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A LINEAR RELATIONSHIP IN STELLAR OPTICAL SPECTRA USING THE NEW INTENSITY FORMULA

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A linear relationship in stellar optical spectra has been found by using the spectroscopical method used with optical light sources. The method is based on a new intensity formula in optical emission spectroscopy (OES).

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1. Introduction

The author and his collegue Dr. Sten Yngström presented a new formula for the intensity of spectral lines in optical emission spectroscopy (OES) in several previous papers and conferences. According to a new theory in Ref. [1], the intensity $I(h\nu)$ is given by

$$I(h\nu) = \frac{C(h\nu)\,\exp(-J/kT)}{\exp(h\nu/kT) - 1}\,, \tag{1}$$

where ν is the frequency of the atomic spectral line, J the ionization energy of the atom and $C(h\nu)$ a product of factors of sample properties (number densities of atoms and electrons) and the transition probability of the atom.

In our earlier papers (Ref. [2] and Ref. [3]), we studied absolute intensities. The intensities were obtained by arc measurements and are tabulated in Ref. [4], which we have used in our studies. In these studies, the new intensity formula was used in the development of this method of analysis. In this method, $\ln(I\lambda^2)$ was plotted versus

$$h\nu\left(1 + \frac{\theta}{h\nu}\ln\left[1 - \exp\left(-\frac{h\nu}{\theta}\right)\right]\right)\ \text{eV}$$

for 17 elements. Each intensity value is the mean value of results of many measurements. By calculating the maxima of the differences between $\ln(I\lambda^2)$ and $\ln(\lambda^2)$, the following formula has been used as the basic equation in this method of analysis,

$$\ln(I_{\max}\lambda^2_{\max}) = const. - \frac{1.6J}{h\nu_{\max}} \tag{2}$$

The graph can be seen in Fig. 1 which shows $\ln(I_{\max}\lambda^2_{\max})$ plotted versus $1.6J/(h\nu_{\max}) = J/\theta$ for 17 elements, where $\theta = kT_e$ (electron temperature in energy units), k is the Boltzmann constant, J denotes table value of ionization energy and $h\nu_{\max} = 1.6\,\theta$. This graph shows a good linear relationship. Therefore, this graph is a strong support of the new intensity formula based on the new theory. It is also possible to measure the internal electron temperature for different elements. We have shown that it is possible to obtain a similar linear relationships when using intensity data of stellar optical spectra.

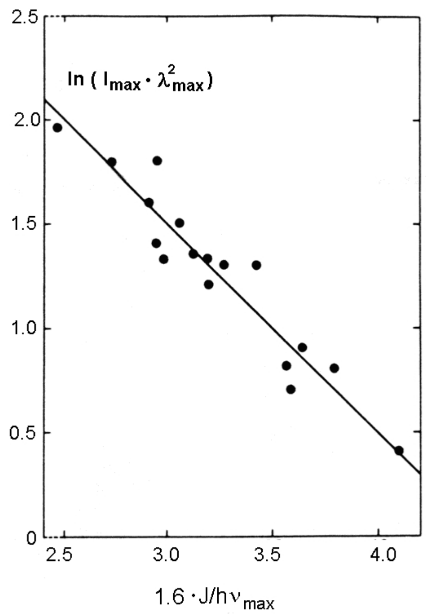

Fig. 1. $\ln(I_{\max}\lambda^2_{\max})$ *plotted versus* $(1.6\,J)/h\nu_{\max}$ *for seventeen elements from the NBS tables of Ref. (4).*

A strong support of this new intensity formula has recently been published in a summary paper Ref. [5] where many different methods from the literature have been used which support the new formula.

2. *Stellar spectra*

The stellar optical spectra extend over the spectral classes O – M, based on the photometrically well-calibrated luminosity measurements from star to star (data from Ref. [6] were used). Good temperature and luminosity coverage have been achieved. The data were digitalized from the main sequence classed O5 – F0 and F6 – K5 and are displayed in term of relative flux as a function of wavelength. The parameters that have been measured in this investigation are maximum luminosities $L_{\max}$ (relative flux maximum) of the Planck curve. In this maximum, the wavelength $\lambda_{\max}$ and the maximum frequency $\nu_{\max}$ were also measured.

TABLE 1. Electron temperature, θ, and $J_{\text{meanvalue}}$ values for stars of different spectral classes.

