THEORETICAL DESCRIPTION OF THE HYPERON NONLEPTONIC DECAYS

DUBRAVKO TADIĆ

Department of theoretical physics, Faculty of Science, University of Zagreb, 41000 Zagreb, Croatia

Received 20 October 1991

UDC 539.12

Original scientific paper

The theoretical description of the hyperon nonleptonic decays has been reexamined by using recently calculated next-to-leading QCD corrections.

1. Introduction

Aim of this article is the critical investigation of our understanding, or lack of it, of hyperon nonleptonic decays. An attempt will be made to pin point the basic theoretical ingredients, in, quoting from a textbook[1]:

»... analysis of the $J_p = 1/2^+$ hyperon decays carried out on this basis [meaning Shifman et al. Hamiltonian[2]] yields results in good agreement with experiment (Tadić and Trampetić 81).«

Above quoted reference[3] was only a finishing step in a walk which has started earlier[4].

Furthermore, once the basic quark structure of the weak Hamiltonian and of the hadrons was understood, such type of approach did suggest itself to any competent researcher in the field[5]. Another texbook[6] gives all the credict to Ref. 5 (whose methods parallel the ones used in Refs. 3 and 4.)

This partial success (See Table 1 below, which illustrates agreement with experimental data) was certainly helped by the octet enhancement[7] contained in Shifman et al. Hamiltonian[2] (See formula (21) below).

However, this took into account only »hard-gluon« exchanges (with energies $\geq$ 1 GeV). It is plausible that »soft-gluon« effects, which might be of central importance, were somehow accounted for by LSZ reduction formalism and by the symmetry (or better anti-symmetry) properties[8] of the hyperon wave functions. The LSZ reduction formalism is associated with often used, but never really theoretically proved, »current-algebra« (See Marshak et al. textbook[9]). Suitable continuation from current-algebra zero-pion-momentum ($q = 0$) limit, was also involved. It is somewhat disturbing that the same combination seems to work poorly in the case of meson decays $K \to 2\pi$[10].

The enhanced influence of the penguin terms[1,2], as discussed below, might improve the theoretical description of the meson decays. Unfortunately the same enhancement (See Table 1 below) seems to unbalance the theoretical description of the hyperon nonleptonic decays. All such statements must always be made with some caution. The theoretical scheme, which will be discussed here, contains numerous terms and contributions, which do combine in a rather complicated way.

Direct, lattice-gauge based calculations of $K \to 2\pi$ decays, seem to stress the importance of the soft-gluon corrections (So called eye-diagrams) which are not properly included in the outlined scheme. Such might be also the case with hyperon nonleptonic decays, where one has gone as far as possible by using approximate methods[2,8,9] to handle QCD corrections further progress will most likely depend on the development of methods for more exact handling of QCD influences.

Present qualitative successes, if they can be called that, followed from the discovery of the basis QCD ingredients: quarks and gluons. Such structural knowledge led to the improvement over older attempts which contained such general dynamical ideas as pole-terms, SU (3)-flavour properties and or current algebra formalism[9]. Here the calculation of QCD effects is still based[3-5] on those old approximative descriptions of strong dynamics.

2. *Brief outline of the theoretical methods*

Only main features and results, which are needed for the discussion of the present understanding of hyperon nonleptonic decays, are described here. For full details the reader is refered to the original papers[2,4,5,11-15].

The effective weak Hamiltonian, valid for the energies small in comparison with the intermediate vector boson mass, is

$$H_W = \sqrt{2} G_F \sin \Theta_C \cos \Theta_C \sum_{i=1}^{6} C_i O_i$$

$$O_1 = :(\bar{d}_L s_L)(\bar{u}_L u_L) - (\bar{d}_L u_L)(\bar{u}_L s_L): (20'', 8, \Delta I = 1/2).$$

$$O_2 = :(\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) + 2(\bar{d}_L s_L)(\bar{u}_L u_L) +$$

$$+ 2(\bar{d}_L s_L)(\bar{s}_L s_L): (84, 8, \Delta I = 1/2). \qquad (2.1)$$

$$O_3 = :(\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) + 2(\bar{d}_L s_L)(d_L \bar{d}_L) -$$

