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A CHARGED FLUID SPHERE ADMITTING SPIN GENERATED TORSION

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By considering a spherically symmetric configuration of charged matter with a spin density we discuss the binding energy of such a configuration when the spin density is tuned so as to enforce the radial component of the metric to be 1 (g_{11}) for $r < R$. Such a condition renders the problem solvable.

1. Introduction

One of the ever present problems in astrophysics and relativity theory entails finding stable solutions to the system of matter and gravity that describes a bound configuration that possibly exists in the cosmos. Configurations of degenerate neutrons[1], charged boson stars[2], mixed fermion boson stars[3] as well as Q-ball configurations[4,5,6] all represent solutions to the Einstein equations plus matter that may have far-ranging astrophysical significance. In addition to these configurations both gauge monopoles[7] and global monopoles[8] may be present in an astrophysical setting with distinct signatures of radiation emitted from them providing a probe to identify them. In fact global monopoles have the interesting property that they generate repulsive gravitational forces. In the study of most of these configurations Einstein's theory of gravity is used, and it would be interesting to ask what modifications would occur if a different theory of gravity was used. There have been numerous modifications to G. R. involving the scalar-tensor theory of Brans Dicke[9], vector metric theory[10], Moffat's theory[11] with a non-symmetric metric as well as theories admitting higher powers of the curvature[12]. One of the most innovative and physically relevant modifications to G. R. involves the realization that there may be a torsion associated with space time that produces a asymmetric path dependence of how fundamental lengths are displaced around a closed curve in space time. There are numerous torsion theories with the most popular being the Einstein Cartan theory[13] wherein the metric is related to the density of mass energy and the torsion is related to spin density[14]. Actually a gauge theory of gravity entails the tetrad e^{α}_{μ} and the spin connection $\omega^{\alpha\beta}_{\mu}$ from which both metric and torsion are derived[15]. Weinberg has made the point that the Einstein Cartan theory and the Poincare gauge theory of gravity involving e^{α}_{μ} and $\omega^{\alpha\beta}_{\mu}$ have never been proven equivalent[16]. Some physical effects generated by torsion include the stabilization of collapsed physical objects[17], the prevention of cosmological singularities[18], as well as the possible stabilization of black holes generated by a zero surface temperature of the horizon[19]. It has also been pointed out that torsion can generate a dispersion of electromagnetic waves that may have far ranging astrophysical consequences[20].

In this paper we choose to study a spherically symmetric configuration of matter admitting a spin density and charge density within the context of the Einstein Cartan theory. We have previously discussed matter with spin density alone and demonstrated that it can be essentially hidden from its surroundings by giving rise to a Minkowski metric outside the configuration[21]. In that study we gave only an approximate calculation of the binding energy. In this note we improve on this result and calculate the binding energy exactly. We also discuss the problem when charge is added to a configuration of matter admitting a spin density. It is hoped that our investigation prods the theoretical community to look again with renewed interest at the problem of torsion and the effects it has on condensed astrophysical objects.

2. *Charged sphere admitting spin and torsion*

We begin by writing the spherically symmetric metric

$$(\mathrm{d}S)^2 = \mathrm{e}^{\nu} (\mathrm{d}x^4)^2 M - \mathrm{e}^{\lambda} (\mathrm{d}r)^2 - r^2 (\mathrm{d}\Theta)^2 - ir^2 \sin^2 \Theta (\mathrm{d}\varphi)^2; \tag{2.1}$$

in the presence of torsion it can be demonstrated that for a spherically symmetric distribution of a perfect fluid the spin torsion effects can be taken into account by writing (Ref. 14)

$$P \to P - \frac{2\pi G}{C^2} S^2, \qquad \varepsilon \to \varepsilon - \frac{2\pi G}{C^2} S^2 \tag{2.2}$$

P = pressure, ε = energy density, S = spin density, C = speed of light, G = gravitational constant.

For the modified Einstein equations of a perfect fluid admitting a spin density we have for the interior of an uncharged sphere

$$\frac{\mathrm{d}}{\mathrm{d}r} (r\mathrm{e}^{-\lambda}) = 1 - \frac{8\pi G}{C^4} r^2 \left(\varepsilon - \frac{2\pi G}{C^2} S^2 \right) \tag{2.3}$$

$$\mathrm{e}^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}=-\frac{8\pi G}{C^4}\left(-P+\frac{2\pi G}{C^2}S^2\right) \tag{2.4}$$

$$\mathrm{e}^{-\lambda}\left(\frac{\lambda''}{2}-\frac{\lambda'\nu'}{4}+\frac{(\nu')^2}{4}+\frac{(\nu'-\lambda')}{2r}\right)=-\frac{8\pi G}{C^4}\left(-P+\frac{2\pi G}{C^2}S^2\right). \tag{2.5}$$

