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PLANE SYMMETRIC COSMOLOGICAL MODELS IN SELF CREATION COSMOLOGY

R. K. TIWARI

Department of Mathematics, Govt. Model (P.G.) Science College, Rewa (M. P.) 486001, India

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Plane symmetric cosmological models are investigated in the framework of Barber's second self-creation theory of gravitation. We solved the Einstein's field equation by using the "gamma law" equation of state $p = m\rho$ where $0 \leq m \leq 1$, in the presence of perfect fluid. Physical consequences of the models have been discussed in the case of Zel'dovich fluid, disordered radiation and vacuum solution.

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1. Introduction

Barber [1] produced two continuous self-creation theories by modifying the Brans-Dicke theory and general relativity [2]. The modified theories create the universe out of self-contained gravitational and matter fields. Brans and Dicke theory [3] develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large-scale distribution of matter in motion. However, Barber [1] included continuous creation of matter in these theories. The universe is seen to be created out of self-contained gravitational, scalar and matter fields. In the Barber's second self-creation theory, the gravitational coupling of the Einstein's field equations is allowed to be a variable scalar on space-time manifold. This second theory is modification of general relativity to a variable G-theory and predicts local effects, and secondly is an adaptation of general relativity to include continuous creation and is within the observational ambit. In this theory, the Newtonian gravitational parameter G is not a constant but a function of time. Further, the scalar field does not gravitate directly, but simply divide the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trace of energy-momentum tensor. The theory predicts the same precession of the perihelia of the planets as general relativity and in that respect agrees with observation to within 1%. In the limit $\lambda \to 0$, this theory approaches the Einstein's theory in every respect. In view of the consistency of Barber's second theory of gravitation, we intend to investigate some of the aspects of this theory in this paper.

Pimentel [4] and Soleng [5] discussed the Robertson-Walker solutions in Barber's second self-creation theory of gravitation by using a power-law relation between the expansion factor of the universe and the scalar field. Carvalho [6] studied a homogeneous and isotropic model of the early universe in which parameter gamma of the 'gamma law' equation of state varies continuously with cosmological time and presented a unified description of early universe for inflationary period and radiation-dominating era. Singh and Singh [7], Reddy [8, 9], and Reddy et al. [10] presented Bianchi type space-time solutions in Barber's second theory of gravitation. Reddy and Venkateswarlu [11] presented Bianchi type-VI_0 cosmological solutions in Barber's second theory of gravitation, both in vacuum as well as in the presence of perfect fluid with pressure equal to energy density. Shanthi and Rao [12] studied Bianchi type-II and III space-times in the second theory of gravitation, both in vacuum as well as in the presence of stiff fluid. Ram and Singh [13, 14] discussed spatially homogeneous and isotropic R-W and Bianchi type-II models of the universe in Barber's second self-creation theory of gravitation in the presence of perfect fluid by using the 'gamma-law' equation of state. Panigrahi and Sahu [15] presented plane symmetric cosmological micro model in this modified theory of Einstein's general relativity.

The Einstein's field equations are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form for the matter content or that space time admits killing-vector symmetries [16]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman [17]. In simple cases, the Hubble's law yields a constant value of deceleration parameter. It is worth observing that most of the well-known models of Einstein's theory and Brans-Dike theory with curvature parameter $k = 0$, including inflationary models, are models with a constant deceleration parameter. In earlier literature, cosmological models with a constant deceleration parameter have been studied by Berman [17], Berman and Gomide [18], Johri and Desikan [19], Singh and Desikan [20], Maharaj and Naidoo [21], Pradhan et al. [22, 23] and others. In recent years, Mohanty et al. [24], Panigrahi and Sahu [25], Sahu and Mohanty [26], Venkateswarlu and Kumar [27], Singh and Kumar [28], Singh et al. [29], Venkateswarlu et al. [30] and Pradhan et al. [31] have studied Barber's second self-creation theory of gravitation in various contexts.

In this paper we obtain cosmological solutions for plane symmetric cosmological model in Barber's second self-creation theory of gravitation with perfect fluid.

2. *The metric and field equations*

We consider the plane-symmetric space-time in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2, \tag{1}$$

where A and B are function of time t. The plane-symmetric metric is spatially homogenous and anisotrophic. The field equations in Barber's second self-creation theory of gravitation are given by [1]

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi T_{ij}}{\phi}\,, \tag{2}$$

and

$$\Box\phi = \frac{8\pi\lambda T}{3}\,, \tag{3}$$

where $\Box\phi = \phi^{k}_{;k}$ is the invariant d'Alemberian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field matter and energy. λ is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of the coupling to $|\lambda| \leq 0.1$. In the limit, $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect, and $G = 1/\phi$.

