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QUARKONIUM IN THE WILSON LOOP FORMALISM: NEW PERSPECTIVES

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We review the present knowledge on the heavy-quark interaction. The framework is the NRQCD effective theory and the interaction is expressed in terms of Wilson loop chromoelectric and chromomagnetic insertions.

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1. *Introduction*

The experimental evidence for heavy-quark bound states, like $b\bar{b}$, $c\bar{c}$, ..., shows that all splittings are considerably less than the masses, suggesting that all dynamical energy scales of these systems are small with respect to the quark masses. As a consequence, the quark velocities v are small and these systems can be considered as nonrelativistic. Hence, the hierarchy of the scales is then a typical one of a nonrelativistic system. We denote the mass of the heavy quark m, the quark momenta scale mv and the quark energies mv^2. The situation in the gluon sector is trickier, but the binding interaction is essentially characterized by the same energy scale distribution. Therefore, the dominant gluon interaction among heavy quarks appears "instantaneous". A potential picture should hold, at least in the first approximation, and the energy levels can be obtained by solving the corresponding Schrödinger equation. In particular, for infinitely heavy quarks, the spin splittings

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vanish, and the system can be described only by means of a single static central potential.

Indeed, potential models have been quite successful [1] in explaining the hadron spectrum. However, since these models are phenomenological, their connection with the QCD parameters is hidden, the scale at which they are defined is not clear, and they cannot be systematically improved. In this paper, we outline the kind of *rigorous description of quarkonium* that can be obtained from QCD.

Several perturbative evaluations of the quarkonium potential have been performed in the last twenty years [2] and recently the complete α_s^2 corrections to the levels have been calculated [3]. The main difficulty in these calculations is the inclusion of nonperturbative contributions which not only set a bound on the reachable precision, but turn out to be relevant for quantitative predictions even on the bottomonium spectrum. A typical approach is to consider all nonperturbative physics to be encoded into few local condensates. This corresponds to the assumption that the typical length associated, for instance, with the nonperturbative gluodynamics is larger than any other length scale of the system and can be put equal to infinity. In a pure analytic calculation, this statement is also dictated by the necessity to have a small number of nonperturbative parameters and to maintain predictability. Even for the ground state, this assumption is doubtful, but surely it does not hold for large-radii quarkonia (i.e. excited heavy mesons) where the nonperturbative gluonic length cannot be considered large with respect to the size of the bound state. Therefore, a proper treatment of nonperturbative effects includes *non-local* condensates. Another way to say the same is that the contribution to the levels associated with the local gluon condensate is proportional to $n^6\langle\alpha_s F^2(0)\rangle$. By increasing the principal quantum number n beyond the ground state, it grows very soon out of control.

A way of including nonperturbative effects in the evaluation of the quarkonium spectrum is lattice QCD. This technique is becoming more and more successful and in the near future it is expected to be the only competitive one (see for instance Ref. 4 and references therein). There are different ways in which lattice QCD calculations can be performed. We mention lattice NRQCD [5] in which the quarkonium spectrum is directly evaluated on the lattice using an effective action derived from QCD by an expansion in the quark velocities. Very close is the approach we will discuss in the following where the QCD Lagrangian is replaced by its effective nonrelativistic formulation *before* doing any lattice evaluation. All nonperturbative physics is encoded in this way in the Wilson loop made up by the quark trajectories and in field strength insertions on it. These (nonlocal) objects are then evaluated on the lattice. We call this the *Wilson loop approach.* The advantage of it is that our expressions are safer to handle for lattice purposes, since they are less affected by finite size effects, and easier to treat also for analytic purposes like the implementation of the vacuum models. In particular, we get an expression for the heavy quark-antiquark potential. In the limit where the insertions of two field strengths on the Wilson loop can be approximated by a local condensate, one gets back the "improved" perturbative expression discussed above.