To begin our analysis we first choose $\varepsilon - \varepsilon_0$ (constant) in Eq. (2.3) and set

$$\varepsilon_0-\frac{2\pi G}{C^2}S^2=0,$$

this gives for the interior of the sphere $r \leq R$

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\mathrm{e}^{-\lambda})=1$$

or $\mathrm{e}^{-\lambda}=1$ for $r \leq R$. (2.6)

From Eq. (2.4) and Eq. (2.5) we have

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(P-\frac{2\pi G}{C^2}S^2\right)+\left(\frac{P+\varepsilon_0-\dfrac{4\pi G}{C^2}S^2}{2}\right)\nu'=0$$

or

$$\frac{\mathrm{d}P}{\mathrm{d}r}+\frac{(P-\varepsilon_0)}{2}\nu'=0. \tag{2.7}$$

Integrating Eq. (2.7) we have

$$P=\varepsilon_0+C\,\mathrm{e}^{-\frac{\nu}{2}}, \tag{2.8}$$

since outside the sphere ($r > R$) we have

$$\mathrm{e}^{-\lambda}=\mathrm{e}^{\nu}=1-\frac{2GM}{rC^2} \tag{2.9}$$

(both spin density and matter density vanish for $r > R$) we must have upon matching Eq. (2.6) and Eq. (2.9) at $r = R$, $M = 0$.

Also $\nu = 0$ for $r \geq R$ from Eq. (2.9). In Eq. (2.8) we set $P(r = R) = 0$ (pressure at surface of sphere $= 0$) giving $C = -\varepsilon_0$ and thus

$$P=\varepsilon_0\left(1-\mathrm{e}^{-\frac{\nu}{2}}\right) \tag{2.10}$$

for $R \geq r$.

From Eq. (2.4) we have using Eq. (2.10)

$$\frac{\nu'}{r} = \frac{8\pi G}{C^4}\left(-\varepsilon_0 e^{-\frac{\nu}{2}}\right)$$

or upon integrating

$$2e^{\frac{\nu}{2}} = -\frac{4\pi G}{C^4}\varepsilon_0 r^2 + K.$$

Setting $e^{\nu/2} r = 1$ at $= R$ gives

$$K = 2 + \frac{4\pi G}{C^4}\varepsilon_0 R^2$$

or

$$e^{\frac{\nu}{2}} = 1 + \frac{2\pi G}{C^4}\varepsilon_0 (R^2 - r^2) \qquad (2.11)$$

for $R \geq r$.

Since the above solution represents a bound configuration of spin and energy density with zero mass we have given it a name »hidden matter« in Ref. 21. We now calculate the exact binding energy of the above configuration which we calculated only in an approximate manner in Ref. 21. For the relation between the proper energy density ε_0, the rest mass energy density $(\varrho_0 C^2)$ and the pressure we have

$$\varepsilon_0 = \varrho_0 C^2 + \frac{3}{2}\beta P + 3(1-\beta) P \qquad (2.12)$$

where β = ratio of gas pressure to total pressure and $(1-\beta) P$ = radiation pressure, thus

$$\varrho_0 C^2 = \varepsilon_0 + \left(\frac{3\beta}{2} - 3\right) P. \qquad (2.13)$$

For the binding energy we have

$$B.E. = \int_0^R 4\pi r^2 e^{\frac{\lambda}{2}} \varrho_0 C^2 \, dr - MC^2$$

here $M = 0$, $e^{\lambda/2} = 1$, and we have for $\beta = 1$ (pure gas sphere)

$$B.E. = \int_0^R 4\pi r^2 \left(\varepsilon_0 - \frac{3P}{2}\right) dr. \qquad (2.14)$$

Inserting Eq (2.10) and Eq. (2.11) in Eq. (2.14) we find upon integrating the exact expression for the binding energy of the gas sphere admitting spin generated torsion,

$$B.E. = \frac{2\pi\, \varepsilon_0\, R^3}{3} - \frac{6\pi\, \varepsilon_0\, R}{B} + \frac{6\pi\, \varepsilon_0\, A^{\frac{1}{2}}}{B^{\frac{3}{2}}} \ln \left| \frac{1 + \sqrt{\frac{B}{A}}\, R}{1 - \sqrt{\frac{B}{A}}\, R} \right|^{\frac{1}{2}} \tag{2.15}$$

where

$$A = 1 + \frac{2\pi\, G\, \varepsilon_0\, R^2}{C^4}, \quad B = \frac{2\pi\, G\, \varepsilon_0}{C^4}.$$

