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HARD EXCLUSIVE ELECTROPRODUCTION OF TWO PIONS AND THEIR RESONANCES[1]

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We study the hard exclusive production of two pions in the virtual photon fragmentation region with various invariant masses including the resonance region. The amplitude is expressed in terms of two-pion light-cone distribution amplitudes (2πDA's). We derive dispersion relations for these amplitudes, which allow to fix them completely in terms of $\pi\pi$-scattering phases and a few low-energy subtraction constants determined by the effective chiral Lagrangian. Quantitative estimates of the resonance as well $\pi\pi$ background DA's at low normalization point are made. We also prove a new soft-pion theorem relating two-pion DA's to the one-pion DA's. Crossing relations between 2πDA's and parton distributions in a pion are discussed. We demontrate that by studying the shape of the $\pi\pi$ mass spectra (not absolute cross section!) in a diffractive electroproduction, one can extract the deviation of the meson (π, ρ, etc.) wave functions from their asymptotic form $6z(1-z)$ and hence, to get important information about the structure of mesons. We suggest an (alternative to Söding's) parametrization of $\pi\pi$ spectra which is suitable for large photon virtuality.

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1. Introduction

An outline of the approach to two-pion hard electroproduction developed in Ref. 1 is given. We summarize the main results; details can be found in Ref. 1.

Hard exclusive electroproduction of mesons is a new kind of hard processes

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calculable in QCD. In Refs. 2 and 3, it was shown that a number of exclusive processes of the type:

$$\gamma_L^*(q) + T(p) \to F(q') + T'(p') \,, \tag{1}$$

at large Q^2 with $t = (p - p')^2$, $x_{Bj} = Q^2/2(pq)$ and $q'^2 = M_F^2$ fixed, are amenable to QCD description. The factorization theorem [2] asserts that the amplitude of the process (1) has the following form:

$$\sum_{i,j} \int_0^1 \mathrm{d}z \int \mathrm{d}x_1\, f_{i/T}^{T'}(x_1, x_1 - x_{Bj}; t, \mu)\, H_{ij}(Q^2 x_1/x_{Bj}, Q^2, z, \mu)\, \Phi_j^F(z, \mu)$$
$$+\text{power-suppressed corrections,} \tag{2}$$

where $f_{i/T}^{T'}$ is $T \to T'$ skewed parton distribution (for review and references see Ref. 4), $\Phi_j^F(z, \mu)$ is the distribution amplitude of the hadronic state F (not necessarily one particle state), and H_{ij} is a hard part computable in pQCD as series in $\alpha_s(Q^2)$. Here we shall study general properties of the distribution amplitudes $\Phi_j^F(z, \mu)$ when the final hadronic state F is a two-pion state ($F = \pi\pi$).

The two-pion light-cone distribution amplitudes (2πDA's) were introduced recently in Ref. 5 in the context of QCD description of the process $\gamma^*\gamma \to 2\pi$. In Refs. 6 and 1, it was suggested that the usages of 2πDA's to describe a hard electroproduction of two pions leads to universal picture of resonant and non-resonant two-pion production.

2. *Properties of 2πDA's*

Below we list main properties of 2πDA's and shortly discuss their applications to hard diffractive production of two pions off a nucleon.

• **Definition**:

$$\Phi^{ab}(z, \zeta, m_{\pi\pi}^2) = \frac{1}{4\pi} \int \mathrm{d}x^- e^{-\mathrm{i}\frac{1}{2} z P^+ x^-}\, {}_{out}\langle \pi^a(p_1)\, \pi^b(p_2)\,|\, \bar\psi(x)\, \hat n\, T\, \psi(0)\, |0\rangle \Big|_{x^+ = x_\perp = 0}, \tag{3}$$

Here, n is a light–like vector ($n^2 = 0$), which we take as $n_\mu = (1, 0, 0, 1)$. For any vector, V, the "plus" light–cone coordinate is defined as $V^+ \equiv (n \cdot V) = V^0 + V^3$; the "minus" component as $V^- = V^0 - V^3$. The outgoing pions have momenta p_1, p_2, and $P \equiv p_1 + p_2$ is the total momentum of the final state. Finally, T is a flavour matrix ($T = 1$ for the isosinglet, $T = \tau^3/2$ for the isovector 2πDA). The generalized distribution amplitudes, Eq. (3), depend on the following kinematical variables: the quark momentum fraction with respect to the total momentum of the two–pion state, z; the variable $\zeta \equiv p_1^+/P^+$ characterizing the distribution of longitudinal momentum between two pions, and the invariant mass, $m_{\pi\pi}^2 = P^2$.