The (Wegner)-Wilson loop formalism has a long story. It was first suggested by

K. Wilson [6] that the object called after him Wilson loop would be the relevant one in order to describe confinement. The strong coupling expansion suggested an area law behaviour, further confirmed by lattice simulations, which were born with this pioneering work. In Ref. 7, it was shown how to relate rigorously the static Wilson loop with the static quark-antiquark potential. A few years later, also spin-dependent corrections to the potentials where expressed in terms of Wilson loop and chromoelectric and chromomagnetic field insertions [8]. In particular, it was proven that a nonperturbative behaviour in the static potential must also give rise to nonperturbative spin-dependent corrections. Non spin-dependent corrections were treated in the same framework some time later [9]. This was the situation at the beginning of the '90 (see Ref. 1). Several problems were still open. No relativistic formulation in terms of the Wilson loop was available. There was an apparent mismatch between the Eichten–Feinberg–Gromes expression for the spin dependent sector of the potential (analytic in the quark masses) and the perturbative one-loop estimate (containing logarithms of the quark masses). The inclusion of non-potential terms was not understood. While not all these problems have been solved, some remarkable progress has been achieved in the last years. Attempts in the direction of a relativistic formulation have been done in Ref. 10. In Ref. 11, it was shown that, by performing properly the matching between the effective theory and QCD, Wilson coefficients carrying logarithms of the quark masses appear in the Eichten–Feinberg–Gromes expression for the potential. The inclusion of non-potential contributions in this framework is now underway [12] and the way to precision calculations in quarkonium seems finally to be open. Here we only mention that this goal is met by a new effective theory built from NRQCD, where explicitly potential and non-potential terms have been separated [13]. Finally, all the Wilson-loop averages relevant for the potential have been calculated on the lattice and the bottomonium and charmonium spectra have been calculated with good agreement with the data [14,15].

In the meantime there has been considerable progress in the building of a non-relativistic effective theory from QCD, mainly due to the success of heavy-quark effective theory (HQET) in describing heavy-light systems [15]. Wilson coefficients have been calculated to higher order and the role played by reparameterization invariance has been better understood.

Here, we summarize the present level of understanding of the heavy-quark potential which contributes to the energy levels of quarkonium to the order v^4. The framework is NRQCD, the tool of the Wilson approach.

2. NRQCD

In order to define an effective theory, we typically need three ingredients: an effective Lagrangian, a regularization scheme and therefore a matching scale, and a power counting set of rules. The effective theory we will use is NRQCD [5]. Let us discuss its key ingredients.

The NRQCD Lagrangian is obtained from QCD by expanding with respect to the heavy-quark masses. The matching with QCD is performed like in HQET [17,18]. We emphasize that in order to build up the effective Lagrangian from the QCD Lagrangian, we have to ignore the specific dynamical problem we are dealing with and expand with respect to the heavy-quark masses which are explicit parameters of QCD. Typically, the effective Lagrangian turns out to be the sum of a pure gauge part $L_{\rm g}$ plus two, four, ... fermion terms ($L_{\rm 2f}$, $L_{\rm 4f}$, ...). Since we are interested in two-body bound states, we will take into account only two- and four-fermion terms (terms involving more fermions will contribute only in intermediate states). Therefore, the NRQCD Lagrangian obtained from QCD by expanding with respect of the mass m_1 of a heavy quark and the mass m_2 of a heavy antiquark is given by [17,18]

$$L = L_{\rm 2f} + L_{\rm 4f} + L_{\rm g}, \tag{1}$$

where to the order $1/m^2$ and up to the field redefinitions

$$\begin{aligned} L_{\rm 2f} \;=\;& Q_1^\dagger \left({\rm i}D_0 + c_2^{(1)} \frac{\mathbf{D}^2}{2m_1} + c_4^{(1)} \frac{\mathbf{D}^4}{8m_1^3} + c_F^{(1)} g \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_1} + c_D^{(1)} g \frac{\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}}{8m_1^2} \right. \\ & \left. + {\rm i}c_S^{(1)} g \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})}{8m_1^2} \right) Q_1 + O\left(\frac{1}{m_1^3}\right) \\ +\;& Q_2^\dagger \left(-{\rm i}D_0 + c_2^{(2)} \frac{\mathbf{D}^2}{2m_2} + c_4^{(2)} \frac{\mathbf{D}^4}{8m_2^3} + c_F^{(2)} g \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_2} - c_D^{(2)} g \frac{\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}}{8m_2^2} \right. \\ & \left. - {\rm i}c_S^{(2)} g \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})}{8m_2^2} \right) Q_2 + O\left(\frac{1}{m_2^3}\right), \end{aligned} \tag{2}$$

Q_j are the heavy-quark fields and the covariant derivative is defined as $D_\mu = \partial_\mu + {\rm i}gA_\mu^a T^a$. The explicit form of $L_{\rm 4f}$ is given in Ref. 18. $L_{\rm g}$ is the SU(3) Yang–Mills Lagrangian modified in order to give rise to an effective $\alpha_{\rm s}$ running with 2 (heavy) flavours less.