$$- 3(\bar{d}_L s_L)(\bar{s}_L s_L): (84, 27, \Delta I = 1/2).$$

$$O_4 = :(\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) -$$

$$- (\bar{d}_L s_L)(\bar{d}_L d_L): (84, 27, \Delta I = 3/2).$$

$$O_5 = :(\bar{d}_L \lambda_a s_L)(\bar{q}^i_R \lambda^a q^i_R): (15, 8, \Delta I = 1/2).$$

$$O_6 = :(\bar{d}_L s_L)(\bar{q}^i_R q^i_R): (15, 8, \Delta I = 1/2).$$

Here SU (4) and SU (3) flavour contents are indicated.

The QCD corrected values[2)] of the C_i coefficients reflect the short range corrections. They are for example[3)]:

$$C_1 = -2.538 \qquad C_2 = 0.080 \qquad C_3 = 0.082$$
$$C_4 = 0.411 \qquad C_5 = -0.080 \qquad C_6 = -0.021. \tag{2.2}$$

They should be compared with the »bare« values, obtained from the basic electroweak Hamiltonian

$$C_1 = -1 \qquad C_2 = 0.2 \qquad C_3 = 0.13\bar{3}$$
$$C_4 = 0.66\bar{6} \qquad C_5 = 0 \qquad C_6 = 0. \tag{2.3}$$

The values (2.2) were obtained[2,7)] in the one loop approximation, which already lead to the discovery of the important »penguin« operators O_5 and O_6. Recent improved calculation[16)], based on two-loop expansion, and including next-to-leading QCD corrections, gave even larger enhancement of the $\Delta I = 1/2$ (SU (3) octet) O_1 operator. A convenient measure for thes enhancement is the ratio $\zeta = |C_1/C_4|$ which is for (2.2) values

$$\zeta = 6.18.$$

According to Ref. 16 the next-to-leading QCD corrections can increase this value by about 40%, i. e. to about

$$\zeta \approx 8.65.$$

The influence of the »penguins« O_5 and O_6 is also increased: A fourfold enhancement ($P = 4$) of the penguins is almost enough to explain $K \to 2\pi$ decays, using the same theoretical schemes. One calculation[1] has obtained satisfactory agreement with the experiment for $P = 5.92$.

However, as it will be discussed below, it might spoil the description of the hyperon nonleptonic decays.

In the calculation of the hyperon-nonleptonic-decay amplitudes one includes various contributions. Parity-violating decay amplitudes A receive contributions A^C from current-algebra commutator terms, as for example

$$A^C \begin{pmatrix} \Xi^-_- \\ \Sigma^+_0 \end{pmatrix} = \frac{-1}{f_\pi} \bar{G}_F \begin{pmatrix} a_{x^0 \Lambda} \\ \frac{1}{\sqrt{2}} a_{\Sigma^+ P} \end{pmatrix}, \qquad \bar{G}_F = \frac{G_F}{2\sqrt{2}} \cos \Theta_C \sin \Theta_C,$$

$$a_{x^0 \Lambda} = \left(\frac{2}{3}\right)^{1/2} \left\{-12 C_1 (a + b) + \left(C_6 - \frac{8}{3} C_5\right) \left[3 (a - a') - (9b - b')\right]\right\}.$$

$$a_{\Sigma^+ P} = \frac{2}{3} \left[-18 C_1 (a + b) + \left(C_6 - \frac{8}{3} C_5\right) (3a - 13b)\right]. \tag{2.4}$$

Here a, b, a' and b' are integrals over quark wave functions which are defined in Ref. 3. It is important to note that only the operators O_1, O_5 and O_6 contribute. This is due to the fact that the baryon (i. e. hyperon) states are antisymmetric in quark operators[18]. Among all left-left operators, which can be transformed in itself by Fierz-transformation, only the operator O_1 has the right symmetry-properties. The vanishing of the matrix elements

$$\langle B_f | Oi | Bi \rangle = a_{BiBf} \qquad i = 2, 3, 4 \tag{2.5}$$

leads to the $\Delta I = 1/2$ selection rule, which in the old times[9] was considered as an empirical property of the hyperon-nonleptonic-decay amplitudes. It turns out that this is somewhat more specific. Only the SU (4) 20″ representation contributes, while the other $\Delta I = 1/2$ pieces (O_2, O_3) do not.