• **The isospin decomposition**:

$$\Phi^{ab} = \delta^{ab}\mathrm{Tr}(T)\Phi^{I=0} + \frac{1}{2}\mathrm{Tr}([\tau^a, \tau^b]T)\Phi^{I=1}. \tag{4}$$

• **Symmetries and normalization**: From the C-parity, one can easily derive the following symmetry properties of the isoscalar ($I = 0$) and isovector ($I = 1$) parts of 2πDA's Eq. (3):

$$\begin{aligned} \Phi^{I=0}(z,\zeta,m_{\pi\pi}^2) &= -\Phi^{I=0}(1-z,\zeta,m_{\pi\pi}^2) = \Phi^{I=0}(z,1-\zeta,m_{\pi\pi}^2), \\ \Phi^{I=1}(z,\zeta,m_{\pi\pi}^2) &= \Phi^{I=1}(1-z,\zeta,m_{\pi\pi}^2) = -\Phi^{I=1}(z,1-\zeta,m_{\pi\pi}^2)\ . \end{aligned} \tag{5}$$

The isospin one parts of 2πDA's Eq. (3) is normalized as follows:

$$\int_0^1 \mathrm{d}z \Phi^{I=1}(z,\zeta,m_{\pi\pi}^2) = (2\zeta - 1)F_\pi^{\mathrm{e.m.}}(m_{\pi\pi}^2)\ , \tag{6}$$

where $F_\pi^{\mathrm{e.m.}}(m_{\pi\pi}^2)$ is the pion e.m. form factor in the time-like region ($F_\pi^{\mathrm{e.m.}}(0) = 1$). For the isoscalar 2πDA $\Phi^{I=0}$, we have the following normalization condition:

$$\int_0^1 \mathrm{d}z(2z-1)\Phi^{I=0}(z,\zeta,m_{\pi\pi}^2) = -2\ M_2^\pi\zeta(1-\zeta)F_\pi^{\mathrm{EMT}}(m_{\pi\pi}^2)\ , \tag{7}$$

where M_2^π is a momentum fraction carried by quarks in a pion, $F_\pi^{\mathrm{EMT}}(m_{\pi\pi}^2)$ is a pion form factor of quark part of energy momentum tensor normalized by $F_\pi^{\mathrm{EMT}}(0) = 1$. In Ref. 1, this form factor was estimated in the instanton model of QCD vacuum at low two-pion invariant mass, with the result:

$$F_\pi^{\mathrm{EMT}}(m_{\pi\pi}^2) = 1 + \frac{N_c m_{\pi\pi}^2}{48\pi^2 f_\pi^2} + \dots\ ,$$

where $f_\pi = 93$ MeV is a pion decay constant.

• **Double decomposition in conformal and partial waves** : Let us decompose 2πDA's in eigenfunctions of the ERBL [8] evolution equation (Gegenbauer polynomials $C_n^{3/2}(2z-1)$) and in partial waves of produced pions (Gegenbauer polynomials $C_l^{1/2}(2\zeta - 1)$). Generically, the decomposition (for both isoscalar and isovector DA's) has the form:

$$\Phi(z,\zeta,m_{\pi\pi}^2) = 6z(1-z)\sum_{n=0}^{\infty}\sum_{l=0}^{n+1} B_{nl}(m_{\pi\pi}^2)C_n^{3/2}(2z-1)C_l^{1/2}(2\zeta-1). \tag{8}$$

Using symmetry properties Eq. (5), we see that the index n goes over even (odd) and index l goes over odd (even) numbers for isovector (isoscalar) 2πDA's. Normalization conditions Eq. (6) correspond to $B_{01}^{I=1}(m_{\pi\pi}^2) = F_\pi^{\rm e.m.}(m_{\pi\pi}^2)$.

