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LETTER TO THE EDITOR

ELASTIC PION SCATTERING ON POLARIZED ^{3}He IN THE ENERGY RANGE OF THE Δ-RESONANCE

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The π-^{3}He interaction for the polarized target is studied in the fixed-centers approximation with all rescatterings included. Only the P_{33} wave is taken for the πN interaction. The nuclear wave function is taken either as a sum of Gaussians or as a Faddeev wave function in the s-wave approximation. The asymmetries for elastic π^+ ^{3}He scattering at the lab energies T_π =142, 180 and 256 MeV are calculated. The results are compared with experimental data.

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The investigation of pion scattering off light nuclei requires reliable theoretical description. The exact relativistic treatment of systems with four or more particles is beyond present calculational possibilities. Therefore, simplifications are unavoidable. Current studies of π-trinucleon reactions are mostly done in the framework of the optical potential model [1]. This may be a good approach at low energies when the pion wavelength λ_π is larger than the internucleon distance R. However, at energies above 100 MeV, when $\lambda_\pi \leq R$, the optical potential model does not seem natural and alternative approaches deserve attention.

From this point of view the energy region of the Δ-resonance exhibits two big advantages. First, at these energies the πN interaction can be well approximated by the P_{33} wave, i.e., it is simple and essentially reduced to a separable form.

Second, the pion energy is still considerably smaller than that of the nucleon. Therefore one can hope that a reasonable approximation might be to neglect the nucleon recoil.

In this limit, the amplitude of the NN interaction vanishes and the problem becomes simplified to the fixed-scattering-centers approximation. It is well-known that this approximation gives an accuracy of about 10 - 20% for πd interaction [2]. In Ref. 3, the πd breakup was studied including the summation of relativistic ladder diagrams, and it has been shown that taking into account of the nucleon recoil, insignificant improvement was obtained as compared with the static approximation. Of course, the accuracy is decreasing with the increase of momentum transfer. However, any attempt to take the nucleon recoil into account inevitably requires an ultraviolet cutoff, so that in principle a better accuracy can be achieved only at the price of an extra parameter.

In Ref. 4, a model based on the fixed-centers approximation for elastic π^3H and π^3He reactions, with full account of multiple pion rescattering, was introduced. This paper reports on the application of the model to polarized targets.

Neglecting the NN interaction and other waves in the πN interaction except the P_{33}, the determination of the amplitude of the interaction with a nucleus is reduced to a summation of the diagrams shown in Fig. 1.

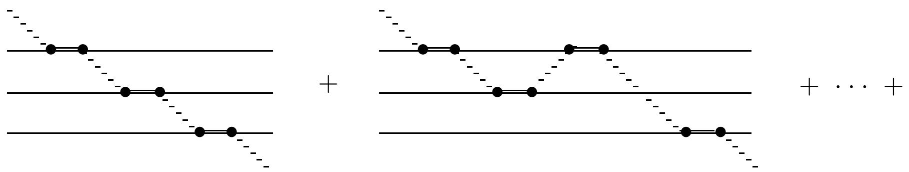


Fig. 1. Graphic representation of successive rescatterings of the π meson with production of the Δ_{33}-resonance. The dotted lines correspond to the π meson, thick lines correspond to the Δ_{33}-resonance, and thin lines correspond to nucleons.

The exact treatment of these graphs requires the introduction of 216 amplitudes. In our numerical calculations, the spin-tensor interaction in the elementary block of Fig. 2 was replaced by an averaged one. As a result, the number of independent amplitudes is reduced to twenty seven.

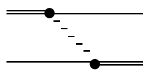


Fig. 2. The potential W_{ik}. The notation is the same as in Fig. 1.

The basic Faddeev-like amplitudes are M_{ik} where i and k refer to the number of the initial and final nucleons $(i, k = 1, 2, 3)$. Each M_{ik} is a 3 ×3 matrix in both spin and isospin. In the fixed centers approximation, they are determined by a system of linear equations

$$M_{ik} = R_1 P^{(i)} \delta_{ik} + R_1 \sum_{l \neq i} P^{(i)} W_{il} M_{lk} \,. \tag{1}$$

Here $R_1(E) = (m_\Delta - m_N + \varepsilon_b - k_0 - \mathrm{i}0)^{-1}$ and k_0 is the pion energy. $P^{(i)}$ are projectors onto states $I = J = 3/2$ for the system of pion plus the i-th nucleon.

