ISSN 1330–0016
CODEN FIZBE7

NUCLEON STATIC PROPERTIES IN A TAMM-DANCOFF INSPIRED APPROXIMATION

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Received 16 June 1998; Accepted 9 November 1998

A version of the chiral bag model, which contains the linear $\sigma-$model dynamics and is quantized by using the constituent quark operators, has been solved using a Tamm-Dancoff inspired approximation. Model gives reasonable values for the axial coupling constant $g_A = \Delta u - \Delta d = 1.28$, for the combination $h_A^S = \Delta u + \Delta d = 0.38$, and for the proton magnetic moment $\mu_p = 2.77$. These values, which match the combinations $\Delta u \pm \Delta d$ of the quark density functions, are consequences of the chiral character of the model.

PACS numbers: 11.30.Rd, 12.39.Ba, 12.39.Fe, 14.20.Dh UDC 539.125, 539.126

Keywords: chiral bag model, the linear $\sigma-$model dynamics, constituent quark operators, Tamm-Dancoff approximation, axial coupling constant, proton magnetic moment, chirality

1. Introduction

The deep inelastic scattering reveals that a proton has a rather complicated composition $|uud;\bar{q}q;g\rangle$ consisting of valence quarks (u,d), quark pairs $(\bar{q}q)$ and gluons (g). The low energy, quasi-static properties of a nucleon, such as the axial-vector coupling constant g_A, or the nucleon magnetic moment μ_n, can be reasonably well understood if the proton

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(neutron) is composed of free quark-like pieces, say $|UUD>$ [1-3]. Here the capital letters U and D are used to distinguish the constituent quarks, which appear in a quark model [4-8], from the valence quarks u and d which are observed in the deep inelastic scattering. As will be illustrated in this paper, one should not identify the valence quark, defined by the corresponding quark density function Δq, with the constituent quark [2]. Valence or current quarks belong to a composite system of partons, i.e. quarks, antiquarks and gluons [9-11]. Nevertheless, the constituent quark models have been employed [10,11] in the discussion [2,9] of the spin of the proton. Besides constituent quarks, such approaches had to include some less schematic features such as a superposition of SU(6) configurations [10] or the modified proton wave function [11] which contains $(3Q)(Q\bar{Q})$ components besides the convential $3Q$ term [12]. Skyrme model has also been used [13-16] with various degrees of success [2].

The internal structure of nucleons is obtained from experiments at high energies [2,3,17-20], whose mass scale differs significantly from the scale at which constituent quark models are formulated [21]. However, a quark model, which uses the constituent quarks to describe static hadronic properties, should be able to reproduce the correct values of certain combination of the quark density functions. These combinations determine [2,3,17-20] certain static properties of hadrons. For example:

$$g_A = \Delta u - \Delta d \tag{1.1}$$

$$h_A^S = \Delta u + \Delta d \tag{1.2}$$

Here g_A is the nucleon axial-vector coupling constant (in units of g_V). The quantity h_A^S can also be calculated in the constituent quark models. It differs slightly from the constant g_A^S which has been discussed in our earlier paper [7]. The new quantity does not include strange quark density Δs, i.e.

$$h_A^S = g_A^S - \frac{2}{5}\Delta s. \tag{1.3}$$

The quark densities Δq can be extrapolated [2,3,17-20,22-24] from the deep inelastic scattering data. Thus the relations (1.1) and (1.2) constrain the constituent quark model values of g_A and h_A^S. As is well known [11], the experimental value of g_A is closely related to the chiral symmetry. Only a model which satisfies these relations can be an acceptable tool for the estimate of other low energy quantities, such as electroweak formfactors.

As an illustration of such a general statement, the quantities g_A and h_A^S will be obtained in the framework of a chiral bag model [4-6]. A particular version, which involves the linear sigma-model outside the bag [4], will be used because of its relative simplicity. This model will be solved using the Tamm-Dancoff [25-32] inspired approximation (TDIA) which was described previously [33]. Even such "simple" approach does lead (see Sect. 2 below) to involved nonlinear systems of differential and algebraic equations which are to be solved simultaneously. It is related to non-linear sigma-models and thus to their Skyrme-soliton solutions [13].