• Relations of 2πDA's to quark distribution in a pion : By crossing symmetry, the 2πDA's are related to the so-called skewed parton distributions (see Ref. 4). The latter are defined as:

$$\int \frac{{\rm d}\lambda}{2\pi} e^{{\rm i}\lambda\tau} \langle \pi^a(p')| T\{\bar\psi_{f'}(-\lambda n/2)\hat n \psi_f(\lambda n/2)\}|\pi^b(p)\rangle =$$
$$\delta^{ab}\delta_{ff'} H^{I=0}(\tau,\xi,t) + {\rm i}\varepsilon^{abc}\tau^c_{ff'} H^{I=1}(\tau,\xi,t)\ , \tag{9}$$

where ξ is a skewness parameter defined as: $\xi = -(n\cdot(p'-p))/(n\cdot(p'+p))$, $t=(p'-p)^2$ and light-cone vector n^μ normalized by $(n\cdot(p'+p))=2$. By crossing symmetry, we can easily express the moments of skewed distributions to coefficients B_{nl} in the expansion (8) (for detailed discussion see Ref. 15):

$$\int_{-1}^{1} {\rm d}\tau \tau^{N-1} H^I(\tau,\xi,t) = \sum_{n=0}^{N-1}\sum_{l=0}^{n+1} B^I_{nl}(t)\xi^N C_l^{1/2}\left(\frac{1}{\xi}\right) \int_{-1}^{1} {\rm d}x \frac{3}{4}[1-x^2]\ x^{N-1} C_n^{3/2}(x)\,. \tag{10}$$

If we take the forward limit in this formula, we obtain the relations between moments of quark distributions in a pion and coefficients B_{nl}:

$$M_N^{(\pi)} \equiv \int_0^1 {\rm d}x x^{N-1}(q_\pi(x)-\bar q_\pi(x)) = B^{I=1}_{N-1,N}(0) A_N \quad \text{for odd } N,$$
$$M_N^{(\pi)} \equiv 2\int_0^1 {\rm d}x x^{N-1}(q_\pi(x)+\bar q_\pi(x)) = B^{I=0}_{N-1,N}(0) A_N \quad \text{for even } N, \tag{11}$$

where A_N are numerical coefficients (e.g., $A_1=1, A_2=9/5, A_3=6/7, A_4=5/3$, etc.) and $q_\pi(x) = u^{\pi^+}(x)$. For example, from Eq. (11), we obtain that $B_{01}^{I=1}(0) = M_1^{(\pi)} = 1$ what corresponds to normalization condition (6). Also, it is easy to see that $B_{12}^{I=0}(0) = 5/9 M_2^{(\pi)}$ corresponds to normalization (7)[2]

• Evolution : The Gegenbauer polynomials $C_n^{3/2}(2z-1)$ are eigenfunctions of the ERBL [8] evolution equation and hence the coefficients B_{nl} are renormalized multiplicatively (for even n and odd l):

$$B_{nl}(m_{\pi\pi}^2;\mu) = B_{nl}(m_{\pi\pi}^2;\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/(2\beta_0)}\ , \tag{12}$$

[2]To see this, one has to use additionally soft-pion theorem (16) for isoscalar 2πDA.

where $\beta_0 = 11 - 2/3N_f$ and the one-loop anomalous dimensions are [9]:

$$\gamma_n \quad = \quad \frac{8}{3}\Big(1 - \frac{2}{(1+n)(2+n)} + 4\sum_{k=2}^{n+1}\frac{1}{k}\Big) \ .$$

From the decomposition Eqs. (8) and (12), we can make a simple prediction for the ratio of the hard $P-$ and $F-$ wave-production amplitudes of pions in the reaction $\gamma^* p \to 2\pi p$ at asymptotically large virtuality of the incident photon $Q^2 \to \infty$ and fixed $m_{\pi\pi}^2$:

$$\frac{F\text{−wave amplitude}}{P\text{−wave amplitude}} \sim \frac{1}{\log(Q^2)^{50/(99-6N_f)}} \ , \tag{13}$$

or generically for the $2k+1$ wave:

$$\frac{(2k+1)\text{−wave amplitude}}{P\text{−wave amplitude}} \sim \frac{1}{\log(Q^2)^{\gamma_{2k}/(2\beta_0)}} \ . \tag{14}$$