The energy-momentum tensor T_{ij} for a perfect fluid distribution is given by

$$T_{ij} = (p + \rho) v_i v_j + p g_{ij}\,, \tag{4}$$

together with the relation $g_{ij} v^i v^j = -1$ and perfect fluid obeys the general equation of state

$$p = m\rho\,, \tag{5}$$

where m $(0 \leq m \leq 1)$ is a constant, p is the pressure in the fluid, ρ is the energy density of the fluid and v^i is the four-velocity vector defined by $v_i = \delta^i_i$, $i = 1, 2, 3, 4$. We assume the coordinate frame to be co-moving so that $v^i = (0, 0, 0, -1)$ in the co-moving coordinate system. We have from (4) and (5)

$$T^1_1 = T^2_2 = T^3_3 = -p\,, \qquad T^4_4 = \rho\,, \qquad T = \rho - 3p\,. \tag{6}$$

The Barber's field equations (2) and (3) for the metric (1) with (6) are

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi p}{\phi}\,, \tag{7}$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -\frac{8\pi p}{\phi}\,, \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} = \frac{8\pi\rho}{\phi}\,, \tag{9}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{8\pi\lambda}{3}(\rho - 3p)\,. \tag{10}$$

The energy conservation equation of general relativity

$$T^{ij}_{;j} = 0$$

takes the form

$$\dot{\rho} + (\rho + p)\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \tag{11}$$

for the space time (1).

Now we use a correspondence to general relativity and define equivalent density and pressure [2]

$$\rho_{\mathrm{eq}} = \frac{\rho}{\phi}\,, \qquad p_{\mathrm{eq}} = \frac{p}{\phi}\,. \tag{12}$$

Using (12) in the energy conservation equation (11), we find that

$$\dot{\left(\frac{\rho}{\phi}\right)} + \left(\frac{\rho}{\phi} + \frac{p}{\phi}\right)\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0\,. \tag{13}$$

3. Solution of the field equations

The field Eqs. (7) – (10) supply only four independent equations in five unknowns, A, B, p, ρ and ϕ. One extra equation is needed to solve the system completely. We assume that the shear is proportional to the expansion scalar, which leads to

$$B = A^n\,. \tag{14}$$

From Eqs. (7) and (8), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} = 0\,. \tag{15}$$

From Eqs. (14) and (15), we obtain

$$\frac{\dot{A}}{A} = \frac{1}{(n+2)t + k_1}\,, \tag{16}$$

where k_1 is a constant of integration Integrating (16), we get

$$A = k_2\big[(n+2)t + k_1\big]^{1/(n+2)}, \tag{17}$$

and therefore

$$B = k_3\big[(n+2)t + k_1\big]^{n/(n+2)}, \tag{18}$$

where $k_3 = k_2^n$ and k_2 is a constant of integration.

Hence the geometry of the universe in Barber's second self-creation theory for the metric (1) is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + k_2^2\big[(n+2)t + k_1\big]^{2/(n+2)}(\mathrm{d}x^2+\mathrm{d}y^2) + k_3^2\big[(n+2)t+k_1\big]^{2n/(n+2)}\mathrm{d}z^2, \tag{19}$$

The model (19) represents an isotropic homogeneous plane-symmetric cosmological model in the presence of a perfect fluid within the framework of Barber's second self-creation theory.

4. *Discussion*

If we assume that all of the perfect fluid relevant to cosmology is represented by the notation p_{eq} and ρ_{eq} to denote the pressure and density of the fluid, respectively, then the general equation of state (5) becomes

$$p_{\mathrm{eq}} = m\rho_{\mathrm{eq}}, \tag{20}$$

where m $(0 \le m \le 1))$ is a constant independent of time. Equation (13) with Eq. (20) can be integrated to obtain

$$\rho = \rho_0 \phi R^{-3(1+m)}. \tag{21}$$

For a complete discussion, we describe the models for radiation-dominated universe model, stiff fluid model and matter dominated solution in Barber's second self-creation theory.

4.1. *Radiation dominated universe model* $(m = 1/3)$

In this model, the equation of state is $p = \rho/3$, which represents the matter distribution with disordered radiation and a universe in which most of the energy density is in the form of radiation. Hence it is called the radiation dominated universe. From (21), the energy density of radiation is

$$\rho = \rho_0 \phi R^{-4}. \tag{22}$$

Using $p = \rho/3$ and above equation in (10), the solution for the scalar field $\phi(t)$ in Barber's second self -creation theory is given by

$$\phi(t) = \log \big[(n+2)t + k_1\big]^{k_5/(n+2)} \phi_0 \,, \tag{23}$$

where ϕ_0 is a constant of integration. The pressure p, energy density ρ, Hubble parameter H, expansion scalar θ and the shear σ are given by

$$\rho = 3p = \rho_0 \log \big[(n+2)t + k_1\big]^{k_5/(n+2)} \phi_0 \; k_2^{-4(n+2)} \big[(n+2)t + k_1\big]^{-4/3}. \tag{24}$$

$$H = \frac{(n+2)k_2}{3[(n+2)t + k_1]} \,, \tag{25}$$

$$\theta == \frac{(n+2)k_2}{(n+2)t + k_1} \,, \tag{26}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 = \frac{2}{3}\frac{(1-n)^2}{\big[(n+2)t + k_1\big]^2} \,. \tag{27}$$

In order to satisfy the reality of energy density and pressure, we require the constants k_1, k_2 to be positive. In this case the universe starts with an infinite rate of expansion from $t = t_0$ where $t_0 = -k_1/(n+2)$. The energy density, scalar field and pressure are infinite at the initial singularity. The space-time exhibits a 'point type' singularity at $t = t_0$. As t increases, spatial volume V increases, but the rate of expansion slows down. All physical parameters decrease with time increases. Spatial volume V becomes infinitely large as $t \to \infty$. The energy density, pressure and directional Hubble factors tend to zero as $t \to \infty$. The shear σ is infinite at $t = t_0$ and tends to zero as $t \to \infty$. Since the model approaches isotropy for large values of t, the model represents non-rotating, shearing and expanding universe with a big-bang start.