In our "simple" description, the chiral invariance is explicitly violated only by the term proportional to the sigma field σ which ensures the correct PCAC relation. All other chiral invariance violations are caused spontaneously, as discussed below in Sect. 2. Thus, the axial-vector current is, even when expressed in term of the model solutions, a well defined chiral vector in the chiral symmetry limit. As a consequence, the coupling constant g_A tends to stay close to its chiral symmetry limit $g_A = 1$ (see Table 1). On the other hand, the model produces very reasonable values of the proton magnetic moment (see Table 1).

TABLE 1. The results for the chiral-quark model in the TDIA. The bag radius R is in GeV^{-1} *units.*

R	ω	magn. moment			ax. const. g_A			m_π^{phys}
		μ_Q	μ_M	μ_{Tot}	$g_{A/Q}$	$g_{A/M}$	$g_{A/Tot}$	
4.00	2.10	1.53	1.41	2.94	1.01	0.12	1.13	0.208
5.00	1.90	1.77	0.44	2.21	1.06	0.23	1.29	0.142
5.00	2.10	1.01	0.91	1.92	1.06	0.06	1.12	0.198
6.00	1.80	2.09	0.28	2.37	1.06	0.05	1.21	0.132
6.00	1.90	2.09	0.34	2.43	0.91	0.25	1.16	0.166
7.00	1.80	3.02	0.25	3.27	1.03	0.18	1.21	0.155
7.00	2.10	2.55	0.06	2.61	1.06	0.21	1.27	0.156
The parameters								
$\lambda = 9.062$		$m_\sigma = 1.2$ GeV		$\mu_{exp} = 2.79$	$m_\pi^{exp} = 0.139$ GeV			
$\nu = 0.092$		$f_\pi = 0.093$ GeV		$g_{A/exp} = 1.26$	$m_\pi = 0.140$ GeV			

All these results are made out of two components which are called generically quark and meson phases. These approximate solutions are more precisely defined in Sect. 2. Roughly, one can say that the quark phase of a current is built up of the constituent quark "fields". The "meson" phase (or "field") is composed of bilinear combinations (i.e. $\bar{U}U$, $\bar{D}D$) of constituent quark operators, vaguely reminescent of the real physical mesons. In short, the model describes a nontrivial dynamics of constituent quarks, and it is closely related to the models of Refs. 5 and 6 which have been often used [6-8]. As it is made of $\bar{Q}Q$ pairs, its physical content has some similarity with the approaches [11,12] which are employing the modified proton wave function.

As it turns out, a particular mixture of the two contributions is needed to reproduce simultaneously the experimental values of g_A, h_A^S and μ_N. The constants g_A and μ_N, which are independently measurable in the low energy experiments, can be explained by several sets of model parameters. The same parameters ought to give a reasonable value of the constant h_A^S. Thus, one can test the compatibility between the constituent quark models and the high energy results [2,3,17-20,22-24,27].

2. Brief outline of the model

The Lagrangian with the linear sigma model embedded in the bag environment has the usual form [4,34-37]:

$$\mathcal{L} = \mathcal{L}_\psi \theta + \mathcal{L}_{\rm int}\delta_S + [\mathcal{L}_{\sigma\pi} - \mathcal{U}(\sigma,\vec{\pi})]\overline{\theta}, \tag{2.1}$$

where

$$\begin{aligned}
\mathcal{L}_\psi &= \frac{\rm i}{2}(\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - \partial_\mu\bar{\psi}(x)\gamma^\mu\psi(x)) - B,\\
\mathcal{L}_{\rm int} &= \frac{g}{2}\bar{\psi}(x)(\sigma(x) + {\rm i}\vec{\tau}\vec{\pi}(x)\gamma_5)\psi(x),\\
\mathcal{L}_{\sigma\pi} &= \frac{1}{2}\partial^\mu\sigma(x)\partial_\mu\sigma(x) + \frac{1}{2}\partial^\mu\vec{\pi}(x)\partial_\mu\vec{\pi}(x),\\
U(\sigma,\vec{\pi}) &= \frac{\lambda^2}{4}[\sigma^2(x) + \vec{\pi}^2(x) - \nu^2]^2 - f_\pi m_\pi^2\sigma(x)
\end{aligned} \tag{2.2}$$