The effective Lagrangian (1) is not renormalizable. Therefore, it is necessary to regularize it. In a given regularization scheme, the reproduction of the correct ultraviolet regime of QCD is obtained by means of the Wilson coefficients. The effective Lagrangian is complete only once these coefficients are given. The Wilson coefficients are evaluated at a matching scale where perturbation theory still holds. They encode the ultraviolet regime of QCD up to a given scale μ, order by order in the coupling constant $\alpha_{\rm s}$. Renormalization group (RG) transformations should be used in order to resum leading log contributions ($\sim \ln m/\mu$). The matching coefficients are known in the literature to different levels of precision (i.e. to different orders in the coupling constant) (see Refs. 17, 18 and 19). Here we remember only that reparameterization invariance [17] (i.e. the invariance of the effective Lagrangian with respect a variation of v) fixes $c_2^{(j)} = c_4^{(j)} = 1$.

Heavy-quark bound states are characterized by a dynamical dimensionless parameter, the quark velocity v. As explained in the Introduction, this parameter is small. From phenomenological potential models [20] and from lattice studies [14], we get the usually accepted values of $\langle v_b^2 \rangle \approx 0.07$ for the bottomonium system and $\langle v_c^2 \rangle \approx 0.24$ for the charmonium system. This allows the ordering of the energy scales of the problem, m, mv and mv^2. The first scale has been explicitly integrated out in the Lagrangian (1) and, as discussed above, the contributions coming from it are carried order by order in α_s by the Wilson coefficients. The last two scales, sometimes known by the names "soft" and "ultrasoft", respectively, are still mixed up. The relation between v and the QCD parameters is generally unknown. It is expected to be the result of the perturbative and nonperturbative effects. For infinitely heavy quarks, v coincides with α_s like in QED where, for instance in the hydrogen atom, the hyperfine structure constant α is equal to the electron velocity. The main point is that the existence of this small dynamical parameter allows to set up power counting rules for the operators in the effective Lagrangian. For the sake of simplicity, we reproduce here the rough power counting argument of Ref. 5, keeping in mind that an exact power counting cannot be done until soft and ultrasoft degrees of freedom have not been completely disentangled. Noticing that the number operator for heavy quarks, $\int \mathrm{d}^3x\, Q^\dagger(x)Q(x)$, does not depend on v, one gets $Q \sim (mv)^{3/2}$. The kinetic energy $\int \mathrm{d}^3x\, Q^\dagger(x)\frac{\mathbf{D}^2}{2m}Q(x)$ is by definition of the order mv^2 and, therefore, $\mathbf{D} \sim mv$ (i.e. the contribution of $\mathbf{D}$ to the levels is of order mv). Finally, using the equation of motion, one gets the other counting rules $D_0 \sim mv^2$, $gA_0 \sim mv^2$ $g\mathbf{A} \sim mv^3$, $gE \sim m^2v^3$ and $gB \sim m^2v^4$. With respect to these rules, the Lagrangian of Eq. (2) is accurate up to the order $O(v^4)$ of the levels (or up to $O(v^2)$ with respect to the leading contribution). This should guarantee a rough 10% accuracy on the absolute value of the levels.

Concluding this section, we stress that the NRQCD power counting defined above is not the same as that one used in HQET. In particular, in HQET, the kinetic energy $\mathbf{D}^2/2m$ is suppressed by a factor Λ_{QCD}/m with respect to the operator D_0, while in NRQCD, the two operators are of the same order. As a consequence, the heavy-quark propagator contains in NRQCD a kinetic part which is absent in HQET where the heavy-quark propagator is static. In a very general way one can say that these differences are due to the fact that, even if the effective Lagrangian is essentially the same, the physical problem is different, and it is the physical problem which defines the counting rules.

3. *The Wilson loop formalism*

The next step is to derive the heavy-quark interaction in the so-called Wilson-loop formalism. In this context, the use of the effective Lagrangian (2) (with tree level matching) was first suggested by L. S. Brown and W. I. Weisberger [7], later by E. Eichten and F. Feinberg [8]. A one loop RG improved calculation was done in Ref. 11. In those papers, an expansion is performed around the static solution. Here, we adopt the approach of Ref. 9, where the kinetic energy was kept during all calculations.