4.2. *Stiff fluid model* $(m = 1)$

In this case the equation of state is $p = \rho$, which is called Zel'dovich fluid distribution. This equation of state is widely used in general relativity to obtain stellar and cosmological models for utterly dense matter (Zel' dovich 1962). From (21), the energy density in stiff fluid is

$$\rho = \rho_0 \phi R^{-6} \,. \tag{28}$$

We obtain and exact solution for the scalar field $\phi(t)$ as

$$\phi(t) = \frac{k_6}{\big[(n+2)t + k_1\big]} + k_7 \,, \tag{29}$$

where k_6 and k_7 are constants of integration.

The pressure, energy density, expansion scalar, Hubble parameter and shear scalar of the fluid in Barber's second self-creation theory are given by

$$p = \rho = \rho_0 \left[\frac{k_6}{\big[(n+2)t + k_1\big]} + k_7 \right] \frac{k_2^2}{\big[(n+2)t + k_1\big]^2} , \tag{30}$$

$$\theta = \frac{(n+2)k_2}{(n+2)t + k_1} , \tag{31}$$

$$H = \frac{n+2}{3} \frac{k_2}{(n+2)t + k_1} , \tag{32}$$

$$\sigma^2 = \frac{2}{3} \frac{(1-n)^2}{\big[(n+2)t + k_1\big]^2} . \tag{33}$$

Clearly, the spatial volume is zero and expansion scalar is infinite at initial singularity $t = t_0$, where $t_0 = -k_1/(n+2)$. The universe starts expanding with zero volume and at infinite rate of expansion. At $t = t_0$ the space time exhibits 'point type' singularity. The scalar field, anisotropy parameter and shear scalar also tend to infinity at initial singularity. As t increases, spatial volume increases, but the rate of expansion slows down. The scalar field and anisotropic parameter decrease as time increases. Expansion scalar θ, shear σ vanish asymptotically.

4.3. Vacuum solution ($\rho = p = 0$)

In this case the scalar field ϕ, Hubble's parameter H and other physical quantities such as spatial volume, expansion scalar and shear scalar are given by the following expressions,

$$\phi(t) = \frac{k_3 \big[(n+2)t + k_1\big]^2}{2} + k_8 ,$$

$$\theta = \frac{(n+2)k_2}{(n+2)t + k_1} ,$$

$$H = \frac{n+2}{2} \frac{k_2}{(n+2)t + k_1} ,$$

$$\sigma^2 = \frac{2}{3} \frac{(1-n)^2}{\big[(n+2)t + k_1\big]^2} .$$

Clearly, the spatial volume is zero and expansion scalar is infinite at initial singularity $t = t_0$, where $t_0 = -k_1/(n+2)$. The universe starts expanding with zero volume and with an infinite rate of expansion. At $t = t_0$ the space time exhibits a 'point type' singularity. Scalar field, anisotrpy parameter and shear scalar also tend to infinity at initial singularity. Expansion scalar θ and shear σ vanish asymptotically.

5. *Conclusion*

In this paper we have obtained exact solutions of the field equations for plane-symmetric cosmological model in Barber's second self creation theory of gravitation. We discuss both the solutions for the stiff fluid, radiation and vacuum cases. We discuss geometrical and kinematical properties of different parameters in detail for each phase. The nature of singularities of the models has been clarified and explicit forms of scale factors have been obtained in each case. In each case, the spatial volume V grows linearly with cosmic time. It has been observed that the models represent shearing, non-rotating and expanding universe with a big-bang start. Since the limit $\lim_{t\to\infty} = \sigma/\theta$ is constant, the model does not approach isotropy for large values of t and the deceleration parameter is 2. It is found that if $\lambda \to 0$, the Barbers self-creation theory tends to general theory of relativity in all respects.

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RAVNI SIMETRIČNI KOZMOLOŠKI MODELI U SAMOTVORBENOJ KOZMOLOGIJI

Istražujemo ravne simetrične kozmološke modele u okviru druge Barberove samotvorbene teorije gravitacije. Rješavamo Einsteinove jednadžbe polja primjenom gama zakona za jednadžbu stanja $p = m\rho$, gdje je $0 \leq m \leq 1$, uz prisustvo perfektne tekućine. Raspravljaju se ishodi teorije za modele poremećenog zračenja, Zel'dovichevu tekućinu i vakuum.