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EVOLUTION OF FRIEDMANN-LIKE SUPERSYMMETRIC SUBSPACES IN EIGHT-DIMENSIONAL THEORY

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We consider the eight-dimensional theory on the basis of general relativity with supersymmetry between four-dimensional subspaces. This theory is contrasting by several time-like dimensions with more usable many-dimensional schemes of the Kaluza-Klein type. The evolution of Friedmann-like models for bosonic and fermionic subspaces is studied and differences in dynamics of these subspaces are determined by means of approximate analytical and numerical computations. Peculiarities of the evolution can be proposed as possible observable consequences of higher dimensions.

1. *Introduction*

Different many-dimensional schemes are thought to be the basis for building the unified theory of all physical interactions. Among very big diversity of versions of these theories in contemporary literature we would like to point out those approaches which include timelike dimensions in extra space rather than purely Kaluza-Klein (KK) type theories[1−5].

From the other side the so-called "wormhole" physics having developed in connection with the euclidean quantum gravity shows the significance of representations on extended space-time manifolds (with event horizons and non-trivial topology) in microworld and the unification problem. In these extended manifolds signs of metrical coefficients are interchanged passing horizons, and there are many-dimensional (complex) versions of such representation in which time-like and space-like coordinates are taken into account by the unified symmetrical way [6−11].

A rather attractive interconnection of extended manifolds with superluminal theories, where also many dimensions appear, makes the perspective to invoke a variety of these ideas in order to construct the theory with different fields in space-time of non-trivial topology.

The many-dimensional theory with extra time-like dimensions attracts the additional interest by taking into account the connection of superluminal transformations and supersymmetric transformations[12−13]. Ranges of time-like dimensions are separated from space-like ones by pseudosingular surfaces (horizons). The superluminal transformations transform bradyonic objects into tachyonic ones and vice versa; the supersymmetric ones do boson states into fermion ones. Moreover, the superluminal transformations can be used succesfully in flat space-time and in general relativistic metrics[14−17].

There are versions of many-dimensional theories in which supersymmetric coordinates correspond to additional dimensions[18−19], however, with no introduction of notions on extended space-time manifolds and superluminal physics. The introduction of additional (extra) dimensions in the spirit of those in the abovementioned theories[6−11] with the unified consideration of bradyonic and tachyonic spaces in the eight-dimensional representation as $D^8 = R^4 \otimes T^4$ for instance[8−9], allows to overcome some problems of trivial construction with timelike coordinates in extra space-time and to obtain new properties of matter fields in such global space-time manifolds.

In the present work we shall construct the eight-dimensional theory with supersymmetric coordinates and study the evolution of bosonic and fermionic subspaces and their interconnection. The evolution of these subspaces appears within the framework of similar KK scheme[18−19] almost the same with some approximations in the solution procedure of corresponding equations. We shall take the another scheme with equal roles of both the subspaces as well as the equal (and symmetrical) ones of bradyonic and tachyonic frames and sub-spaces in many-dimensional theories with superluminal objects[8−11]. The latter correspondence will not be discussed in this paper specially, since our conclusions are independent of this interpretation itself, though very interesting from the point of view of the "democracy" between space and field variables[9]. Bosonic and fermionic dimensions in our present approach form the global eight-dimensional space-time manifold, and we shall consider the case of maximally symmetric subspaces as one of simplest possible models, however, the construction can be performed in more general way to include other fields, matter and more complicated geometry.

2. Supersymmetric higher-dimensional space-time and field equations

We consider the eight-dimensional space-time with coordinates

$$(M, N) = (1, 2, 3, 4, 5, 6, 7, 8) = (t, y^1, y^2, y^3, \bar{t}, \bar{y}^1, \bar{y}^2, \bar{y}^3),$$

where (5-8)-coordinates will be fermionic ones, and they will be transformed from (l-4)-bosonic coordinates by means of

$$\bar{y}^\alpha \bar{y}^\alpha = y^\alpha h_{\alpha\beta} y^\beta$$

$$h_{\alpha\beta} = \begin{pmatrix} \sigma_2 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & \sigma_2 \end{pmatrix}, \qquad \text{and } \sigma_2 \text{ is Pauli matrix.}$$