Such ideal $\Delta I = 1/2$ selection rule is to some extent broken by the separable terms in which two by two quark fields are sandwiched between baryon states, as for example

$$A^s \begin{pmatrix} \Xi^-_- \\ \Sigma^+_0 \end{pmatrix} = \left(\frac{2}{3}\right)^{1/2} \bar{G}_F f_\pi \begin{pmatrix} m_{\Xi_-} - m_\Lambda)\, a^s_{\Xi^- \Lambda} \\ \frac{-1}{\sqrt{3}} (m_\Sigma - m_p)\, a^s_{\Sigma^+ p} \end{pmatrix},$$

$$a^s_{\Xi^- \Lambda} = \left[C_1 - 2 (C_2 + C_3 + C_4) - \left(C_6 + \frac{16}{3} C_5\right) \frac{m_\pi^2}{(m_s - m_u)(m_d + m_u)}\right],$$

$$a^s_{\Sigma^+ p} = \left[-C_1 + 2 (C_2 + C_3 - 2 C_4) + \left(C_6 + \frac{16}{3} C_5\right) \frac{m_\pi^2}{(m_s - m_d)(2 m_d)}\right], \tag{2.6}$$

$$B^s \begin{pmatrix} \Xi_-^- \\ \Sigma_0^+ \end{pmatrix} = \frac{\sqrt{2}}{3} \bar{G}_F f_\pi \begin{pmatrix} \frac{1}{\sqrt{3}} (\bar{D} - 3\bar{F}) (m_\Xi + m_\Lambda) b^s_{\Xi-\Lambda} \\ (\bar{D} - \bar{F}) (m_\Sigma + m_p) b^s_{\Sigma+p} \end{pmatrix}$$

$$b^s_{\Xi-\Lambda} = \left[-C_1 + 2(C_2 + C_3 + C_4) - \left(C_6 + \frac{16}{3} C_5 \right) \frac{m_\pi^2}{(m_s + m_u)(m_d + m_u)} \right],$$

$$b^s_{\Sigma+p} = \left[C_1 - 2(C_2 + C_3 - 2C_4) + \left(C_6 + \frac{16}{3} C_5 \right) \frac{m_\pi^2}{(m_s + m_d)(2m_d)} \right].$$

Current quark masses, used by Ref. 3, do break the SU (2) (isospin) symmetry

$$m_u = 4.2 \text{ MeV}; \qquad m_d = 7.5 \text{ MeV}; \qquad m_s = 150 \text{ MeV}. \tag{2.7}$$

It should be mentioned that this separable terms can be replaced by vector meson pole terms (for A) and by K — meson pole terms (for B)[3,11] leading to somewhat different estimate of essentially the same quantity.

The main contributions to the B amplitudes comes from baryon pole terms, which depend on the same matrix elements (2.5) as the A contributions. For example

$$B^p (\Xi_-^-) = -g\bar{G}_F (m_x + m_\Lambda) \left[\frac{a_{\Xi^0\Lambda}}{\sqrt{2} m_x} \frac{f - d}{(m_x - m_\Lambda)} + \frac{a_{\Xi^-\Sigma^+}}{\sqrt{3}} \frac{2d}{(m_\Xi - m_\Sigma)(m_\Sigma + m_\Lambda)} \right],$$

$$B^p (\Sigma_0^+) = +g\bar{G}_F \frac{m_\Sigma + m_p}{m_\Sigma - m_p} a_{\Sigma^+p} \left(\frac{1}{2m_p} - \frac{f}{m_\Sigma} \right). \tag{2.8}$$

The calculations based on various quark models[3,18] reproduced always correctly the relative signs of the hyperon-decay amplitudes. This was also the case with the relative magnitudes of the various amplitudes, as shown in Table 1.

TABLE 1.