The "potentials" W_{ik} describe intermediate pion propagation. They are complex:

$$W(x) = \frac{\lambda^2}{3(2\pi)^2} \int \mathrm{d}^3 q \frac{\mathbf{q}^2 \exp(\mathrm{i}\mathbf{q}\mathbf{x})}{\mathbf{q}^2 + m_\pi^2 - (k_0 - \varepsilon_b)^2 - \mathrm{i}\,0}, \quad \lambda^2 = \frac{6\pi\Gamma_\Delta}{k_{\mathrm{res}}^3}. \tag{2}$$

Here ε_b is the nucleus binding energy, Γ_Δ is the resonance width and k_{res} is the pion momentum at the resonant energy. Once the system (1) is solved, the $\pi\,^3\mathrm{He}(^3\mathrm{H})$ amplitudes with a given value of the total isospin T are found as

$$\mathcal{A}^{(T)} = \mathcal{A}_1^{(T)} + \mathcal{A}_2^{(T)} = \frac{3\lambda^2}{\sqrt{2k_0 2k_0^{'}}} \sum_{S=1/2,3/2} (\mathbf{k}^{'} P^{(S)} \mathbf{k}) \int \mathrm{d}^3 x_1 \mathrm{d}^3 x_2 |\Psi_A(\mathbf{x}_{ik})|^2$$

$$\times (\exp(\mathrm{i}(\mathbf{k} - \mathbf{k}^{'})\mathbf{x}_1) \langle M_{11(T,S)} \rangle + 2\exp(\mathrm{i}(\mathbf{k}\mathbf{x}_1 - \mathbf{k}^{'}\mathbf{x}_2)) \langle M_{21(T,S)} \rangle). \tag{3}$$

Here $\mathcal{A}_{1(2)}^{(T)}$ refer to the 1st(2nd) term in the brackets; $\langle M_{ik} \rangle$ are the amplitudes averaged over spin and isospin of the $^3\mathrm{He}(^3\mathrm{H})$ nuclei. The standard spin-non-flip and spin-flip amplitudes f and g are expressed via $\mathcal{A}_{1(2)}^{(T)}$ as follows

$$\begin{aligned} f^{(T)} &= \frac{1}{3} \left(\mathcal{A}^{(T)}{}_{1,S=1/2} + \mathcal{A}^{(T)}{}_{2,S=1/2} + 2(\mathcal{A}^{(T)}{}_{1,S=3/2} + \mathcal{A}^{(T)}{}_{2,S=3/2} \right) \\ g^{(T)} &= \tfrac{1}{3} \left(\mathcal{A}^{(T)}{}_{1,S=1/2} + \mathcal{A}^{(T)}{}_{2,S=1/2} - (\mathcal{A}^{(T)}{}_{1,S=3/2} + \mathcal{A}^{(T)}{}_{2,S=3/2}) \right). \end{aligned} \tag{4}$$

Here we used the approximation $m_N \to \infty$. However, both the kinematical correction for the finite m_N and the contribution from the s-wave πN interaction can in principle be taken into account.

In our normalization, the differential cross-section on the unpolarized target is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta_\pi} = \frac{k_0^2}{4\pi^2} \left(|f^{(T)}|^2 + |g^{(T)}|^2 \right). \tag{5}$$

The asymmetry for the polarized target is

$$A_y = \frac{\sigma\uparrow - \sigma\downarrow}{\sigma\uparrow + \sigma\downarrow} = \frac{2\,\mathrm{Im}(f^{(T)} g^{*(T)})}{|f^{(T)}|^2 + |g^{(T)}|^2}. \tag{6}$$

To see the influence of the ground state structure of the nucleus, we performed calculations with two different wave functions Ψ. The first choice was a sum of two Gaussians

$$|\Psi|^2 = \sum_{j=1,2} N_j \exp(-\alpha_j \sum_{i<k} x_{ik}^2) \tag{7}$$

with the parameters α_j chosen to give the best fit for the elastic cross-section at large angles (see Ref. 5).

Our second choice was the Faddeev wave function obtained as a solution of the Faddeev 3-nucleon equation with an NN potential taken as a sum of the Coulomb potential and a strong interaction potential in the *s*-wave approximation [6,7].