Spectral class	θ (eV)	$J_{\text{meanvalue}}$ (eV)
K5V	1.44	15.5
K4V	1.47	15.6
G9-K0	1.50	15.8
G6-G8	1.53	16.0
G1-G2	1.56	16.2
F8-F9V	1.63	16.7
F6-F7V	1.63	16.5
A9-F0V	1.72	16.9
A8	1.75	17.1
A5-A7	1.81	17.5
A1-A3	1.84	17.6
B6V	1.88	17.8
B3-B4V	1.94	18.0
O7-B0V	1.97	18.1
O5V	2.00	18.2

The $L_{\max}\lambda_{\max}^2$ values are plotted versus $(1.6J_{\text{meanvalue}}/\nu_{\max})$ where $J_{\text{meanvalue}}$ is the mean value of the ionization energies of elements of the stars. To obtain a linear relationship for the stellar data as in Fig. 1 from the spectroscopical method from Refs. [2] and [3], the data from Table 1 were used and plotted according to the equation

$$\ln(L_{\max}\lambda_{\max}^2) = const. - \frac{1.6J_{\text{meanvalue}}}{h\nu_{\max}} \tag{3}$$

which is similar to Eq. (2) for atomic spectra. To obtain the values of Table 1 it is necessary to use a two step procedure. In the first step it is necessary to define the graph by calculating the $J_{\text{meanvalue}}$ of the G2-star. The $J_{\text{meanvalue}}$ can be expressed in the following way,

$$J_{\text{meanvalue}} = \sum c_n J_n \tag{4}$$

where c_n is the normalized content of element n of a star. The c_n-values of the Sun have been used here. J_n is here the ionization energy of an element of a star. The $J_{\text{meanvalue}}$ has been calculated for the Sun (G2 star), which gave $J_{\text{meanvalue}} = 16.2$ eV according to the linear graph in Fig. 2. This value is 16.2 eV, too, for the Sun when using Eq. (4) together with established chemical composition values of the Sun. This means that we now have one point determined in Fig. 2.

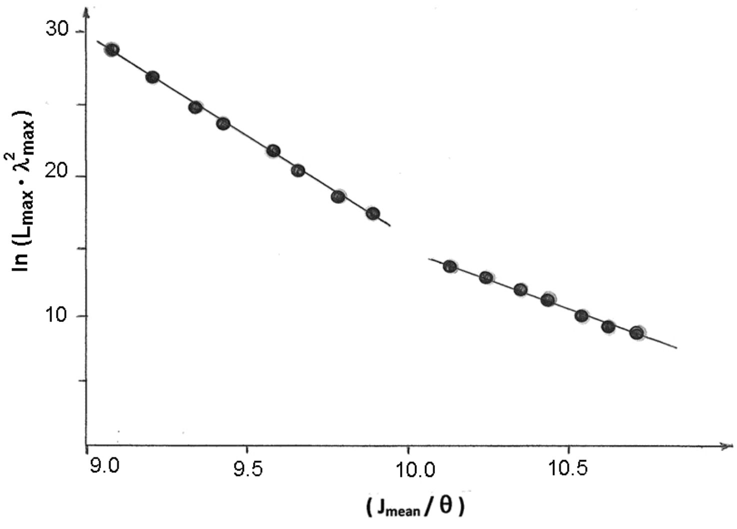

Fig. 2. $\ln(L_{\max}\lambda_{\max}^2)$ *values plotted versus* $(1.6\,J_{\text{meanvalue}})/h\nu_{\max}$ *for different stars of spectral classes O - M.* T_e *increases to the left and* λ *to the right.*

The second step is to add in Fig. 2 the results for stars from other classes. In these stars, we do not know the chemical composition very well. Therefore, we cannot use Eq. (4), and we must use the slope of Fig. 2 instead, for the determination of the $J_{\text{meanvalue}}$. Therefore, it was necessary to determine the $J_{\text{meanvalue}}$ graphically for other stars. For the atoms, on the other hand, usual J-tables were used. The slope of this graph is determined by the individual differences in $\ln(L_{\max}\lambda_{\max}^2)$ and wavelength differences between the individual classes. Therefore, it is possible to determine the individual $J_{\text{meanvalue}}$ from the slope in order to obtain a linear graph of Eq. (3). This equation constitutes a straight line. These $J_{\text{meanvalue}}$-values and θ-values are tabulated in Table 1. The θ-values (electron temperatures) are calculated using the expression $\theta = h\nu_{\max}/1.6$ mentioned above.

The data in Fig. 2 constitute a straight line in the classes O5 – F0 and F6 – K5.