• **Soft pion theorems for two-pion distribution amplitudes** relate 2πDA's to distribution amplitude of one pion:

$$\Phi^{I=1}(z,\zeta=1,m_{\pi\pi}^2=0) = -\Phi^{I=1}(z,\zeta=0,m_{\pi\pi}^2=0) = \varphi_\pi(z)\,, \tag{15}$$

where $\varphi_\pi(z)$ is a one-pion DA. The analogous theorem for an isoscalar part of the 2πDA's has the form:

$$\Phi^{I=0}(z,\zeta=1,m_{\pi\pi}^2=0) = \Phi^{I=0}(z,\zeta=0,m_{\pi\pi}^2=0) = 0 \ . \tag{16}$$

The soft-pion theorem Eq. (15) allows to relate the coefficients $B_{nl}^{I=1}(m_{\pi\pi}^2)$ (see Eq. (8)) and the coefficients of expansion of the pion DA in Gegenbauer polynomials

$$\varphi_\pi(z) = 6z(1-z)\Bigg(1 + \sum_{n=\text{even}} a_n^{(\pi)} C_n^{3/2}(2z-1)\Bigg). \tag{17}$$

The relation has the form:

$$a_n^{(\pi)} = \sum_{l=1}^{n+1} B_{nl}^{I=1}(0) \ . \tag{18}$$

• **Dispersion relations and their solution for 2πDA's**: The 2π DA's are generically complex functions due to the strong interaction of the produced pions. Above the two-pion threshold $m_{\pi\pi}^2 = 4m_\pi^2$, the 2πDA's develop the imaginary part corresponding to the contribution of on-shell intermediate states (2π, 4π, etc.). In the region $m_{\pi\pi}^2 < 16m_\pi^2$, the imaginary part is related to the pion-pion scattering amplitude by Watson theorem [10]. This relation can be written in the following form (see Ref. 1):

$$\mathrm{Im}\, B_{nl}^I(m_{\pi\pi}^2) = \sin[\delta_l^I(m_{\pi\pi}^2)] e^{\mathrm{i}\delta_l^I(m_{\pi\pi}^2)} B_{nl}^I(m_{\pi\pi}^2)^* = \tan[\delta_l^I(m_{\pi\pi}^2)] \mathrm{Re} B_{nl}^I(m_{\pi\pi}^2)\ . \quad (19)$$

Using Eq. (19) one can write an N-subtracted dispersion relation for the $B_{nl}^I(m_{\pi\pi}^2)$

$$B_{nl}^I(m_{\pi\pi}^2) = \sum_{k=0}^{N-1} \frac{m_{\pi\pi}^{2k}}{k!} \frac{d^k}{dm_{\pi\pi}^{2k}} B_{nl}^I(0) + \frac{m_{\pi\pi}^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{\tan\delta_l^I(s)\, \mathrm{Re} B_{nl}^I(s)}{s^N(s - m_{\pi\pi}^2 - \mathrm{i}0)}\ . \quad (20)$$

Solution of this type of dispersion relation was found long ago in Ref. 11; the solution has the exponential form:

$$B_{nl}^I(m_{\pi\pi}^2) \quad (21)$$
$$= B_{nl}^I(0) \exp\left\{ \sum_{k=1}^{N-1} \frac{m_{\pi\pi}^{2k}}{k!} \frac{\mathrm{d}^k}{dm_{\pi\pi}^{2k}} \log\, B_{nl}^I(0) + \frac{m_{\pi\pi}^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{\delta_l^I(s)}{s^N(s - m_{\pi\pi}^2 - \mathrm{i}0)} \right\}\ .$$

A great advantage of the solution Eq. (22) is that it gives the 2πDA's in a wide range of energies in terms of known $\pi\pi$ phase shifts and a few subtraction constants (usually two is sufficient). The key observation is that these constants (non-perturbative input) are related to the *low-energy* behaviour of the 2πDA's at $m_{\pi\pi}^2 \to 0$. In the low-energy region, they can be computed using the effective chiral Lagrangian.