In order to investigate a matter dynamics in the present model we shall use the uniform and isotropic spherical symmetric metrics for subspaces which are reduced in the four-dimensional case to the Friedmann-Robertson-Walker (FRW) metric. It, evidently, is one of the simplest models for beginning study, and it admits either matter or fields in the right part of Einstein (Einstein-Maxwell) equations. Thus, the general form of our metric will be the following

$$g_{MN} = \begin{pmatrix} -1 & & & \\ & g_{ij}R^2 & & \\ & & +1 & \\ & & & g_{mn}\bar{R}^2 \end{pmatrix}, \tag{1}$$

where R and $\bar{R}$ are maximally symmetric subspaces, and the line element then may be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\bar{t}^2 + b_b \mathrm{d}y^\alpha \mathrm{d}y^\alpha (1 + y^2/4)^{-2} - b_f^2 \mathrm{d}\bar{y}^\alpha \mathrm{d}\bar{y}^\alpha (1 + \bar{y}^2/4)^{-2}. \tag{2}$$

The metric in the "supersymmetric" representation can be written as

$$\mathrm{d}s^2 = E^M G_{MN} E^N, \tag{3}$$

where the vielbein E^M is

$$E^1 = \mathrm{d}t, \qquad E^5 = -\mathrm{d}\bar{t},$$

$$(E^\alpha, E^\alpha) = \frac{(\mathrm{d}y^\alpha b, \mathrm{d}\bar{y}^\alpha f)}{(1 + y^2/4)},$$

and the non-zero components of G_{MN} are

$$G_{00} = -1, \quad G_{\alpha\alpha} = 1, \quad G_{55} = +1, \quad G_{\alpha\beta} = h_{\alpha\beta}.$$

In this construction of the "supersymmetric" space-time we follow in some measure the work in Ref. 19 but with the equal number of bosonic and fermionic dimensions. In contrast to the Ref. 19 our space-time is not a KK type: spatial dimensions themselves are supersymmetric, and they are not "extra" ones. This circumstance is similar to the versions of many-dimensional theories including bradyons and tachyons on equal footings[6−11] when the space-time sector associated with additional dimensions itself possesses the "tachyonic" character.

Further, for the derivation of field equations we shall calculate the curvature invariant which will give the action S and Lagrangian L:

$$S = \int \mathrm{d}^8 z \; G^{(1/2)} R = \int \mathrm{d}t L \, . \tag{4}$$

We obtain

$$\begin{aligned} R/3 = 2(\ddot{b}/b - \ddot{f}/f) - 2\dot{b}^2/b^2 + 6\dot{b}\dot{f}/bf - 4\dot{f}^2/f^2 \\ -(1/4)(y^2 + \bar{y}^2 + 1)(b^{-2} + f^{-2}) \, . \end{aligned} \tag{5}$$

Instead of b and f we introduce for the sake of simplicity the following values: $Q = (1/2)\log(bf)$ and $q = (1/2)\log(b/f)$, then the equations of motion get

$$3q + (27/4)\dot{q} + (3/2)\dot{q}\dot{Q} \, , +(3/2)\exp(-2Q)\sinh q = 0 \tag{6}$$

$$\ddot{q} + 3\dot{q}^2 + 2\exp(-2Q)\sinh q = 0 \, , \tag{7}$$

$$\ddot{q} - (1/3)\ddot{Q} + (3/2)\dot{q}^2 + (1/3)\exp(-2Q)(3\sinh q + \cosh q) = 0 \, . \tag{8}$$

This system of non-linear differential equations determines the evolution laws for bosonic and fermionic subspaces in the metric (2), but its exact complete solution is impossible by analytical methods. Firstly we consider some approximate solutions, which are representable in the analytical form and can give rather clearly the physical meaning of the model.

3. Analytical solutions for particular cases

As the "zero" approximation we take the case of coincidence of bosonic and fermionic scale factors, i.e. $b = f$ and $q = 0$. Then we obtain the following equation of evolution

$$\ddot{Q} = \exp(-2Q) \, . \tag{9}$$

with the solution in the form

$$Q = -\log\left[\frac{C_1}{2}\left(1 - \left(\frac{1 + \exp\sqrt{C_1 t}}{1 - \exp\sqrt{C_1 t}}\right)^2\right)\right] , \tag{10}$$

where C_1 =const. It is a meaningless dependence for b- and f-factors, since for reasonable values of C_1 ($C_1 > 0$) it goes to infinity ($0 < t < \infty$).

If we take $Q = 0$, i.e. $bf = 1$, our system of equations (6-8) will not be essentially simpler, but one may conclude that the occurrence of time-dependence for q will mean the different speeds of variations of b and f. That can evidence that b- and f-factors initially are very different, and supersymmetry in the sense of scales of these subspaces can not appear.