and $f_\pi = 0.093$ GeV. The self-interaction potential U includes the symmetry-breaking term $f_\pi m_\pi^2\sigma(x)$. The values of other constants are fixed by the creation of mass terms for the $\vec{\pi}$ and σ fields, by the PCAC and by the requirement $U^{(min)} = 0$. Their values are given in Sect. 3. In the framework of this particular model, m_σ and m_π are not necessarily equal to the physical sigma and pion masses, but play the role of model parameters.

In the Heisenberg picture [38] and in the leading order of TDIA [33], the bosonic entities σ and $\vec{\pi}$ are not the elementary fields. The *Ansatz* for the quark field is

$$\psi_f^c(x) = \frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0 \\ {\rm i}(\vec{\sigma}\hat{r})j_1\end{pmatrix}\chi_m^f b_{m,f}^c + \frac{N}{\sqrt{4\pi}}\begin{pmatrix}(\vec{\sigma}\hat{r})j_1 \\ {\rm i}j_0\end{pmatrix}\chi_m^f d_{m,f}^{c\dagger}. \tag{2.3}$$

Here c is a quark colour and f is a quark flavour, whereas m is the spin projection. $b_{m,f}^c$ and $d_{m,f}^c$ are constituent quark and antiquark annihilation operators, respectively. The quantities $j_{0,1}(r)$ are spherical Bessel functions of the order (0,1) and χ_m^f is the quark isospinor ($\tilde{\chi}^f$) - spinor (χ_m) product

$$\chi_m^f = \tilde{\chi}^f \cdot \chi_m. \tag{2.4}$$

The σ and $\vec{\pi}$ solitons are determined by the boundary conditions

$$(\partial^\mu\pi^a(r))n_\mu\delta_S - \frac{g_\pi}{2}\overline{\psi}(r)i\tau^a\gamma_5\psi(r)\delta_S = 0 \tag{2.5}$$

and

$$(\partial^\mu\sigma(r))n_\mu\delta_S - \frac{g_\sigma}{2}\overline{\psi}\psi\delta_S = 0. \tag{2.6}$$

One finds

$$\begin{aligned}\pi^a(r) = \pi_s(r)(b^{c\dagger}_{m,f}d^{c\dagger}_{m',f'} + d^c_{m,f}b^c_{m',f'}) \cdot [\chi^\dagger_{m,f}\tau^a\chi_{m',f'}] \\ + \pi_p(r)(b^{c\dagger}_{m,f}b^c_{m',f'} \\ + d^c_{m,f}d^{c\dagger}_{m',f'}) \cdot [\chi^\dagger_{m,f}(\vec{\sigma}\hat{r})\tau^a\chi_{m',f'}]\end{aligned} \tag{2.7}$$

and

$$\sigma(r) = \sigma_s(r) \cdot (b^{c\dagger}_{m,f}b^c_{m,f} + d^{c\dagger}_{m,f}d^c_{m,f}) - f_\pi. \tag{2.8}$$

Here the radial functions $\sigma_s(r)$, $\pi_s(r)$ and $\pi_p(r)$ outside the bag $(1-\theta)$ correspond to the solutions of the Euler-Lagrange equations:

$$\partial^\mu\partial_\mu\pi^a(r) + \lambda^2\pi^a(r)[\sigma(r)^2 + \vec{\pi}(r)^2 - \nu^2] = 0 \tag{2.9}$$

and

$$\partial^\mu\partial_\mu\sigma(r) + \lambda^2\sigma(r)[\sigma(r)^2 + \vec{\pi}^2 - \nu^2] + f_\pi m_\pi^2 = 0. \tag{2.10}$$