In Eq. 3, $h\nu_{\max} = 1.6\theta$, where θ = internal electron temperature in eV. This means that the classes O5 – F0 have higher temperature than the classes F6 – K5, which is also in accordance with the usual Hertzsprung-Russell (HR) diagram. The linear relationship between electron temperature θ versus $J_{\mathrm{meanvalue}}$ can be seen in Fig. 3. θ was determined from the data on light emanating from the optical layers of the star (photosphere) and constitutes a mean value in those layers. For example, a G2 star (the Sun) has $\theta = 1.56$ eV ($T_e = 18110$ K).

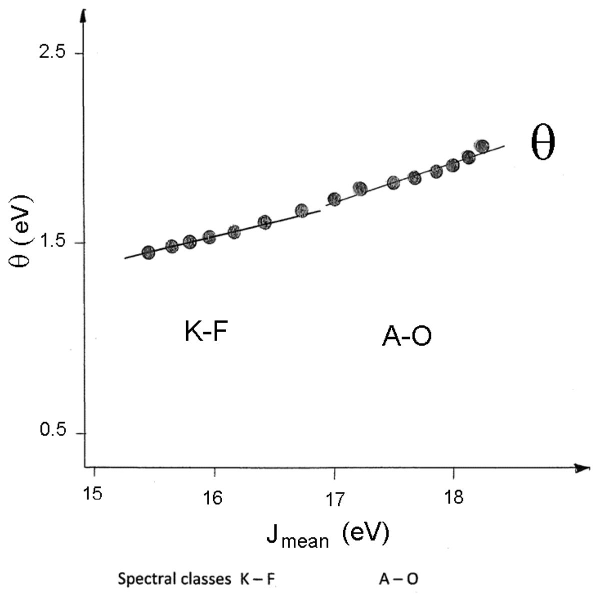

Fig. 3. Electron temperature values, θ, plotted versus $J_{\mathrm{meanvalue}}$ for different stars of spectral classes O - M.

A comparison of the graphs in Figs. 1 and 2 for atoms and stars shows the following differences: In Fig. 1 for atoms, θ is increasing to the right in the graph. The ionization energy J is here varying more than θ for different elements. On the other hand, in Fig. 2 for stars, θ is increasing to the left in the graph. θ is varying here more than $J_{\mathrm{meanvalue}}$ among the different stars.

The reason why the slope of the curve of the O5 – F0 stars of Fig.. 2 is larger than the slope of the F6 – K5 stars is the difference in density. The density of the younger O5 – F0 stars is higher in comparison with the F6 – K5 stars, which gives a higher intersecting value between the extrapolated line and the $\ln(I_{\max}\lambda^2_{\max})$-axis, in comparison with the F6 – K5 stars. This is in accordance with Eqs. (1), (2) and (3) where the C-factor of Eq. (1) and the constants of Eqs. (2) and (3) are proportional to number densities of atoms and electrons. The electron temperature results of Table 1 are also in accordance with the earlier measurements of electron temperature of planetary nebulae. The central stars and nebulae in Ref. [7] were studied by using photoionization methods.

The present method of analysis has shown to be a simple method of determining the mean electron temperature of the optical layers of a star without knowing so much about the chemical composition of the star. The method also gives an organizing method for stars similar to the established HR-diagram. The $J_{\mathrm{meanvalue}}$ has shown to be a kind of "signum" for every star. Figures 2 and 3 are a valuable and simple method of organizing and classifying the stars without knowing many other details about the stars. It is, however, fascinating for a humble scientist to detect a method common to both atoms and stars.

3. *Summary*

In this paper a linear relationship in stellar optical spectra has been found by using a spectroscopical method used with optical light sources. This method is based on a new intensity formula in optical emission spectroscopy (OES). Like the HR-diagram, it seems to be possible to organize the luminosity of stars from different spectral classes. It is also possible to determine the mean electron temperature of the optical layers (photospheres) of the stars. The mean value of the ionization energies of different elements of the stars has been shown to be very significant for each star.

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LINEARNA OVISNOST OPTIČKIH SPEKTARA ZVIJEZDA S NOVIM IZRAZOM ZA INTENZITET

Našli smo linearnu ovisnost u optičkim spektrima zvijezda poput one koja se rabi u optičkim spektrima atomskih izvora svjetlosti. Metoda se zasniva na novoj formuli za intenzitete u optičkoj emisijskoj spektroskopiji.