The 4-point gauge-invariant Green function G associated with the Lagrangian (1) is defined as

$$G(x_1,y_1,x_2,y_2) = \langle 0|Q_2^\dagger(x_2)\phi(x_2,x_1)Q_1(x_1)Q_1^\dagger(y_1)\phi(y_1,y_2)Q_2(y_2)|0\rangle,$$

where $\phi(x_2,x_1) \equiv \exp\left\{-\mathrm{i}g\int\limits_0^1 \mathrm{d}s\,(x_2-x_1)^\mu A_\mu(x_1+s(x_2-x_1))\right\}$ is a Schwinger line added to select the singlet-state contribution. For large time separations the string vanishes. After integrating out the heavy-quark fields, Q_j and $Q_j^\dagger$, G can be expressed as a quantum-mechanical path integral over the quark trajectories [10]:

$$G=\int\limits_{y_1}^{x_1}\mathcal{D}z_1\mathcal{D}p_1\int\limits_{y_2}^{x_2}\mathcal{D}z_2\mathcal{D}p_2\,\exp\left\{\mathrm{i}\int\limits_{-T/2}^{T/2}\mathrm{d}t\sum_{j=1}^{2}\mathbf{p}_j\cdot\mathbf{z}_j-\frac{p_j^2}{2m_j}+\frac{p_j^4}{8m_j^3}-\mathrm{i}\int\limits_{-T/2}^{T/2}\mathrm{d}t\,U\right\}$$

where $z_j = z_j(t)$ and $y_2^0 = y_1^0 \equiv -T/2$, $x_2^0 = x_1^0 \equiv T/2$. The function U describes the heavy-quark interaction. Since the kinetic energy has been properly isolated, it is consistent with the counting rules given in the previous section to expand the interaction around the static quark trajectories $z_1 = (t,\mathbf{r})$ and $z_2 = (t,\mathbf{0})$. Moreover, we define the heavy quark-antiquark potential as $V = \lim\limits_{T\to\infty}\frac{1}{T}\int\limits_{-T/2}^{T/2}\mathrm{d}t\,U$, with the warning that, since the soft and ultrasoft degrees of freedom have not been disentangled in NRQCD, non-potential terms could still contribute to some extent to V.

Working out this expression [9,10], we get

$$\begin{aligned}
V &= V_0 + \text{spin independent terms} - \frac{1}{m_1m_2}(d_{ss}+C_F\,d_{vs})\,\delta^3(r)\\
&+\frac{1}{8}\left(\frac{c_D^{(1)}}{m_1^2}+\frac{c_D^{(2)}}{m_2^2}\right)(\Delta V_0(r)+\Delta V_{\mathrm{a}}^E(r))+\frac{1}{8}\left(\frac{c_F^{(1)}}{m_1^2}+\frac{c_F^{(2)}}{m_2^2}\right)\Delta V_{\mathrm{a}}^B(r)\\
&+\left(\frac{\mathbf{S}^{(1)}\cdot\mathbf{L}^{(1)}}{m_1^2}+\frac{\mathbf{S}^{(2)}\cdot\mathbf{L}^{(2)}}{m_2^2}\right)\frac{2c_F^+V_1'(r)+c_S^+V_0'(r)}{2r}+\frac{\mathbf{S}^{(1)}\cdot\mathbf{L}^{(2)}+\mathbf{S}^{(2)}\cdot\mathbf{L}^{(1)}}{m_1m_2}\frac{c_F^+V_2'(r)}{r}\\
&+\left(\frac{\mathbf{S}^{(1)}\cdot\mathbf{L}^{(1)}}{m_1^2}-\frac{\mathbf{S}^{(2)}\cdot\mathbf{L}^{(2)}}{m_2^2}\right)\frac{2c_F^-V_1'(r)+c_S^-V_0'(r)}{2r}+\frac{\mathbf{S}^{(1)}\cdot\mathbf{L}^{(2)}-\mathbf{S}^{(2)}\cdot\mathbf{L}^{(1)}}{m_1m_2}\frac{c_F^-V_2'(r)}{r}\\
&+\frac{c_F^{(1)}c_F^{(2)}}{m_1m_2}\left(\frac{\mathbf{S}^{(1)}\cdot\mathbf{r}\,\mathbf{S}^{(2)}\cdot\mathbf{r}}{r^2}-\frac{\mathbf{S}^{(1)}\cdot\mathbf{S}^{(2)}}{3}\right)V_3(r)\\
&+\frac{\mathbf{S}^{(1)}\cdot\mathbf{S}^{(2)}}{3m_1m_2}\left(c_F^{(1)}c_F^{(2)}V_4(r)-12\,(d_{sv}+C_F\,d_{vv})\,\delta^3(r)\right).
\end{aligned}\tag{3}$$