Amplitude	MIT-bag[3] model	HO quark[3] model	Chiral[18]-bag model	Higher--order[3,16] QCD	Expt.[18]
$10^6 A (\Lambda_-^0)$	0.21	0.21	0.25	0.49	0.32
$10^6 A (\Xi_-^-)$	−0.43	−0.42	−0.46	−0.78	−0.44
$10^6 A (\Sigma_0^+)$	−0.38	−0.38	−0.42	−0.66	−0.32
$10^6 A (\Sigma_-^-)$	0.49	0.49	0.43	0.84	0.42
$10^6 A (\Sigma_+^+)$	0	0	$3 \cdot 10^{-3}$	0	$10 \cdot 10^{-3}$
$10^6 B (\Lambda_-^0)$	1.81	1.81		3.68	2.16
$10^6 B (\Xi_-^-)$	0.94	1.18		0.062	1.45
$10^6 B (\Sigma_+^0)$	2.00	1.93		2.77	2.60
$10^6 B (\Sigma_-^-)$	−0.41	−0.29		−0.90	−0.14
$10^6 B (\Sigma_+^+)$	2.53	2.54		3.12	4.13

Hyperon nonleptonic decay amplitudes.

All technical details can be found in the appropriate references which are indicated in the table. Other formally related approaches[5,20] did lead to the equivalent results.

It is interesting to estimate the influence of the next-to-leading QCD corrections and enhanced penguins. Instead of the set (2.2) of C_i coefficients one can use

$$\begin{array}{lll} C_1 = 2.96 & C_2 = 0.071 & C_3 = 0.062 \\ C_4 = 0.342 & C_5 = -0.32 & C_6 = -0.084. \end{array} \tag{2.9}$$

The results are shown in the fifth column (labeled: Higher order QCD) in Table 1. In general the agreement is somewhat poorer (especialy for A ampltudes) than in the earlier estimates[3,4,5,18].

The calculational scheme, which is presented in this chapter, is not the most general one. There might be and there have been investigated[12–15], the contributions due to $1/2^{+*}$, $1/2^{-*}$ and $3/2^{+*}$ baryon-pole terms. Probably one should not neglect the instanton induced terms[21,22]. All these additional corrections and associated theoretical difficulties will be discussed below. It is not imposible that a combination of all contributions and corrections provides the right answer. Such an elaborate construction is not an aesthetically appealing theory.

3. *Baryon poles*

The theoretical scheme[3,4,5] which was outlined in the previous chapter can serve as a basis for semiempirical fit of the experimental data. In that case the quantities appearing in formulae (2.4), (2.6) and (2.8) such as $a_{B_iB_f}$ are suitably parametrized and used as fitting parameters[13–15]. Such scheme, the formula (2.5) in particular, includes the $1/2^+$ baryon poles. Besides those poles, the $1/2^{+*}$ resonance poles can also contribute[12]. The same holds for $1/2^{-*}$, $3/2^{+*}$ etc. resonance poles[13–15]. The existing approaches have been roughly summarized in Table 2. The contributions of such poles can be either calculated in quark models, or parametrized by using suitable fitting parameters. One also encounters some combinations of both methods, i. e. the quark models are used as an inspiration for the selection of fitting parameters.

Inclusion of pole terms becomes very interacting when the higher-order QCD corrections[16] are taken into the account. (They lead to the results shown in the fifth column in Table 1). Unfortunately the description of baryon resonances is not sufficiently developed in quark models. Models work probably the best for $1/2^{+*}$ resonances[12]. Calculation shows that $1/2^{+*}$ poles significantly improve the theoretical predictions for B amplitudes, when those predictions are based on the one-loop estimate of the hard gluon QCD corrections. With next-to-leading QCD corrections[16] one will obviously experience the same problems (See Table 1) as with $1/2^+$ poles only.

In the case of $1/2^{-*}$ resonances, the quark models did not lead[12] to any very definitive predictions as the results were to dependent on the type of the model

TABLE 2.

Theoretical decay*) amplitude	Remarks	References
$A = A^C(a_{BiBf}) + A^S + A^P(1/2^{-*})$ $B = B^P(a_{BiBf}) + B^S + B^P(1/2^{+*})$	$A^P(1/2^{-*})$ uncertain $A(\Sigma^+_+)$ is not explained Ω^- decay works	3, 4, 5, 12, 17
$A = A^C(a_{BiBf}) + A^P(1/2^{-*})$ $B = B^P(a_{Bi,Bf})$ $a_{BiBf} \sim$ fitting parameters	$A(\Sigma^+_+)$ is not explained	14, 15
$A = A^C(a_{BiBf}) + A^P(3^+/2^*)$ $B = B^Z_C(a_{BiBf}) + B^P(3^+/2^*)$ five fitted parameters	$A(\Sigma^+_+)$ is not explained Ω^- decay works $\Sigma^+ \to p\gamma$ is explained	13, 15

*) QCD enhancement coefficients (2.2) or (2.3).