• **Amplitudes of $\pi\pi$ resonances production**: We give here an example how ρ meson distribution amplitude can be expressed in terms of 2πDA. General formula for DA's of resonances with any spin can be found in Ref. 1. In Ref. 1, we obtained the following expression for the coefficients of the expansion of ρ meson DA in Gegenbauer polynomials

$$\varphi_\rho(z) = 6z(1-z)\left(1 + \sum_{n=\mathrm{even}} a_n^{(\rho)} C_n^{3/2}(2z-1)\right),$$

in terms of only subtraction constants entering Eq. (22)

$$a_n^{(\rho)} = B_{n1}^{I=1}(0) \exp\Big(\sum_{k=1}^{N-1} c_k^{(n1)} m_\rho^{2k}\Big)\ , \quad (22)$$

where the subtraction constants $c_k^{(nl)}$ can be expressed in terms of $B_{nl}^{I=1}(m_{\pi\pi}^2)$ at low energies:

$$c_k^{(nl)} = \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}m_{\pi\pi}^{2k}} [\log\, B_{nl}(m_{\pi\pi}^2) - \log\, B_{l-1\,l}(m_{\pi\pi}^2)] \Big|_{m_{\pi\pi}^2 = 0}\ . \quad (23)$$

The normalization constants f_ρ can be computed as:

$$f_\rho = \frac{\sqrt{2}\Gamma_\rho \mathrm{Im} F_\pi^{\mathrm{e.m.}}(m_\rho^2)}{g_{\rho\pi\pi}} \,. \tag{24}$$

Here, we quote the result for the chirally-even ρ meson DA extracted from 2πDA using Eqs. (22) and (24), and the values of low-energy subtraction constants calculated using effective chiral Lagrangian (see details and results of these calculations in Refs. 6 and 1):

$$\varphi_\rho(z) = 6z(1-z)\big[1 - 0.14\, C_2^{3/2}(2z-1) - 0.01\, C_4^{3/2}(2z-1) + \ldots\big] \,, \tag{25}$$

with normalization constant $f_\rho = 190$ MeV according to Eq. (24), in a good agreement with the experimental value $f_\rho = 195 \pm 7$ MeV [14]. The ρ-meson DA's were the subject of the QCD sum-rule calculations [12,13], our result Eq. (25) is in a qualitative disagreement with the results of QCD sum-rule calculations, the sign of a_2 obtained here is opposite to the QCD sum-rule predictions $a_2^{(\rho)} = 0.18 \pm 0.1$ [12] and $a_2^{(\rho)} = 0.08 \pm 0.02$, $a_4^{(\rho)} = -0.08 \pm 0.03$ [13] (these results refer to normalization point $\mu = 1$ GeV). Let us note that actually our sign of $a_2^{(\rho)}$ follows from: 1) soft-pion theorem (Eq. (15)) 2) the relation of $B_{N-1,N}$ to moments of quark distribution in a pion (Eq. (11)) 3) relation $a_2^{(\rho)} = B_{21}^{I=1}(0)\exp(c_1^{(21)} m_\rho^2)$ (see Eq. (22)) and 4) the fact that $a_2^{(\pi)} \approx 0$. Combining all this, we can derive relations between $a_2^{(\rho)}$, $a_2^{(\pi)}$ and the third moment of quark distribution in a pion:

$$\begin{aligned} a_2^{(\rho)} &= B_{21}^{I=1}(0)\exp(c_1^{(21)} m_\rho^2) \\ &= (a_2^{(\pi)} - B_{23}^{I=1}(0))\exp(c_1^{(21)} m_\rho^2) = (a_2^{(\pi)} - \tfrac{7}{6} M_3^{(\pi)})\exp(c_1^{(21)} m_\rho^2) \,. \end{aligned} \tag{26}$$

Now, if we assume that $a_2^{(\pi)} < \frac{7}{6} M_3^{(\pi)}$, what is satisfied in the model of instanton vacuum (and seems phenomenologically), we get negative sign of $a_2^{(\rho)}$.