To extract the equations for the σ_s π_s and π_p components from the TDIA operator equations of motion, the equations (2.7)-(2.10) are "sandwiched" between the final state $\langle f| = \langle q^c_{f,t}| = \langle 0|b^c_{f,t}$ and the initial state $|i\rangle = |q^c_{i,u}\rangle = b^{c\dagger}_{i,u}|0\rangle$. This choice yields the equation for $\sigma_s(r)$:

$$\begin{aligned}\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\sigma_s(r) = \lambda^2[\sigma_s(r) - f_\pi][(\sigma_s(r) - f_\pi)^2 \\ + 3\pi_p^2(r) - \nu^2] + f_\pi m_\pi^2\end{aligned} \tag{2.11}$$

and for $\pi_p(r)$:

$$\begin{aligned}\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{2}{r^2}\right]\pi_p(r) = \lambda^2\pi_p(r)[(\sigma(r) - f_\pi)^2 \\ + 3\pi_p^2(r) - \nu^2].\end{aligned} \tag{2.12}$$

The other choice, i.e. $\langle f| = \langle 0|$ and $|i\rangle = |q^c_{i,u}\overline{q}^c_{i',u'}\rangle = d^{c\dagger}_{i',u'}b^{c\dagger}_{i,u}|0\rangle$, gives the π_s component

$$\left[\frac{\mathrm{d}^2}{dr^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\pi_s(r) = \lambda^2\pi_s(r)\left[f_\pi^2 + 36\pi_s^2(r) - \nu^2\right]. \tag{2.13}$$

Now, one can specify the boundary conditions (2.5) and (2.6) using the same initial/final-state combinations:

$$
\begin{aligned}
\frac{\partial}{\partial r}\sigma_s(r)\Big|_{r=R_{\rm bag}} &= -\frac{N^2}{4\pi}\frac{g_{\sigma/s}}{2}[j_0^2(\omega)-j_1^2(\omega)],\\
\frac{\partial}{\partial r}\pi_s(r)\Big|_{r=R_{\rm bag}} &= -\frac{N^2}{4\pi}\frac{g_{\pi/s}}{2}[j_0^2(\omega)+j_1^2(\omega)],\\
\frac{\partial}{\partial r}\pi_p(r)\Big|_{r=R_{\rm bag}} &= -\frac{N^2}{4\pi}\frac{g_{\pi/p}}{2}[j_0(\omega)\cdot j_1(\omega)].
\end{aligned}
\tag{2.14}
$$

At spatial infinity, the σ and π ”fields” (i.e. solitons) must vanish:

$$
\sigma_s(r)\Big|_{r\to\infty}=0 \qquad \pi_s(r)\Big|_{r\to\infty}=0 \qquad \pi_p(r)\Big|_{r\to\infty}=0. \tag{2.15}
$$

For the fermion field, one obtains the boundary condition

$$
\begin{aligned}
\mathrm{i}(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{\rm bag}} &= \mathrm{i}g_\sigma\sigma(r)(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{\rm bag}}\\
&\quad - g_\pi\vec{\tau}\vec{\pi}(r)(\vec{\gamma}\hat{r})\gamma_5\psi(r)\Big|_{r=R_{\rm bag}}.
\end{aligned}
\tag{2.16}
$$

This boundary condition is ”sandwiched” between the quark (Fock) states, as was done with the equations of motion.

Several equations valid at $R=R_{\rm bag}$ follow from that expression:

$$
\begin{aligned}
&j_0(R)g_\sigma(f_\pi-\sigma_s(R))-j_1(R)(1-3g_{\pi/p}\pi_p(R))=0\\
&j_0(R)(1+3g_{\pi/p}\pi_p(R))-j_1(R)g_\sigma(f_\pi-\sigma_s(R))=0\\
&j_0(R)-j_1(R)(g_\sigma f_\pi+3g_{\pi/s}\pi_s(R))=0\\
&j_0(R)(g_\sigma f_\pi-3g_{\pi/s}\pi_s(R))-j_1(R)=0.
\end{aligned}
\tag{2.17}
$$