In order to determine $Q(t)$ and $q(t)$ let us transform the system (6-8) into the two following equations which contain these functions:

$$\ddot{q} - 6\dot{q}^2 - 2\dot{q}\dot{Q} - 4q = 0\,. \tag{10}$$

and

$$\ddot{Q} + (9/2)\dot{q}^2 + 3\sinh q \exp(-2Q) - -\cosh q \exp(-2Q) = 0\,. \tag{11}$$

In order to find an example of solution we take the power-law dependence on t for logarithms of scale factors

$$q = Bt^{\beta} - Ft^{\varphi}, \qquad Q = (1/2)(Bt^{\beta} + Ft^{\varphi})\,, \tag{12}$$

i.e. we take such particular case, where B and F are positive constants and let β and φ be integer constants.

Substituting these expressions in the preceding equations of motion and derivating we can obtain

$$B\beta(\beta-1)t^{(\beta-2)} - 7B^2\beta^2t^{2(\beta-1)} + 4Bt^{\beta} - F\varphi(\varphi-1)t^{(\varphi-2)}$$
$$-5F^2\varphi^2t^{2(\varphi-1)} - 4Ft^{\varphi} + 12BF\beta\varphi t^{(\beta+\varphi-2)} = 0 \tag{13}$$

and consider some particular values for exponents β and φ.

In the case $\beta = \varphi = 1$ the rather simple connection between B and F gets expressed by $B = F$ (i.e. for every t): bosonic and fermionic factors equal at the beginning are connected further according to

$$B = (4t + 5F)/7\,. \tag{14}$$

Thus, the difference in scales of bosonic and fermionic subspaces increases with time, but in this particular case rather slowly. However, the connection between them, which appears as the universal one, is of special interest itself, since it may give the determination of properties of one subspace through another. The supersymmetry introduced by the construction of this model, evidently, is the cause of this connection.

Let us consider further $\beta = \varphi = 2$, and from (13) we have

$$B\beta(\beta-1) - 7B^2\beta^2t^2 + 4Bt^2 - F\varphi(\varphi-1) - 5F^2\varphi^2t^2$$
$$-4Ft^2 + 12BF\beta\varphi t^2 = 0\,. \tag{15}$$

It can be seen that due to absence of linear terms in t we have again the rather simple connection of B and F

$$(B-F)/2=(B+F+12BF+7B^2+5F^2)t^2, \tag{16}$$

i.e. the difference between scales of b- and f-subspaces decreases almost quadratically.

We get to the another particular case when $q \ll \dot{q}$. It realizes when the difference in the expansion speeds of bosonic and fermionic subspaces increase very essentially. For Q we shall then have the approximation $Q=0 \rightarrow bf=1$, i.e. b- and f- factors are reciprocal, and the solution for q is obtained from the equation (10) simplified to

$$\ddot{q}-6\dot{q}^2=0 \tag{17}$$

in the form

$$b/f=t^{(-1/6)}. \tag{18}$$

Thus, there are the reversed 6th-degree root dependence both for bf and b/f. In a more general form for some $\gamma_1>\gamma_2$ we can get the time dependences by the type of $b/f=t^2$

$$b=C_1C_2^{(1/2)}\,t^{(\gamma_1+\gamma_2)/2}, \quad f=(C_1/C_2)^{(1/2)}t^{(\gamma_1-\gamma_2)/2}. \tag{19}$$

In this approximation, consequently, the difference between b and f can increase fast in spite of their closeness at the beginning. Generally speaking, we have the possibility to determine the evolution of one subspace through parameters of another up to multiplicative constants. A difference between their evolution can mean the breakdown of starting supersymmetry; even in the above approximate solutions results with equal scale factors ($q=0$) for all the evolution period $0<t<\infty$ is very little probable. That shows that in such a many-dimensional model for the Universe a broken supersymmetry is more natural state and is consistent with observable statement.