From Eq. (2.15) we can see that if

$$\frac{2\pi\, G\, \varepsilon_0\, R^2}{C^4} < 1$$

(which is a statement that the bound configuration without spin would not have a horizon for $r = R$), the binding energy can be expanded to give approximately

$$B.E. \cong \frac{2\pi\, \varepsilon_0\, R^3}{3} - \frac{6\pi\, \varepsilon_0\, R}{B}$$

$$+ 6\pi\, \varepsilon_0 \frac{A^{\frac{1}{2}}}{B^{\frac{3}{2}}} \left[\left(\frac{2\pi\, G \varepsilon_0}{C^4}\right)^{\frac{1}{2}} R + \frac{1}{3} \left(\frac{2\pi\, G\, \varepsilon_0}{C^4}\right)^{\frac{3}{2}} R^3 \right] \tag{2.16}$$

or

$$B.E. \cong \frac{4}{3}\, \pi\, R^3\, \varepsilon_0 \text{ assuming } A \approx 1.$$

However, this does not take into account the full nature of the natural log. If we approximate

$$A \cong 1$$

in Eq. (2.15) we look for a maximum of the binding energy by setting

$$\frac{\mathrm{d}(B.E.)}{\mathrm{d}R} = 0$$

or

$$R^4\, (B) + R^2\, (2) + \frac{3}{B} = 0 \tag{2.17}$$

which has no real root. Thus the binding energy rises for all R and we would expect the configuration to be stable for all R provided the condition

$$\frac{2\pi G \varepsilon_0 R^2}{C^4} < 1$$

is met (assuming $A \cong 1$).

We now turn to the problem of a charged sphere with spin density. For the metric we again assume

$$(\mathrm{d}S)^2 = \mathrm{e}^{\nu} (\mathrm{d}x^4)^2 - \mathrm{e}^{\lambda} (\mathrm{d}r)^2 - r^2 (\mathrm{d}\Theta)^2 - r^2 \sin^2 \Theta (\mathrm{d}\varphi)^2, \qquad (2.18)$$

for the Maxwell equations we have

$$\frac{\partial}{\partial x^{\nu}} (\sqrt{-g} F^{\mu\nu}) = 4\pi \sqrt{-g}\, J^{\mu} \qquad (2.19)$$

here

$$J^4 = \varrho_E \frac{\mathrm{d}x^4}{\mathrm{d}S} = \varrho_E\, \mathrm{e}^{-\frac{\nu}{2}}$$

$\varrho_E =$ proper charge density. If we approximate $\mathrm{e}^{\lambda} \approx \mathrm{e}^{\nu} \approx 1$ in Eq. (2.19) we have using $F_{14} = E(r)$

$$\frac{\partial}{\partial r} (r^2 E) = 4\pi \varrho_E r^2$$

or

$$E(r) = \frac{4\pi r \varrho_E}{3} \qquad (2.20)$$

($\varrho_E =$ const.)

For t he energy momentum tensor of the electromagnetic field we have

$$T_{\mu\nu} = \frac{1}{16\pi} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - \frac{1}{4\pi} F_{\mu\alpha} F^{\alpha}_{\nu} \qquad (2.21)$$

giving

$$T^4_4 = T^1_1 = \frac{E^2}{8\pi} = - T^2_2 - T^3_3. \qquad (2.22)$$

Thus for the total effective energy momentum tensor (including spin density) we have

$$T^4_4 = \varepsilon_0 - \frac{2\pi G}{C^2} S^2 + \frac{E^2}{8\pi}$$

($\varepsilon_0 =$ const.)

$$T^1_1 = -P + \frac{2\pi G}{C^2} S^2 + \frac{E^2}{8\pi} \tag{2.23}$$

$$T^3_3 = T^2_2 = -P + \frac{2\pi G}{C^2} S^2 - \frac{E^2}{8\pi}$$

with

$$E = \frac{4\pi \varrho_E r}{3}. \tag{2.24}$$

For the Einstein equations in the presence of a perfect fluid with spin density and charge density ϱ_E we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\,\mathrm{e}^{-\lambda}) = 1 - \frac{8\pi G}{C^4}\left(\varepsilon_0 - \frac{2\pi GS^2}{C^2} + \frac{E^2}{8\pi}\right) \tag{2.25}$$

$$\mathrm{e}^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = -\frac{8\pi G}{C^4}\left(-P + \frac{2\pi GS^2}{C^2} + \frac{E^2}{8\pi}\right) \tag{2.26}$$

$$\mathrm{e}^{-\lambda}\left(\frac{\lambda''}{2} - \frac{\lambda'\nu'}{4} + \frac{(\nu')^2}{4} + \frac{\nu' - \lambda'}{2r}\right) = -\frac{8\pi G}{C^2}\left(-P + \frac{2\pi GS^2}{C^4} - \frac{E^2}{8\pi}\right). \tag{2.27}$$

If in Eq. (2.25) we set

$$\varepsilon_0 - \frac{2\pi GS^2}{C^4} + \frac{E^2}{8\pi} = 0 \tag{2.28}$$

we have $\mathrm{e}^{-\lambda} = 1$ for $r \leq R$ by solving

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\,\mathrm{e}^{-\lambda}) = 1$$

and insisting on a finite metric at $r = 0$.