The quark eigenenergy ω will be determined from the compatibility of the boundary conditions (2.14) and (2.16). In this case, instead of a common meson coupling constant g (Eq. (2.2)), flavour- and angular-momentum dependent couplings $g_{\sigma/s}$, $g_{\pi/s}$ and $g_{\pi/p}$ appear. This reflects chiral symmetry breaking. As shown in (2.18) below, this appears naturally when the non-linear system (2.2) is solved using the *Ansätze* (2.3), (2.7) and (2.8), which lead to the system of equations (2.17). One solution for $g_{\pi/p}=\pi_p(R)/3$ gives a trivial solution for g_σ, i.e. $g_\sigma=0$. The other gives

$$
\begin{aligned}
&g_\sigma = J^2+\frac{1}{2}f_\pi J, \qquad g_{\pi/s}=\frac{1-J^2}{6J\pi_s(R)},\\
&g_{\pi/p}=J^2-\frac{1}{3}(J^2+1)\pi_p(R), \qquad \sigma_s(R)=f_\pi\frac{J^4-4J^2+1}{(1+J^2)^2},\\
&J=j_1(R)/j_0(R).
\end{aligned}
\tag{2.18}
$$

The problem is to find a set of solutions of the differential equations for $\{\sigma(r), \pi_s(r), \pi_p(r)\}$, which satisfy the *mathematical* boundary conditions (2.14) and (2.15). The solutions must be compatible with (2.18) which is independent of r. As J presents information on the system of differential equations, one has a strongly correlated algebraic system (2.18) and the system of differential equations (2.11)-(2.13).

The parameters (λ, ν), which enter $\mathcal{L}$ (2.2), are restricted by the symmetry-breaking behaviour of the theory. Usually [4,39], the σ particle is considered to be a 1.2 GeV resonance, whereas the pion "mass" is a parameter which, for simplicity (and lack of knowledge), is assigned the value of the physical pion mass (0.137 GeV). In the present application, these values have also been used, although m_σ and m_π can, in principle, be considered as additional parameters.

This resulted in a system which determined fermion and boson radial functions appearing in *Ansätze*, for example in (2.7) and (2.8).

In Eq. (2.15), the normalization constant N can be expressed in terms of Bessel functions and quark eigenfrequencies ω:

$$N^2 = \frac{1}{R^3}\left[j_0^2(\omega) + j_0^2(\omega) - \frac{2j_0(\omega)j_1(\omega)}{\omega}\right]. \tag{2.19}$$

The radial parts of the quark wave functions appearing in (2.3) are Bessel functions $j_\ell(\omega r/R)$ for any spherical bag with the radius R. At the bag boundary, where $r = R$, these functions have to satisfy the relations (2.18) which combine the quark frequency ω with the coupling constants g_σ, g_π, f_π, etc.

This complex system has been solved using the code COLSYS, the **col**location **sy**stem **s**olver, developed by U. Ascher, J. Christiansen and R. D. Russel from the University of British Columbia and Simon Fraser University, Canada [40]. From the asymptotic behaviour and some earlier experience, the input was simple and a fast convergence has been achieved.

The problem is rather sensitive to the derivative boundary conditions which in all cases involve the coupling constant(s). Although the asymptotic behaviour of the solutions can be inferred from the system itself (see also Ref. 39), the COLSYS is able to handle rather general initial (guess) solutions. The problem parameters can be gradually changed (increased) by using a continuation method in COLSYS.

There are additional chiral-bag-model parameters, the same as those used in the MIT bag, i.e. B, Z_0 and α_s [4-6,29,36,41]. They are related to the bag properties (B, Z_0) and with the effective gluon exchange (α_s), which removes the nucleon (N) - resonance (Δ) mass degeneracy. Some earlier experience (see Ref. 4) suggested that these parameters would remain within typical chiral-bag-model values. Here these parameters are used to fix the N and Δ masses within a 1% accuracy. The numerical values depend on the particular *Ansätze* used. Thus, for example, for one solution displayed in Table 1 one finds: R =5.0, ω=2.10, Z_0=2.6, $B^{1/4}$=0.148 and α_s=1.2.