$\mathbf{S}^{(j)} = \boldsymbol{\sigma}^{(j)}/2$ and $\mathbf{L}^{(j)} = \mathbf{r} \times \mathbf{p}_j$ are the spin and orbital angular-momentum operators, respectively. The matching coefficients d_ℓ come from the 4-fermion sector $L_{4\mathrm{f}}$ [18] and $2c^{\pm}_{F,S} \equiv c^{(1)}_{F,S} \pm c^{(2)}_{F,S}$. The static potential V_0 is

$$V_0(r) = \lim_{T\to\infty} \frac{\mathrm{i}}{T} \ln\langle W(\Gamma_0)\rangle, \qquad W(\Gamma) \equiv \mathrm{P}\,\exp\left\{-\mathrm{i}g \oint_\Gamma dz^\mu A_\mu(z)\right\}. \quad (4)$$

where Γ is the loop made up by the quark trajectories z_1 and z_2 and the endpoint Schwinger strings, and the static loop Γ_0 is a $r\times T$ rectangle. The bracket means the colour trace of the average over the gauge fields weighted by the gluon Lagrangian L_{g}. The "potentials" V_1, V_2, ... are scale-dependent gauge field averages of electric and magnetic field-strength insertions in the static Wilson loop (see Refs. 14 and 19).

Since the spin-independent corrections come from the terms D_0 and $\mathbf{D}^2/2m$ of the Lagrangian (2), whose matching coefficients are protected by reparameterization invariance, they are scale independent. An evaluation of the so-called momentum-dependent corrections can be found in Ref. 9. Terms involving logarithms of the quark masses are present. As a consequence of the matching procedure, they are all encoded in the matching coefficients. Therefore, as argued in Ref. 11, the correct handling of the matching allows finally the agreement between the potential derived here, evaluated in the perturbative regime, with the traditional QCD one-loop perturbative calculation [3,21]. For a careful discussion of this point see Ref. 19.

The "potentials" are known exactly (up to a given order in α_{s}) only in the perturbative regime, i.e. in the short-range behaviour. Nevertheless, there exist some exact relations between them which hold at any range. For instance, from Lorentz invariance it follows that [8]

$$V_0'(r) + V_1'(r) - V_2'(r) = 0. \quad (5)$$

Since for reparameterization invariance $c^{(j)}_S = 2c^{(j)}_F - 1$ [17], equation (5) holds at any scale μ. Similar relations exist for the momentum dependent "potentials" of Ref. 9. Moreover, the scale independence of the potential (3) (i.e. $\mu \mathrm{d}V/\mathrm{d}\mu = 0$) establishes several relations between quantities at different renormalization scales [14].

The potentials V_0, V_1, ... are suitable for direct lattice computation and for *analytic evaluation inside a QCD vacuum model*. The first possibility relies on the fact that all dynamical quantities are expressed in terms of field-strength insertions on a static Wilson loop which is an object traditionally measured on the lattice. Such an analysis has been performed in Ref. 14. All "potentials" have been measured on the lattice and the previously mentioned exact relations have been used to check the accuracy of the results. Using the parameterized form obtained in this way for the "potentials", the bound state equation has been solved and the bottomonium and charmonium spectra evaluated. The agreement with the experimental data is found to be quite good. On the other hand, an analytic evaluation of the heavy-quark potential inside a QCD vacuum model turns out to be very convenient in this framework since by means of functional derivatives, all averages of the field-strength insertions on the Wilson loop can be expressed in terms of the average of the non-static Wilson loop alone. This is usually a quantity provided by QCD vacuum models. A study of different models in this framework has been done in Refs. 15 and 22, where a comparison with the existing lattice data has also been provided. Up to now, they are not accurate enough to really discriminate between different models. More precise lattice measurements will be performed in the near future providing in this way strong constraints on all the infrared QCD models with predictability on the long-range quarkonium interaction.

