[30] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83** (1999) 4922.

[31] C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D **62** (2000) 045015.

[32] C. Csaki, J. Erlich, T. J. Hollowood and J. Terning, Phys. Rev. D **63** (2001) 065019.

[33] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Rev. Lett. **84** (2000) 5928.

[34] V. Gorini, A. Kamenshchik and U. Moschella, Phys. Rev. D **67** (2003) 063509.

[35] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. **344** (2003) 1057.

[36] R. A. El-Nabulsi, Astrophys. Space Scie. **324**, 1 (2009) 71.

[37] R. A. El-Nabulsi, Braz. J. Phys. **40**, 2 (2010).

[38] A. I. Arbab, Chin. Phys. Lett. **25**, 10 (2008) 3834.

[39] I. L. Shapiro and J. Sola, Phys. Lett. B **475** (2000) 236.

[40] I. L. Shapiro and J. Sola, JHEP **0202** (2002) 006.

[41] I. L. Shapiro, J. Sola, C. Espana-Bonet, and P. Ruiz-Lapuente, Phys. Lett. B **574** (2003) 149.

[42] C. Espana-Bonet, P. Ruiz-Lapuente, I. L. Shapiro and J. Sola, JCAP **0402** (2004) 006.

[43] B. Guberina, R. Horvat and H. Stefancić, Phys. Rev. D **67** (2003) 083001.

[44] M. Reuter, Phys. Rev. D **57** (1998) 971.

[45] R. Horvat, Phys. Rev. D **70** (2004) 087301.

[46] S. D. Hsu, Phys. Lett. B **594** (2004) 13.

[47] M. Li, Phys. Lett. B **603** (2004) 1.

[48] R. A. El-Nabulsi, Braz. J. Phys. **39** (2009) 1.

[49] G. P. Singh, R. V. Deshpande and T. Singh, PRAMANA J. Phys. **63**, 5 (2004) 937.

[50] J. D. Anderson et al., Acta Astron. **5** (1978) 43.

UBRZANO ŠIRENJE VIŠEDIMENZIJSKOG PROSTORNO RAVNOG SVEMIRA

Proučavamo dvije neovisne postavke za višedimenzijski prostorno ravan svemir. U prvoj postavci uzimamo da se kozmološka konstanta mijenja kao $\Lambda = \rho^\alpha$, $\alpha \in \mathbb{R}$, gdje je ρ gustoća energije čija je ovisnost dana s $\rho = DH^m$, $(D, m) \in \mathbb{R}$. U drugoj postavci uzimamo da se kozmološka konstanta mijenja kao $\Lambda = G^\beta$, $\beta \in \mathbb{R}$, gdje je G gravitacijska konstanta vezanja. Obje postavke otkrivaju zanimljive i podsticajne ishode koji se podrobno raspravljaju.