Schematical comparison among theoretical alternatives.

used. However they could not satisfy some expected sum-rules. It is possible that $1/2^{-*}$ poles might resolve some problems illustrated in fifth column in Table 1. New QCD corrections seem to lead to too-large A amplitudes. Such too large A amplitudes, by containing large a_{BiBf} matrix elements, give larger (as needed) B amplitudes. (For example $B(\Lambda^-_0)$ is too large, $B(\Sigma^+_0)$ almost exact, and $B(\Sigma^+_+)$ is improved). $1/2^{-*}$ poles, which contribute to A amplitudes only, can destructively interfere with A^C and A^S terms, leading to smaller values. Such semiempirical schemes have been already put forward[13,15,23]. With $3/2^{+*}$ pole terms even larger successes were claimed. Ref. 13 could explain major features of all hyperon decay amplitudes, all $B(\Omega)$ decay amplitudes and all $\Sigma^+ \to p\gamma$ decay amplitudes in terms of five fitted parameters. With assortment of baryon (including resonances) and meson pole terms[15] even better fitting successes might be possible.

Some additional theoretical possibilities will be discussed in the following chapter.

4. *Outlook*

Some additional theoretical investigations can be performed even if staying within the general theoretical framework[3-4] which contains quark-models, simple, actually somewhat ancient[9], dynamical »ansatz«-es and hadron-gluon QCD corrections. A general, »strategic« aim must be the simultaneous explanation of both hyperon-nonleptonic and $K \to 2\pi$ decays.

First step must be inclusion of instantons[21], which have been used for $K \to 2\pi$ decays[22]. In that way one has included some additional, nonperturbative, QCD corrections.

Next should be the improvement in kinematic. The quark models which were used[3–5,10,12] give static description of a hadron. Roughly speaking hadron is represented as an immovable ball inside which quarks might have a relativistic dynamics. In such picture hadron states are not the hadron-momentum eigenstates[24] and one cannot calculate recoil effects, i. e. momentum dependent effects. Some additional contributions like separable terms (2.6) must be included. The meson momentum dependence is especially important for $K \to 2\pi$ decays[10], where it has to be guessed by a suitable continuation from the current algebra results[5,10]. It was based on the factorizable diagrams or on the chiral Lagrangian calculations.[25].

With the static hadron states one cannot calculate the weak vertices containing $3/2^{+*}$ states, which appear in the corresponding pole diagram . The matrix element of the scalar weak Hamiltonian H_W between static spin 1/2 and spin 3/2 states does vanish

$$\langle 3/2 \vec{P} = 0 \,|H_W|\, 1/2\, \vec{P} = 0 \rangle \equiv 0. \tag{4.1}$$

However, by using the momentum eigenstates one can calculate the theoretical quark-model based, and momentum dependent, value for the vertex used by Ref. 13. It would be interesting to find out wether the empirically found values, obtained as fitted parameters, could be reproduced by quark models.

The usage of momentum eigenstates is indispensable for the systematic description of $K \to 2\pi$ decays.

More difficult task is to include other Fock-states in the description of hadrons[26]. Recently the results of the experiments[27] in deep inelastic scattering induced theoretical speculations[28] about the importance of higher Fock-states containing gluons and $\overline{qq}$ pairs. Once that is completely understood, one has to generalize it to the whole baryon octet involved in the hyperon nonleptonic decays.

Even if such approximate procedures, as have been discussed here, are once superseeded by some more elaborate and fundamental appıoach, they could, if successful, retain some value as a quick way to estimate some effects, whose very precise calculation would require lengthy procedure starting from the fundamentals of QCD and of electroweak theory.