The solutions are compared against the consistency conditions (2.18) and the iterative procedure is used. The iteration consists in performing a self-consistent calculation: the coupling constants are set to be equal at the beginning, and after every iteration new coupling constants are calculated from (2.18). These new values are replaced in the boundary conditions to calculate new solutions until the matching is achieved.

3. *Results and comments*

From various analyses [3,17-20,23,24,42,43] of the deep inelastic scattering data, one can extract the following values of the quark densities:

$$\begin{aligned} \Delta u &= 0.78 \pm 0.06 \\ \Delta d &= -0.48 \pm 0.06. \end{aligned} \tag{3.1}$$

This leads to the following prediction of the axial vector constants

$$g_A = 1.26 \pm 0.12 \tag{3.2}$$

$$h_A^S = 0.30 \pm 0.12. \tag{3.3}$$

The results (3.2) and (3.3) are in a sense a continuation, or a bridge, which connects the high energy region, in which one encounters current quarks, quark pairs, gluons, etc., with a region in which an effective description in terms of constituent quarks might be possible.

In the constituent quark model (2.1) which was used here, one can calculate both axial vector constants and the proton magnetic moment μ_p. The quantities g_A and μ_p are also known from the low energy electroweak data which give [44]

$$g_A = 1.26 \pm 0.0028 \tag{3.4}$$

$$\mu_p = 2.79284739 \pm 0.00000006. \tag{3.5}$$

The magnetic moment operator is

$$\vec{\mu}(\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{j}_{EM}(\vec{r})). \tag{3.6}$$

Here

$$j^{\mu}_{EM}(r) = \overline{\psi}(r)\gamma^{\mu} Q \psi(r) + \varepsilon_{3ij}\pi_i(r)\partial^{\mu}\pi_j(r) \tag{3.7}$$

and

$$Q = \frac{2}{3} \cdot \frac{1+\tau_3}{2} - \frac{1}{3} \cdot \frac{1-\tau_3}{2}. \tag{3.8}$$

The proton magnetic moment is given by

$$\mu_p = \mu^{(Q)} + \mu_p^{(M)}. \tag{3.9}$$

Here the quark contribution to μ_p is

$$\mu^{(Q)} = \frac{2}{3} \cdot \frac{R}{\omega^4} \cdot \frac{(\omega/2) - (3/8)\sin 2\omega + (\omega/4)\cos 2\omega}{j_0^2(\omega) + j_1^2(\omega) - 2 j_0(\omega) j_1(\omega)/\omega} \tag{3.10}$$

and the "meson" contribution is

$$\mu_p^{(M)} = \frac{16\pi}{3} \cdot \frac{11}{3} \int_{R_{\mathrm{bag}}}^{\infty} r^2 \, \mathrm{d}r [\pi_p(r)]^2 \mu_p. \tag{3.11}$$

The axial-vector coupling constant g_A has the quark contribution

$$\begin{aligned} g_A^{(Q)} &= \langle p \uparrow | \int \mathrm{d}^3\vec{r} \overline{\psi}(\vec{r}) \gamma^3 \gamma^5 \frac{\tau^3}{2} \psi(\vec{r}) | n \uparrow \rangle \\ &= \frac{5}{3} \cdot \frac{1}{3} \cdot \frac{j_0^2(\omega) + j_1^2(\omega)}{j_0^2(\omega) + j_1^2(\omega) - 2 j_0(\omega) j_1(\omega)/\omega}. \end{aligned} \tag{3.12}$$

For the neutron-proton transition one obtains the "meson" contribution:

$$g_A^{(M)} = \frac{5}{3} \cdot \frac{4\pi}{3} \cdot \int_{R_{\mathrm{bag}}}^{\infty} \mathrm{d}r \, r^2 \left\{ (\sigma_s(r) - f_\pi) \left[\pi_p'(r) + \frac{2\pi_p(r)}{r} \right] - \pi_p(r) \sigma_s'(r) \right\}. \tag{3.13}$$