4. *Numerical solutions*

In this section we present results of numerical calculations of the problem under consideration, which is expressed by the system of equations of evolution for b- and f-subspaces. The system of differential non-linear equations (6-8) we reduce to the following two equations with no loss of generality, evidently, with second derivatives for q and Q separatelly:

$$\ddot{q}-6\dot{q}^2-2\dot{q}\dot{Q}-4q=0 \tag{20}$$

$$\ddot{Q}+(9/2)\dot{q}^2+(3\sinh q-\cosh q)\exp(-2Q)=0 \tag{21}$$

and further transform them to four equations of the first order and use the familiar Runge-Kutta method. Results of computer (IBM PC/AT, PC-MAT LAB package) calculations are presented in the graphic form (Figs. 1-3) as plots of scale factors versus time and dependence of scale factors one on another. We take some particular boundary conditions, which show various types of solutions since a priori they are nc

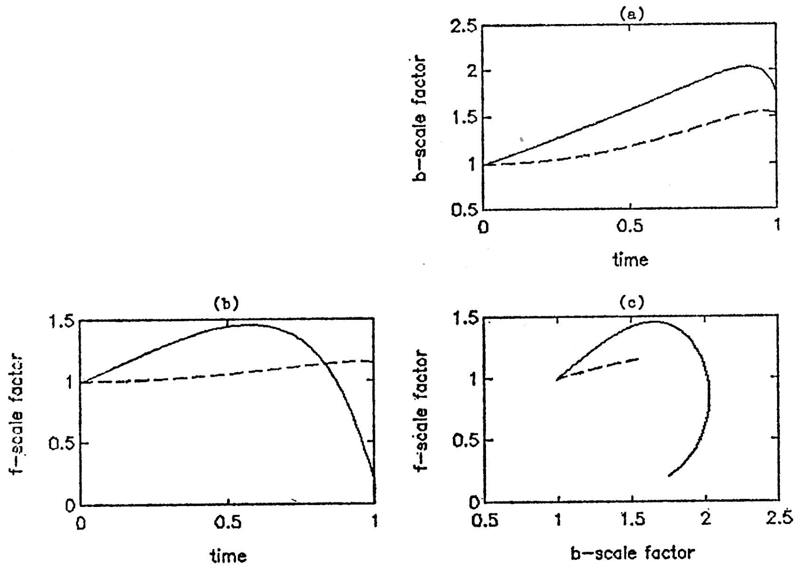

Fig. 1. Time evolution of bosonic (a) and fermionic (b) subspaces at the following initial conditions: $q = q_0 = 0$ when $t = 0$ and $b = b_0 = f = f_0$. Plot (c) indicates the dependence of these scale factors one on another in arbitrary units. $Q_0 < 0$.

It should be remarked from these results that the numerical solutions give also similarity for evolution of bosonic and fermionic subspaces in time, i.e. in the sense of scales b- and f-sectors the supersymmetry appears to be broken rather weakly. Of course it is quite true if initial bosonic and fermionic scales are suppresed to be not very different in contrast with the conventional KK schemes when the scale of extra space can be small initially[1)]. In this connection the unobservability of more-than-four dimensions of space-time in the present epoch we propose to associate with "event horizon" effects taking into account in the present and in similar models the interconnection between bosonic-fermionic symmetry and bradyon-tachyon transformations[12)]. The more detailed formulation of such theory is in progress. Here we would like to notice only, that in more complicated space-times than considered above the interconnection between bosonic and fermionic subspaces can appear by the more special way, but it would be conditioned by properties of space-time symmetries, matter state, influence of fields, etc. The contribution of the supersymmetric connection itself must be seen explicitly in simple models by the type of described here.