If we equate the expressions for the pressure in Eq. (2.26) and Eq. (2.27) we have (using $\mathrm{e}^{-\lambda} = 1$)

$$\frac{(\nu')^2}{4} - \frac{(\nu')}{2r} - \frac{2GE^2}{C^4} = 0$$

or

$$\frac{(\nu')^2}{4} - \frac{(\nu')}{2r} - \frac{2G}{C^4}\left(\frac{4}{3}\pi\varrho_E r\right)^2 = 0. \tag{2.29}$$

Eq. (2.29) is a non-linear first order inhomogeneous equation whose solution can be found, the exterior metric is for $r \geq R$

$$e^{-\lambda} = e^{\nu} = 1 - \frac{2GM}{r\,C^4} + \frac{Ge^2}{r^2C^4}, \tag{2.30}$$

where both spin density and matter density are 0 for $r > R$. Here e = total charge and also since we have $e^{-\lambda} = 1$ for $r = R$ from the interior solution we have upon matching Eq. (2.30) to $e^{-\lambda} = 1$ at $r = R$,

$$M = \frac{e^2}{2RC^2}.$$

Now since $e^{-\lambda} = e^{\nu} = 1$ at $r = R$ from Eq. (2.30), $\nu = 0$ at $r = R$. Thus Eq. (2.29) is solved subject to $\nu = 0$ at $r = R$, To calculate the pressure we substitute $e^{-\lambda} = 1$ and ν found by solving Eq. (2.2) back into Eq. (2.26) using

$$\frac{2\pi\,GS^2}{C^2} = \varepsilon_0 + \frac{E^2}{8\pi}$$

from Eq. (2.28).

$$\left(\varepsilon_0 = constant,\; E = \frac{4\pi\,\varrho_E\,r}{3}\right).$$

Eq. (2.26) can then be solved for P. For the binding energy in the presence of spin and charged density we have

$$B.E. = \int_0^R 4\pi\,r^2\,e^{\frac{\lambda}{2}}\left(\varepsilon_0 - \frac{3P}{2}\right) dr - MC^2 \tag{2.31}$$

$$B.E. = \int_0^R 4\pi\,r^2\left(\varepsilon_0 - \frac{3P}{2}\right) dr - \frac{e^2}{2R}. \tag{2.32}$$

Here P is calculated from Eq. (2.26) and the integral in Eq. (2.32) is evaluated from 0 to R.

3. *Conclusion*

The above model of a charged sphere admitting spin density is the simplest possible model that we can imagine, but it represents a starting point from which all subsequent models may find their primitive origin. The original calculation of the G. R. binding energy was done by J. Wright[22] in an effort to study Quasar stability for a gas sphere. His calculation gave the same results as Chandrasekhar[23] who developed a stability theory of gravitational objects using the method of time dependent perturbations. The fact that a configuration of matter admitting spin density always seems to generate a positive binding energy suggests that spin may provide an excellent means of self-binding for matter in the early universe. It should also be mentioned that the restriction

$$\frac{2\pi G \varepsilon_0 R^2}{C^4} < 1$$

need not be enforced since the object has zero mass (to a distant observer) and thus there are no horizons to such a configuration. A study of the binding energy for the case

$$\frac{2\pi G \varepsilon_0 R^2}{C^4} > 1$$

when charge is absent will be given in a future paper. We might also remark that the calculation of the binding energy for a charged spin carrying sphere might shed light on certain anomalies of the wealth of extra-galactic radiation[24] presently being observed, wherein a burst of energy would result when such a charged spin carrying sphere is formed. Torsion to date has not been found either in an astrophysical setting or in a conventional laboratory experiment[25], it still remains both curiosity and imaginative extension of the original Einstein theory wherein both geometry and matter take on asymmetric characteristics. Certainly the presence of torsion effects in the cosmos may shed light on some of the fundamental unresolved mysteries of early universe cosmology.

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NABIJENA KUGLA TEKUĆINE KOJA DOPUŠTA GENERIRANJE TORZIJE SPINOM

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Promatramo sferno simetričnu konfiguraciju nabijene materije s gustoćom spina i diskutiramo energiju vezanja takve konfiguracije kada se gustoća spina tako prilagodi da prisili radijalnu komponentu metrike da bude 1 (g_{11}) za $r < R$. Takvi uvjeti omogućuju rješivost problema.