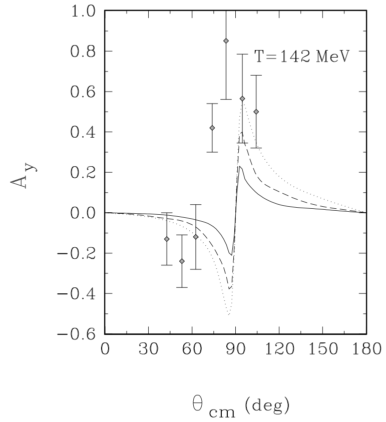
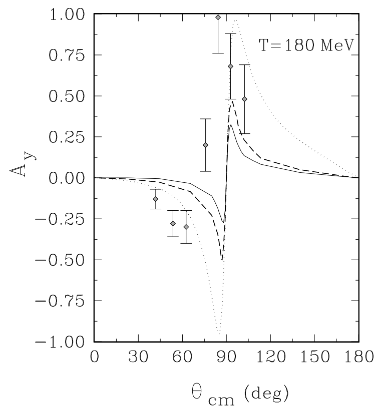


Fig. 3. Asymmetry for elastic π^+ *scattering from polarized* ^{3}He. *The solid line corresponds to the Faddeev wave function, dashed line corresponds to a sum of two Gaussians with all rescatterings of pion taken into account. The dotted line corresponds to a sum of two Gaussians for two rescatterings of pion taken into account. The experimental data are taken from Ref. 8.*



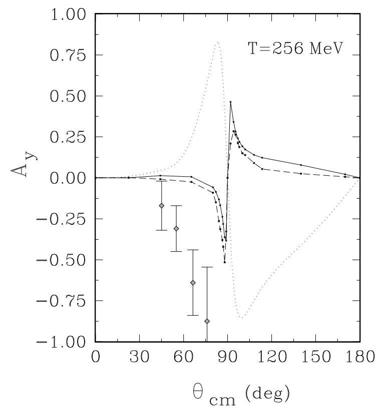


Fig. 4. Asymmetry for elastic π^+ *scattering from polarized* ^{3}He *at 180* MeV. *The notation is the same as in Fig. 3.*

Fig. 5 (right). Asymmetry for elastic π^+ *scattering from polarized* ^{3}He *at 256* MeV. *The notation is the same as in Fig. 3.*

In Figs. 3–5, the calculated asymmetries are shown for the reaction $\pi^+\,^3$He at the pion lab. energy T_π =142, 180 and 256 MeV together with the experimental data [8]. To demonstrate the influence of multiple π rescatterings, also the results which take into account only two rescatterings are shown (for the wave function as a sum of two Gaussians).

The agreement with the experimental data for the asymmetry is found to be considerably worse than for the differential cross-sections. This is not astonishing, since, as stressed in all previous publications on the problem, the asymmetry is extremely sensitive to the details of the interaction.

For energies 142 and 180 MeV, the calculated asymmetry is following the experimental pattern, but it is shifted toward larger angles. This is not surprising. In our approximation (pure P_{33} πN interaction, infinite m_N), the amplitude f vanishes at $\theta = 90^\circ$. Addition of the s-wave interaction and kinematical corrections for the finite m_N would both shift this zero towards smaller angles and thus would improve the agreement with the experimental data. At 256 MeV, there seems to be no agreement with the experimental data at all. However, as found in Ref. 1, the seemingly drastic change in the angular dependence of the asymmetry at this energy is in fact due to a quite small shift in the location of the zeros of the real and imaginary parts of the amplitude f. It was also found there that this shift could be effectively achieved by the inclusion of a small angle-independent term to f. Thus, we expect that inclusion of the s-wave πN interaction will improve the situation at 256 MeV considerably.

From the figures, one can conclude that one observes a very strong effect of multiple rescatterings. For lower energies, the inclusion of multiple rescatterings substantially reduces the asymmetry, and at 256 MeV even completely changes its behaviour. Introduction of a very sophisticated Faddeev wave function does not seem to change the results in any significant manner. Application of our model to the study of the asymmetry requires additional improvements, which include introduction of the s-wave πN interaction and finite m_N kinematics. Work in this direction is now in progress.

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ELASTIČNO RASPRŠENJE PIONA NA POLARIZIRANOM ^{3}He U ENERGIJSKOM PODRUČJU Δ-REZONANCIJE

Razmatra se π-^{3}He interakcija s polariziranom metom u približenju nepomičnih središta, uključujući sva višestruka raspršenja. Za πN interakciju uzima se samo P_{33} val. Nuklearna se valna funkcija uzima bilo kao zbroj Gaussovih funkcija ili kao Faddeeva valna funkcija u aproksimaciji s-vala. Izračunali smo asimetrije za elastično π^{+3}He raspršenje za laboratorijske energije T_π =142, 180 i 256 MeV. Rezultati se uspoređuju s eksperimentalnim podacima.