• **QCD parametrization of two pion spectrum in diffractive two-pion production**: The dependence of the two-pion hard production amplitude on the $m_{\pi\pi}$ factorizes into the factor:

$$A \sim \int_0^1 \frac{dz}{z(1-z)}\, \Phi^{I=1}(z, \zeta, m_{\pi\pi}^2; \bar{Q}^2) \,, \tag{27}$$

for the two pions in the isovector state, and

$$A \sim \int_0^1 dz \frac{(2z-1)}{z(1-z)}\, \Phi^{I=0}(z, \zeta, m_{\pi\pi}^2; \bar{Q}^2) \,, \tag{28}$$

for the pions in the isoscalar state (e.g., for the $\pi^0\pi^0$ production). In the above equations, we showed also the dependence of the 2πDA's on the scale of the process $\bar{Q}^2$, which is governed by the ERBL evolution equation [8].

For the hard exclusive reactions off nucleon at small x_{Bj} (see, e.g., recent measurements [7]), the production of two pions in the isoscalar channel is strongly suppressed relative to the isovector channel because the former is mediated by C-parity odd exchange. At asymptotically large Q^2, one can use the asymptotic form of isovector 2πDA:

$$\lim_{\bar{Q}^2\to\infty} \Phi^{I=1}(z,\zeta,m_{\pi\pi}^2;\bar{Q}^2) = 6F_\pi^{\rm e.m.}(m_{\pi\pi}^2) z(1-z)(2\zeta-1)\;, \tag{29}$$

where $F_\pi^{\rm e.m.}(m_{\pi\pi}^2)$ is the pion e.m. form factor in the time-like region. Therefore, the shape of $\pi^+\pi^-$ mass spectrum in the hard electroproduction process at small x_{Bj} and asymptotically large Q^2 should be determined completely by the pion e.m. form factor in the time-like region:

$$\lim_{Q^2\to\infty} A \propto e^{{\rm i}\delta_1^1(m_{\pi\pi}^2)} |F_\pi^{\rm e.m.}(m_{\pi\pi}^2)| (2\zeta-1)\;. \tag{30}$$

Deviation of the $\pi^+\pi^-$ mass spectrum from its asymptotic form Eq. (30) ("skewing") can be parametrized at small x_{Bj} and large $\bar{Q}^2$ in the form:

$$\begin{aligned}
A \sim\, & e^{{\rm i}\delta_1^1(m_{\pi\pi}^2)} |F_\pi^{\rm e.m.}(m_{\pi\pi}^2)| \left[1 + B_{21}(0;\mu_0)\exp\{c_1^{(21)} m_{\pi\pi}^2\}\left(\frac{\alpha_s(\bar{Q}^2)}{\alpha_s(\mu_0)}\right)^{50/(99-6N_f)}\right] \\
& \times(2\zeta-1) + e^{{\rm i}\delta_3^1(m_{\pi\pi}^2)} B_{23}(0;\mu_0)\exp\{b_{23}m_{\pi\pi}^2 + R_3^1(m_{\pi\pi}^2)\}\left(\frac{\alpha_s(\bar{Q}^2)}{\alpha_s(\mu_0)}\right)^{50/(99-6N_f)} \\
& \times C_3^{1/2}(2\zeta-1) + \text{higher powers of } 1/\log(\bar{Q}^2)\;,
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
& R_l^I(m_{\pi\pi}^2) \\
& = {\rm Re}\frac{m_{\pi\pi}^4}{\pi}\int_{4m_\pi^2}^\infty {\rm d}s \frac{\delta_l^I(s)}{s^2(s-m_{\pi\pi}^2-{\rm i}0)} \quad \text{and} \quad b_{nl} = \frac{\rm d}{{\rm d}m_{\pi\pi}^2}\log\, B_{nl}(m_{\pi\pi}^2)\bigg|_{m_{\pi\pi}^2=0}\;.
\end{aligned} \tag{32}$$