Finally, we emphasize that until soft and ultrasoft degrees of freedom have are not disentangled, an exact value in v cannot be assigned to each term of the effective Lagrangian and the power counting has to be interpreted at the leading order. For instance, the $O(1)$ NRQCD Lagrangian, $L = Q_1^\dagger \left(\mathrm{i}D_0 + \partial^2/2m_1\right) Q_1 + Q_2^\dagger \left(-\mathrm{i}D_0 + \partial^2/2m_2\right) Q_2$, does not contribute to (3) only with the static potential V_0. Since the corresponding Wilson loop $\mathrm{P}\exp\left\{-\mathrm{i}g\oint_\Gamma \mathrm{d}z^0 A_0(z)\right\}$ is a function of the non-static loop Γ, its expansion produces also spin independent terms of the order $O(v)$.

4. Comments and outlook

In the framework of NRQCD, we have shown how to get the complete order $O(v^4)$ expression of the heavy quark-antiquark potential in terms of the field-strength insertions on a static Wilson loop. This has several advantages. Non-perturbative contributions can be evaluated by means either of traditional lattice calculations or of different QCD vacuum models. Having worked out the matching procedure, we find that the potential of Eq. (3) is consistent in the short range with the existing perturbative calculations and with the lattice data [19]. These are sensitive to one loop and in some cases to the next to leading correction, too. We stress here that terms proportional to the static potential in Eq. (3) have to be protected from the running. For instance, using Eq. (5) and reparameterization invariance, the spin-orbit interaction term $\sim 2C_F^+ V_1' + C_S^+ V_0'$ can be written as $2C_F^+ V_2' - V_0'$ where it is apparent that no Wilson coefficient multiplies the static potential. Moreover, we can draw the following consequence about V_2': either V_2' does not contain any nonperturbative contribution at all (as present lattice data seem to suggest) or, if V_2' contains some nonperturbative contributions, they have to satisfy the RG equation $\dfrac{\mathrm{d}}{\mathrm{d}\mu} C_F^+ V_2' = 0$. The same argument suggests for the Darwin term that $\Delta V_{\mathrm{a}}^E = -\Delta V_0 +$ perturbative contributions. This gives some constraints on the QCD vacuum models which, introducing at some point some approximations, lose scale invariance.

All corrections to the effective Lagrangian discussed here are relevant in order to obtain the quarkonium spectrum with an accuracy of $O(v^4)$. For some Wilson coefficients, only the leading α_s contribution needs to be taken into account. This is not the case if we aim to reach a 10% accuracy in the quarkonium spin splittings. Since these quantities are an order $O(v^4)$ effect, a 10% accuracy is achievable only if $O(v^6)$ and $O(\alpha_s v^4)$ effects are calculated as well. Therefore, operators of the order $O(v^6)$ should be added to the effective Lagrangian [5]. The inclusion of such operators in Eq. (3) in terms of (two and three) field-strength insertions on a static Wilson loop, is only a technical problem, but has not be done so far. The main reason is that *non-potential contributions* are expected to become even more important. Ultrasoft gluonic degrees of freedom, characterized by a time scale $1/mv^2$, exist. In perturbative QCD, they are responsible, for instance, for the Lamb-shift (which is an $\alpha_s v^4$ effect). In the language of the Wilson loop, this means that the interaction U (see Sect. 3) can be affected by non-potential contributions which have to be subtracted from V in order to define properly a heavy-quark potential. Since α_s is not a small parameter at the ultrasoft scale, these non-potential terms could be an effect of the order $O(v^4)$. Therefore, precision studies of quarkonium have to take into account it. An approach was recently proposed in Refs. 12 and 13. The ultrasoft degrees of freedom are integrated out directly from the NRQCD Lagrangian giving rise to another effective theory, called potential NRQCD, where all energy scales of the bound state are disentangled explicitly. The advantages are enormous, since in the new theory, potential and non-potential contributions are clearly separated. The novel feature is that the matching this time takes place in a energy region dominated by the non-perturbative physics.

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KVARKONIUM U FORMALIZMU WILSONOVE PETLJE: NOVI IZGLEDI

Daje se pregled današnjeg znanja o međudjelovanju teških kvarkova. Efektivna se nerelativistička QCD teorija i međudjelovanje izražavaju pomoću kromoelektričnih i kromomagnetskih umetaka Wilsonove petlje.