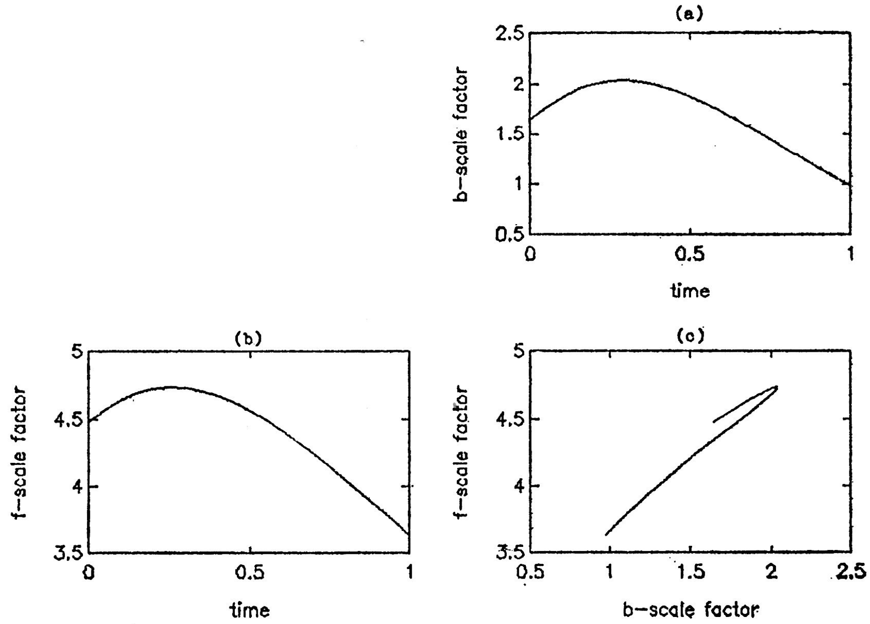

Fig. 2. As Fig. 1 with the initial conditions: $q_0 = -1$, $b_0 f_0 = e^2$, $Q_0 = 1$.

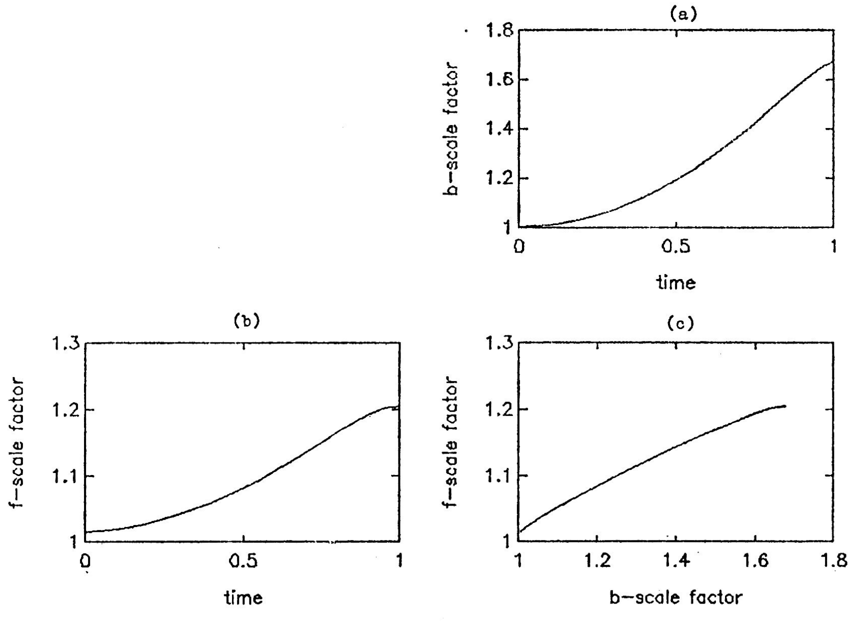

Fig. 3. As Figs. 1 and 2 with the initial conditions: $q_0 > 0$, $b_0 = f_0$, $Q_0 = -0.01$.

5. Discussion

A study of space-time evolution in different higher-dimensional theories is of great interest in the connection with a diversity of many-dimensional models proposed at present[1−10] (these references, of course, do not exhaust all models which exist up to date). Some interconnections of extra (additional) dimensions with our (basic) space-time can give restrictions for selection of a certain theory or in the best cases one could obtain "observable" consequences of higher dimensions. Some steps in this direction we made in this paper.

Evidently, it is rather easy to speak about some quantitative results for comparison with reality of astrophysical observations, however, the theory with supersymmetric connection of our and higher dimensions ought to be perspective in order to invoke the deeply developed theory of supersymmetry in realistic physics, for example, to explain absence of many supersymmetrical partners for ordinary particles. Various "inos", possibly, exist in additional dimensions and can manifest themselves only in regions where different dimensions are "mixed".

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EVOLUCIJA FRIEDMANNOVIH SUPERSIMETRIČNIH PODPROSTORA U OSAM-DIMENZIONALNOJ TEORIJI

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Na bazi opće relativnosti promatramo osam-dimenzionalnu teoriju sa supersimetrijom između četvero-dimenzionalnih podprostora. Ova teorija s nekoliko vremenskih dimenzija suočava se s češće korištenim više dimenzionalnim šemama tipa Kaluza-Klein. Studirana je evolucija Friedmannovih modela za bozonske i fermionske podprostore i određene su razlike u dinamici tih podprostora pomoću približnih analitičkih i numeričkih računa. Posebnosti evolucije mogu se predložiti kao moguće vidljive posljedice viših dimenzija.