We see that the deviation of the $\pi\pi$ invariant mass spectrum from its asymptotic form Eq. (30) in this approximation is characterized by a few low-energy constants ($B_{21}(0)$, $B_{23}(0)$, $c_1^{(21)}$, b_{23}), while other quantities (the pion e.m. form factor and the $\pi\pi$ phase shifts) are known from low-energy experiments. In principle, using the parametrization (33) one can extract the values of these low-energy parameters from the shape of the $\pi\pi$ spectrum (not absolute cross section!) in diffractive production experiments. Knowing them, one can obtain the deviation of the π meson DA (see Eq. (18))

$$a_2^{(\pi)} = B_{21}(0) + B_{23}(0)\;,$$

and the ρ meson DA (see Eq. (22))

$$a_2^{(\rho)} = B_{21}(0)\exp(c_1^{(21)} m_\rho^2) \, ,$$

from their asymptotic form $6z(1-z)$, as well as the normalization constants for the DA of isovector resonances of spin three.

In analysis of experiments on two pion diffractive production off nucleon (see, e.g., Ref. 7), the non-resonant background is described by Söding parametrization [16], which takes into account rescattering of produced pions on final nucleon. Let us note, however, that in a case of hard ($Q^2 \to \infty$) diffractive production, the final state interaction of pions with residual nucleon is suppressed by powers of $1/Q^2$ relative to the leading twist amplitude. Here we proposed alternative leading-twist parametrization (33) describing the so-called "skewing" of two-pion spectrum.

3. Conclusion

We showed that the two-pion distribution amplitudes are the most general object to describe the hard electroproduction of two pions. Using Watson final-state-interaction theorem and the soft-pion theorems proven here, we can determine the 2πDA's in a wide region of pion invariant masses in term of the pion phase shifts and a few low-energy (subtraction) constants. The former are know from $\pi\pi$ scattering data (soft physics), the latter are non-perturbative input which we estimated here using the instanton model of the QCD vacuum. We have shown that these non-perturbative characteristic can, in principle, be extracted from the shape of $\pi\pi$ mass spectra in diffractive pion-production experiments. Then they can be used to determine the deviation of the π-meson DA (see Eq. (18)) and ρ-meson DA (see Eq. (22)) from their asymptotic form $6z(1-z)$, and hence to obtain non-perturbative information about structure of mesons. We demonstrated that DA's of resonances of any spin can be determined from 2π DA's.

We have derived the crossing relation which relate various distribution amplitudes to the quark distributions in a pion.

The soft-pion theorems considered here give us an example of how low-energy (chiral) physics can be studied in hard processes.

The methods developed here can be easily generalized for more complicated cases. An example are generalized $N \to \pi N$ skewed parton distributions [17,18] entering the QCD description of the "non-diagonal" deeply–virtual Compton scattering $\gamma_L^* + p \to \gamma + \pi N$.

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TVRDA EKSKLUZIVNA ELEKTRO-TVORBA DVAJU PIONA I NJIHOVE REZONANCIJE

Proučava se tvrda ekskluzivna tvorba para piona u području virtualne diobe fotona s raznim invarijantnim masama, uključujući rezonantno područje. Amplituda se izražava pomoću dvopionskih distribucijskih amplituda na svjetlosnom stošcu (2πDA). Izvode se disperzijske relacije za te amplitude koje dozvoljavaju njihovo potpuno određivanje preko faza $\pi\pi$ raspršenja i nekoliko nisko-energijskih stalnica koje određuje efektivna kiralna Lagrangeova funkcija. Načinjene su kvantitativne ocjene rezonancija i pozadinskih $\pi\pi$ DA u točki niske normalizacije. Dokazuje se nov teorem za meke pione koji povezuje dvopionske i jednopionske DA. Raspravljaju se unakrsne relacije između 2πDA i partonskih raspodjela u pionu. Pokazuje se kako se proučavanjem *oblika* masenih spektara $\pi\pi$ (ne apsolutnih udarnih presjeka!) u difraktivnoj elektro-tvorbi mogu izvesti odstupanja valnih funkcija mezona (π, ρ, etc.) od njihovog asimptotskog oblika $6z(1-z)$ i tako dobiti važne podatke o strukturi mezona. Također se sugerira druga parametrizacija $\pi\pi$ spektara, različita od Södingove, koja je pogodna za visoko virtualne fotone.