References

1) E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks,* Cambridge University Press, Cambridge (1983);

2) M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B120** (1977) 316; A. I. Vainshtein, V. I. Zakharov and M. A. Shifman, Soviet. Phys. JETP**45** (1978) 760;

3) D. Tadić and J. Trampetić, Phys. Rev. **D23** (1981) 144;

4) H. Galić, D. Tadić and J. Trampetić, Nucl. Phys. **B158** (1979) 306;

5) J. F. Donoghue, H. Golowich, W. R. Ponce and B. R. Holstein, Phys. Rev. **D21** (1980) 186;

6) L. B. Okun, *Leptoni i Kvarki,* Nauka Moskva (1990); *Leptons and Quarks,* North Holland, Amsterdam;

7) M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33** (1974) 108; G. Altarelli and L. Maiani, Phys. Lett. **52B** (1974) 351;

8) M. Miura and T. Minakawa, Progr. Theor. Phys. **38** (1967) 954; J. G. Korner, Nucl. Phys. **B25** (1970) 282; J. C. Pati and C. W. Woo, Phys. Rev. **D3** (1971) 2920;

9) R. E. Marshak, Riazuddin and C. P. Ryan, *Theory of Weak Interactions in Particle Physics*, Wiley — Interscience, New York (1969);

10) M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. **B187** (1981) 514; D. Horvat and D. Tadić, Z. Phys. **C25** (1987) 231; D. Horvat, Z. Narančić and D. Tadić, Z. Phys. **C38** (1988) 431;

11) M. Gronau, Phys. Rev. **D5** (1972) 118;

12) M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. **B207** (1982) 461; D. Palle and D. Tadić, Z. Phys. **C23** (1984) 301;

13) M. D. Scadron and M. Visinescu, Phys. Rev. **D28** (1983) 1117;

14) A. Le Younc, L. Oliver, O. Pene and J. C. Raynal, Nucl. Phys. **B149** (1979) 321;

15) M. Bonvin, Nucl. Phys. **B238** (1984) 231;

16) A. J. Buras, M. Jamin, M. E. Lautenbacher and P. H. Weisz, Max-Planck Institute preprint, MPI-PAE/PTh 56/91;

17) D. Horvat and D. Tadić, Z. Phys. **C25** (1987) 231;

18) In literature this is known as Pati-Woo theorem. The relevant references are listed in 8.

19) D. Horvat and D. Tadić, Z. Phys. **C31** (1986) 311;

20) Review of Particle Properties: Phys. Lett. **111B** (1982);

21) J. A. Oteo and S. Noguera, Orsay preprint IPNO/TH 85—7; P. Gonzales, S. Noguera, J. Bernabeu and V. Vento, Nucl. Phys. **A423** (1984) 477;

22) K. Konishi and S. Ranfone, Phys. Lett. **172B** (1986) 417;

23) D. Horvat, Z. Narančić and D. Tadić, Z. Phys. **C38** (1988) 431;

24) D. Tadić and J. Trampetić, Fizika **18** (1986) 107;

25) A. Ilakovac, D. Tadić, S. Žganec and E. Fischbach, Fizika **22** (1990) 629;

26) H. Y. Cheng, Int. Journ. Mod. Phys. **A4** (1989) 495;

27) L. C. L. Hollenberg and B. H. J. Mc Kellar, Phys. Rev. **D40** (1989) 145; Int. Journ. Mod. Phys. **A4** (1989) 1949;

28) G. Baum et al., Phys. Rev. Lett. **51** (1983) 1135; J. Ashman et al., Phys. Lett. **B206** (1988) 364; V. W. Hughes et al., Phys. Lett. **B212** (1988) 511;

29) H. Fritzsch, Max Planck Institute preprint, (1991) MPI-Ph/91-34.

TEORETSKI OPIS HIPERONSKIH NONLEPTONSKIH RASPADA

DUBRAVKO TADIĆ

Zavod za teorijsku fiziku, Prirodoslovno-matematički fakultet, Zagreb

UDK 539.12

Originalni znanstveni rad

Ponovno je istražen teoretski opis hiperonskih nonleptonskih raspada upotrebljavajući nedavno izračunate QCD korekcije višeg reda. Suprotno no što je nađeno kod $K \to 2\pi$ raspada slaganje sa eksperimentom postane nešto lošije. Potrebno je ponovno ispitati teoretski pristup.