Finally:

$$g_A^{(p)} = g_A^{(Q)} + g_A^{(M)}. \tag{3.14}$$

It is important to note that the axial vector current operator

$$A_3^3(r) = \bar{\psi}(r) \gamma^3 \gamma_5 \frac{1}{2} \tau_3 \psi(r) + \partial_3 \sigma(r) \pi^3(r) - \sigma(r) \partial_3 \pi^3 \tag{3.15}$$

behaves as a vector as long as the chiral symmetry is not broken. From (2.1), one can easily see that it satisfies PCAC as long as all coupling constants (2.18) are equal. The selfconsistent numerical procedure used in solving our model introduces some inequality among coupling constants in (2.18). As this is some kind of the dynamical symmetry breaking, the chiral symmetry is still sufficiently preserved, so that chirality protects against too large g_A values. As one can see in Table 1, g_A does stay relatively close to its chiral symmetry value.

The nonlinear sigma model is usually solved [5-7] using the approximation

$$\exp\left(\frac{\mathrm{i}}{f_\pi} \pi^a \tau^a\right) \cong 1 + \frac{\mathrm{i}}{f_\pi} \pi^a \tau^a. \tag{3.16}$$

These results, obtained in TDIA, seem to indicate that such an approximation destroys the chiral vector current property of the axial vector current, although it preserves the PCAC. The chirality does not sufficiently protect the g_A value, which is usually too large, $g_A > 1.45$, in such models [5,6].

The chirality, which protects g_A in the present model, leads also to the reasonable μ_p values. In both cases, the "meson" phase, which was introduced in (2.1) in order to construct a chiral-invariant model, works in the right direction. The "meson" phase contributions are proportionally much larger in the case of μ_p, as it should be. On the other hand, the "meson" phase, which is an isovector, does not contribute to the isoscalar h_A^S. This lowers h_A^S values, as is visible in Table 1.

The nucleon states used in our model are the well-known SU(6) spin-isospin symmetry eigenvectors. Therefore, the quark phase contributions have to satisfy the naive quark model ratio [3], according to which

$$\eta = \frac{h_A^S(Q)}{g_A(Q)} = \frac{3}{5} = 0.6. \tag{3.17}$$

Thus, the MIT-bag model [5] would always predict a too large value for h_A^S.

The chirality of the model (2.1) is its most important feature. Its "meson" phase, introduced to satisfy the chiral symmetry, leads to a lower ratio (3.17). For the last column in Table 1 one finds

$$\eta = 0.296 \tag{3.18}$$

what is quite close to the experimental value (see (3.2), (3.3))

$$\eta = 0.238 \pm 0.070. \tag{3.19}$$

It is probably not accidental that our best h_A^S and/or η values in Table 1 are paired with the best g_A and μ_p values which differ from the experimental values by 1.5 % and 0.7 %, respectively.

It seems that a constituent quark model might be compatible with the deep inelastic scattering data as long as its chiral invariance [1] is only dynamically and gently (term $f_\pi m_\pi \sigma$ in (2.2) for example) broken.

Acknowledgements

One of us (D.H.) wishes to thank Andrew Kurn (Simon Fraser University - Computing Department) and Davor Grgić (University of Zagreb) for their assistance in the software application. D.T. would like to thank for the hospitality of the Theory Group (professor L. Fonda), University of Trieste, Italy. This work was supported by the Ministry of Science and Technology of the Republic of Croatia, under the contract number 119222.

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STATIČKA SVOJSTVA NUKLEONA U PRISTUPU POTAKNUTOM TAMM-DANCOFFOVOM APROKSIMACIJOM

Primjenom postupka potaknutog Tamm-Dancoffovom aproksimacijom, riješena je inačica modela kiralne vreće koja sadrži dinamiku linearnog σ modela, i kvantizirana je primjenom operatora konstitutivnih kvarkova. Model daje dobre vrijednosti za konstantu aksijalnog vezanja uz vrijednost kao i za magnetski moment protona. Te vrijednosti, koje su u suglasju s vrijednostima funkcija kvarkovske gustoće, posljedica su kiralnog značaja modela.